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THE USE OF MULTI-STATE REAL OPTIONS IN PROJECT EVALUATION AND PROJECT MANAGEMENT[1]

Introduction

Traditional project evaluation is based on discounted cash flow method (DCF). Main measure of effectiveness is the Net Present Value (NPV). When this value is positive, the project is approved, when negative rejected. But this approach sometimes leads to the abandonment of profitable projects, as shown in many works. The reason for this is that the DCF method does not take into account the role of managerial flexibility. Real Options Valuation (ROV) method takes into account future situations in the valuation, assuming that the project is properly managed. The Project Manager shall have the right to take action as appropriate. This situation is called a real option. Using the ROV, we can provide a quantitative measurement of the situation.

The term real option was proposed by S. Myers (1974). He noted some similarities between financial options and opportunities that arise in the course of project management. The option can be defined as some right, but not the obligation, which means that the holder of that right can determine when to exercise his right, depending on the current market situation. In the case of financial options is the right to a certain amount, in the case of a real option right to a certain action. This approach was then developed by A.K. Dixit and R.S. Pindyck (1994), later covered by L. Trigeorgis (1993). The most important element in the Real Options Analysis (ROA) is the valuation (ROV –Real Option Valuation). The first were used methods known from financial market, such as the Black-Scholes Model (Black, Scholes 1973) or Cox-Ross-Rubinstein Model (CRR) (Cox, Ross, Rubinstein 1979). It is also used an approach based on Monte Carlo simulations (Boyle 1977). CRR model is based on the binomial tree.

This approach was also adopted in the Guthrie (2009) work and in this study. Despite the simplicity of the method, however, is computationally difficult method.

The methodology of real options can be used not only to the valuation of projects, but also for risk management. Kumar (2002) proposed the creation of the portfolio of real options to manage the risks of the project. At the beginning of Project Manager (PM) may be able to delay the start of the project. During his life-time PM may have the option free choice of the beginning the next phase, but also extend the scope of the project or its abandonment, depending on market conditions. After successfully completion of the project, the PM may be able to start another.

The paper presents a method for decision support in project management, an approach based on Real Option Methodology. The method can be used in multi-stage projects where there is free choice of the start of the next phases of the project, which creates a Real Option situation. The starting the next phase is chosen according to the market situation characterized by the two stochastically independent variable factors. The method will allow not only better than does the DCF to assess the value of the project, but also to calculate the best moments of the start of the next phases.

1. Problem formulation

We will consider multi-stage project, which is for a longer time than it takes to perform all of its phases. It follows from this possibility (option) free choice of the start of the next phase. This freedom has its limits, of course. This issue is discussed in the literature by Guthrie (2009, p. 168). Choosing the start of the next phase depends on market factors that affect the value of the project. Guthrie is considering one such factor. This article extends these considerations, the binding results of the project with two market factors.

Consider the simple situation where we have the right to begin the phase of the project or to wait. Decision tree for this situation is shown in Figure 1 Decision maker may start phase of the project (decision A), then with probability q the project moves from the current state b to another state a and which is a desirable state of the project (the project is ready). But with probability $1-q$ phase of the project could fail, and therefore returns to the b. Decision maker has the option to wait for positive changes (decision W), but then the project will remain in state b.
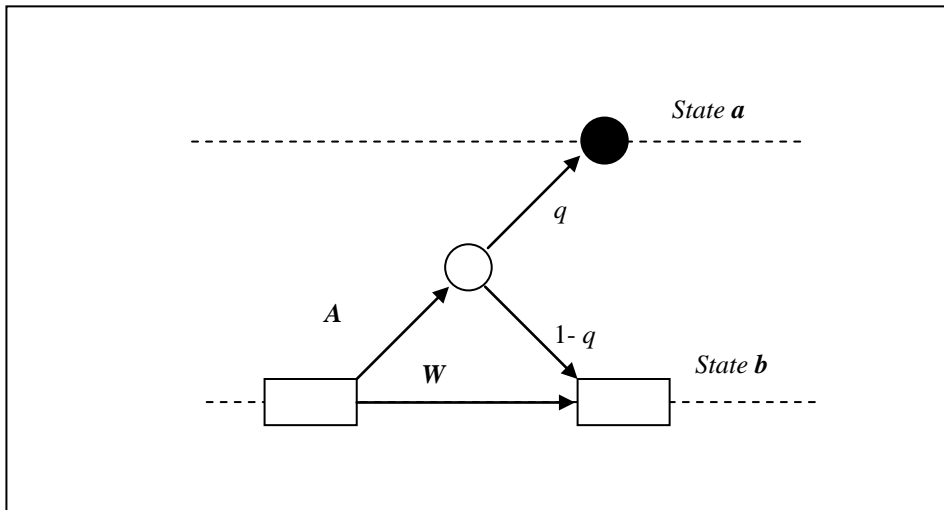
Figure 1. Possible decisions

