GOSPODARKA NARODOWA

11-12
(243-244)
Rok LXXX/XXI
listopad-grudzień
2011
s. 31-45

Dariusz LEŚKO*

A Model of Firm Growth

Introduction

Problems related to firm size and growth, especially its drivers and boundaries, have long been of interest to economists and other scholars working in the field of organisational theory. Yet, despite the existence of many profound works, there still is no common view on the mechanisms of the growth of the firm.

Both Polish and international statistics show that today's business environment is dominated by small enterprises. For instance, as outlined in Table 1, in Poland, 95.9 per cent of all firms employ fewer than 10 employees. At the same time, only 0.2 per cent of all EU firms are classified as large, while 91.8 per cent of all European firms are microenterprises [Eurostat, 2009]. Furthermore, this size distribution of firms seems to be consistent not only across countries, but also across years.

In this article, we aim to contribute to the debate on the observable firm size distribution and the lack of firm growth, firstly by reviewing the most seminal models of firm growth to date, and secondly by suggesting an alternative model of firm growth. We show that it may be a seemingly rational behaviour of a value-maximising agent not to let his firm grow in size. In addition, by providing a theoretical model of firm growth, we aim to stimulate discussion and further empirical research into this trend.

---

* The author is affiliated with the Cass Business School, City University London, and can be reached at dariusz.lesko@cemsmail.org. This paper was written when at the Department of Economics I of the Warsaw School of Economics (SGH). The author would like to thank Professors Urszula Grzelońska, Adam Noga and Stanisław Kasiewicz, as well as the Anonymous Reviewers and the Editors of this Journal for their helpful comments and suggestions regarding the earlier versions of the paper. The article was submitted in September 2011.

Table 1

Breakdown of the non-financial business economy by size-class, 2006 (% share of total)

People employed	Total SMEs 1 to 249	Micro 1 to 9	Small 10 to 49	Medium 50 to 249	Large 250 and over
Belgium	:	:	:	0.9	:
Austria	99.7	87.5	10.5	1.6	0.3
Bulgaria	99.7	89.5	8.5	1.7	0.3
Czech Republic	99.8	95.1	3.9	0.8	0.2
Cyprus	:	:	:	:	:
Denmark	99.7	86.8	11	1.9	0.3
Estonia	99.6	83.1	13.8	2.7	0.4
Finland	99.7	92.8	5.8	1.1	0.3
France	99.8	92.3	6.5	1	0.2
Germany	99.5	83.1	14.1	2.3	0.5
Greece	:	:	:	0.4	:
Hungary	:	:	:	0.8	:
Ireland	:	:	:	:	:
Italy	99.9	94.6	4.8	0.5	0.1
Latvia	99.7	83.3	13.7	2.6	0.3
Lithuania	99.7	88.4	9.3	2	0.3
Luxembourg	99.6	86.7	10.7	2.1	0.4
Malta	:	:	:	:	:
Netherlands	99.7	89.5	8.7	1.5	0.3
Norway	99.8	91.6	7.2	1	0.2
Poland	**99.8**	**95.9**	**2.9**	**1**	**0.2**
Portugal	99.9	94.6	4.7	0.7	0.1
Romania	99.6	88.1	9.4	2.1	0.4
Slovakia	98.8	72.7	21	5.1	1.2
Slovenia	99.7	92.8	5.7	1.3	0.3
Spain	99.9	92.2	6.8	0.8	0.1
Sweden	99.8	94.2	4.8	0.8	0.2
UK	99.6	87.5	10.5	1.7	0.4
EU-27	99.8	91.8	6.9	1.1	0.2

Notes: 2005 figures for EU-27, Bulgaria, Poland, the Netherlands and Slovakia; 2004 figures for Belgium.

Source: Eurostat [2009].

The theory of firm growth to date

Despite the fact that the modern firm emerged in its present form as early as in the second half of the nineteenth century [Chandler, 1977], mainstream economics has been paying relatively little attention to problems related to the growth of the firm. In its most basic form, neoclassical microeconomics

views the firm as a means for transforming inputs into outputs. Furthermore, under the base case conditions of perfect competition, there is an infinitely large number of firms operating in the economy and offering exactly the same products or services. As a result, each and single firm is reduced in its planning activity to deciding on the optimal level of output. Consequently, firms grow automatically to reach their optimal size levels.

A similar mechanism behind the growth of the firm has been proposed by Ronald Coase [1937], who was seeking to find the rationale behind the existence of the firm. From his seminal article it is possible to infer that firms grow in fact as long as it is cheaper to internalise the transaction costs associated with using the markets for the exchange of goods and services. This means that firms grow as long as their cost functions and the market mechanism allow them to.

Robert Gibrat and the law of proportionate effect

One of the first theories materially different from the neoclassical approach to the growth of the firm was *the law of proportionate effect* advocated by Robert Gibrat [1931]. According to data analysed by Robert Gibrat, the size distribution of firms in Germany from 1882 to 1907 was in principle highly skewed towards the small firm, with only a minor share of the largest firms in the total business enterprise population. At the same time, Gibrat notes that the probability and the way in which a small firm grows in time closely resembles both the probability and growth patterns shown by large companies; that is the ease with which a firm of 10 people grows to 11 people is equal to the ease with which a firm of 1,000 grows to 1,100 [Gibrat, 1931]. Therefore, in order to explain this observation, Robert Gibrat advocates the usage of *the law of proportionate effect*, which can be expressed by the following formula: $x_t = (1 + \varepsilon