This simple situation can occur in a real project multiple times. Consider a project consisting of two phases, each of which occupies a period of time, and the execution of the project we have available four periods. Upon successful completion of phase one, we can start the next. This situation creates options for free choice of the start of the project and the start of the second phase. The decision tree of this situation is presented in Figure 2. As previously each phase can result in a success or failed (with fixed probabilities).
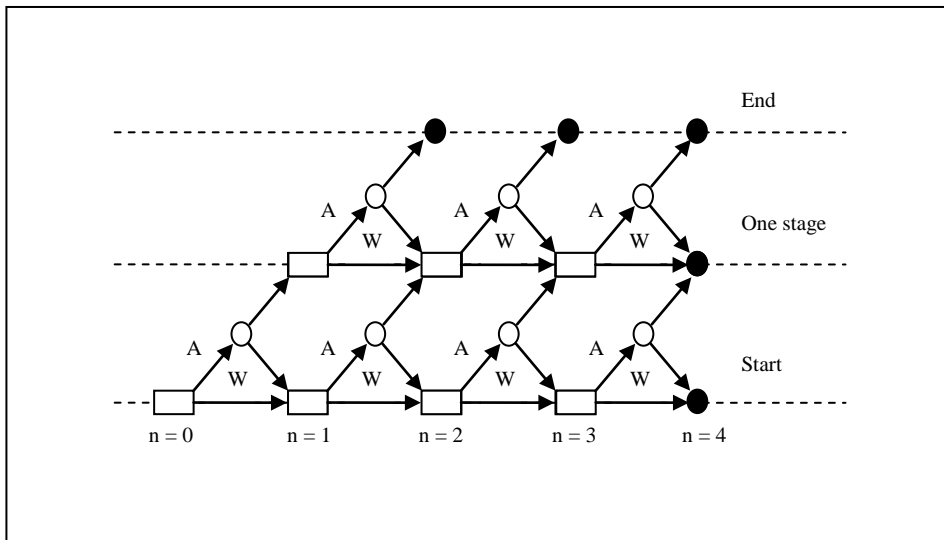

Figure 2. The decision tree

The results of the project, as its value depends on certain market factors, which may be commodity prices or exchange rates. Observing the changes, the decision maker decides on the validity take up the next phase of the project. These factors vary stochastically according to a certain random process. For these changes to take into account stochastic process is a binomial tree covered including future changes to this factor. Figure 3 shows such a tree for one variable. Each node of the tree, is related to the value of the project.
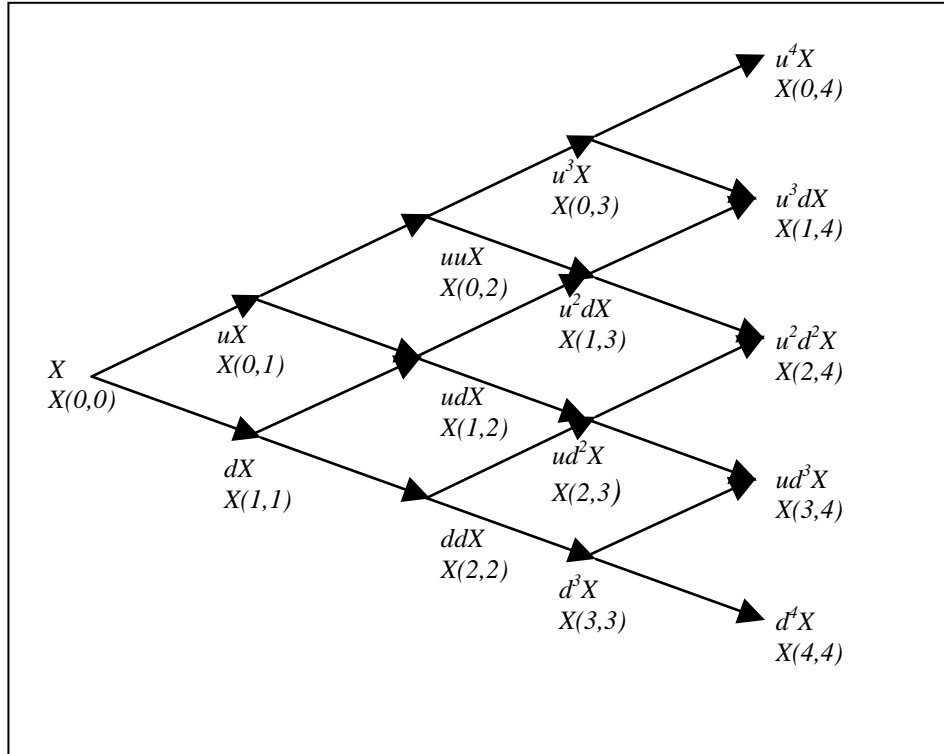

Figure 3. The binomial tree

The project depends not only on future cash flows, as is the traditional method of DCF, but also on the variable factor called state variable. The binomial tree, we assume that the variable state in one period may be increased u-time and fall d-times. This assumption leads to the tree of possible state variable values consists of nodes marked with indices (i, n) where i – means number of falls and n – means period number.

With each node is connected state variable and cash flow. We denote it as (in each node (i,n)):

- $X^k(i, n)$ – state variable in period n
- $Y_m(i, j, n)$ – cash flow in period n

We assume that we have M periods, the present value of state variable denoted by $X^k(0,0)$ and u and d are the values of growth and decline. Value of u can be obtained from historical data by the calibration procedure (Guthrie 2009, p. 263).

2. Proposed method

We will propose a procedure for the valuation of option situation that may arise in investment projects. The problem is multi-stage project. Due to the fact that until the project is more than that implementation takes time, appear in the additional options (real options), choose the appropriate time to begin the next steps. Decisions as to their start will be made based on observations of two market factors. The procedure will evaluate the validity of the start of the project under consideration and determine the best, in terms of efficiency, the moments of start of the subsequent phases. The proposed procedure for the valuation of an option position consists of the following steps:

Step 1 – Build a decision tree (D-Tree)

In this step, we recognize the possible states of the project. They may be different phases or specific stages. We recognize the possible decisions that we take in consideration the state. To take such a decision leads to a transition from one project to another state. Identify all possible transitions. The result of this step is to create a D-tree. An example of such a tree for a two-stage design problem under consideration is shown in Figure 2.

Step 2 – Build lattice of state variables (X-Tree)

In this step you need to identify quantifiable economic magnitudes, from which it may depend on the result of the project (state variables). The method the currently proposed will does not include the correlation between these variables, assuming it does not exist.

The tree starts from a known present value of state variables. Based on the history of changes of this magnitude can be determined by the values u and d is done in the calibration process (Guthrie 2009, p. 324). Formed is a tree of possible changes in state variable, as a possible scenarios of the situation.

Calibration is an appropriate choice of the number of steps and the choice of parameters, d, u, so as to best meet the future value of the variable state.

We assume the arithmetic Brownian Motion (BM) defined differential equation (Weron, Weron 1998, p. 166):

$$dX^k_t = \mu^k \cdot dt + \sigma^k \cdot dW_t \tag{1}$$

where:

k - state variable index $k = \{1, 2\}$
W_t –Wiener process
X^k_t – state variable
μ^k – drift of the process
σ^k – process volatility

Drift of the BM process, we can estimate the arithmetic average (v^k) of historical data:

$$\hat{\mu}^k = \frac{v^k}{\Delta t_d} \tag{2}$$

Process volatility is assessed on the basis of the standard deviation (ϕ^k) of historical data:

$$\hat{\sigma}^k = \frac{\phi^k}{\sqrt{\Delta t_d}} \tag{3}$$

where:

Δt_d – part of the year, which corresponds to the owned historical data.

Assuming the arithmetic Brownian Motion, we can compute grid nodes binomial tree, the following formula (Guthrie 2009, p. 327):

$$X^k(i,n) = X^k_0 + (n - 2i) \cdot \hat{\sigma}^k \sqrt{\Delta t} \tag{4}$$

where:

$\hat{\sigma}^k$ – volatility estimator
i – number of falls
n – period number

hence derive formulas for the parameters u and d

$$u^k = \hat{\sigma}^k \sqrt{\Delta t} \tag{5}$$

$$d^k = -\hat{\sigma}^k \sqrt{\Delta t} \tag{6}$$

The condition that $d^k = -u^k$, that yields a binomial tree.

Figure 4 shows the adjusted binomial tree (in order not to lose the legibility is shown only to the same nodes) changes in the Euro exchange rate for PLN in January 2011 (solid line). The parameters were estimated based on the Euro to PLN exchange rate changes in 2010.
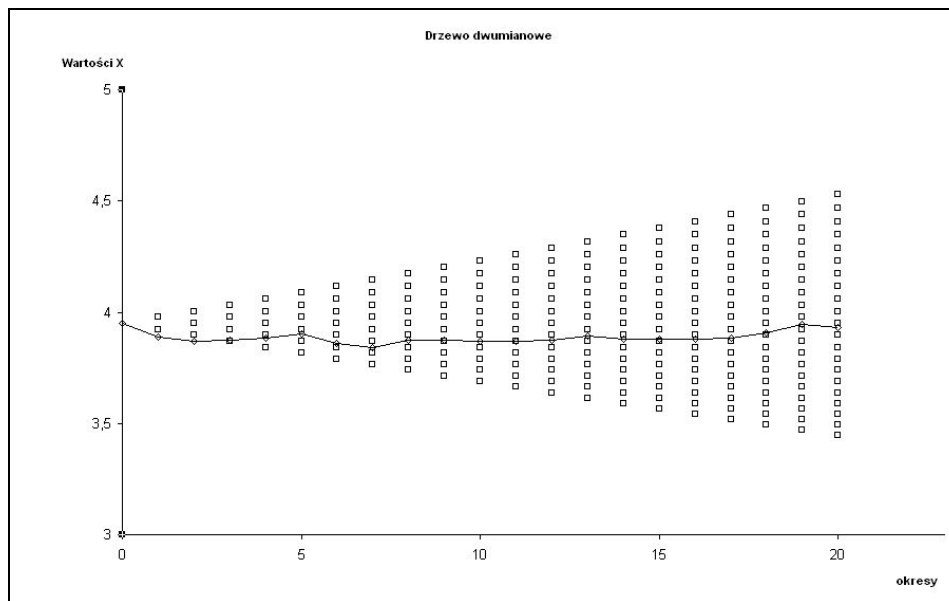

Figure 4. The adjusted binomial tree

Step 3 – Build a tree of the project values (V-Tree)
First, we denote the present value of the project, which is dependent on both state variables, as:

$V_m(i, j, n)$ – discounted present value of project in period n

where:

i – number of falls of first state variable
j – number of falls of second state variable
n – period number
m – project state index

Taking account in the consideration of two state variables, it makes the V-Tree grow in two dimensions. An example of such a tree after four periods are shown in Figure 5 For clarity, not shown on it all the links between nodes.

Calculation of the V-Tree starts from the project results in a final. We assume that the final value of the project is a function of state variables:

$$V_m(i, j, N) = F_m(X^1(i, N), X^2(j, N)) \tag{7}$$

On their basis, moving from the final set the value of the project in other previous nodes. Trees are constructed for each state project. Calculation of value is done by backward induction. Knowing the value of the project after its completion (which is usually equal to the state variable or using the right formula for this variable is calculated), we calculate the value of the project in the preceding nodes.

$$V_m(i, j, n) = max\left\{ Q_m^W(i, j, n),\ Q_m^A(i, j, n) \right\} \tag{8}$$

where:

$m = a, b, c$ – project state index

$n = N - 1, ..., 0$

$i = 0, ..., n$

$j = 0, ..., n$

$Y_m(i, j, n) = - I_m$

$$
\begin{aligned}
Q_m^W(i, j, n) = (\quad & \pi_u^1 \pi_u^2 \cdot V_m(i, j, n+1) \\
& + \pi_u^1 \pi_d^2 \cdot V_m(i, j+1, n+1) \\
& + \pi_d^1 \pi_u^2 \cdot V_m(i+1, j, n+1) \\
& + \pi_d^1 \pi_d^2 \cdot V_m(i+1, j+1, n+1)\ ) e^{-r\Delta t}
\end{aligned}
$$

– is discounted present value when we decide to <u>W</u>ait

$$
\begin{aligned}
Q_m^A(i, j, n) = &\ Y_m(i, j, n) \\
+ e^{-r\Delta t} \cdot (\quad & \pi_u^1 \pi_u^2 \cdot (q V_{m-1}(i, j, n+1) + (1-q) V_m(i, j, n+1)) \\
& + \pi_u^1 \pi_d^2 \cdot (q V_{m-1}(i, j+1, n+1) + (1-q) V_m(i, j+1, n+1)) \\
& + \pi_d^1 \pi_u^2 \cdot (q V_{m-1}(i+1, j, n+1) + (1-q) V_m(i+1, j, n+1)) \\
& + \pi_d^1 \pi_d^2 \cdot (q V_{m-1}(i+1, j+1, n+1) + (1-q) V_m(i+1, j+1, n+1))\ )
\end{aligned}
$$

is the discounted present value when we decide to Attempt to the next phase phase.

Subsequent values are weighted by the probability of achieving those values, which are calculated from the formulas (Seydel 2009, p. 15):

$$\pi_u^k = \frac{e^{r\Delta t} - d^k}{u^k - d^k} \qquad (9)$$

for the growth of the k-th state variable,

$$\pi_d^k = \frac{u^k - e^{r\Delta t}}{u^k - d^k} \qquad (10)$$

for the fall the k-th state variable.

where:

r – risk free interest rate

Because we assume that the project is properly managed, will be selected favorable decision, hence the calculations we use the maximum value.
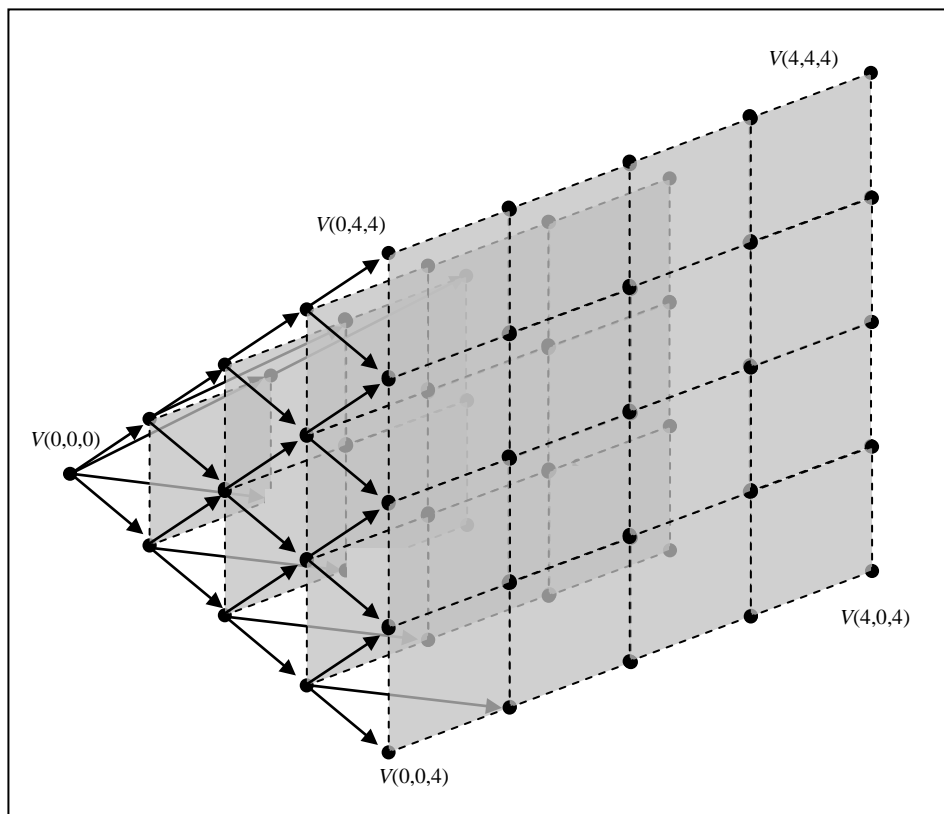

Figure 5. The V-Tree (Not all connections are shown)

When we deal with the situation of decision-making, the problem is a dynamic programming problem, because we have to choose the best decision.

Step 4 – Determine effective transitions (decisions)

Since we are considering a project in which, depending on market conditions, the decision maker may decide to move the project to another state, it is necessary to determine the moments of these transitions. In the case of one state variable, using the methods of dynamic programming is possible to determine the optimal decision. When we consider the two state variables, using multiple criteria dynamic programming methods will also be content with the efficient decisions, i,e, such for which there is no better decision.

$$\text{IF } Q_m^W(i,j,n) > Q_m^A(i,j,n) \quad \text{THEN } Wait$$
$$\text{ELSE } Attempt \tag{9}$$

In many cases it is sufficient to compare the value of Q for different decisions.

Conclusions

This paper presents the outline of the valuation method of development projects in which there are real option situations. The proposed method takes into account the dependence of the project as a result of two independent random economic factors, which are called state variables. Based on binomial trees, and uses multicriteria dynamic programming. Its use should allow not only to make the right decisions about the project, but also to support decision making during its implementation.

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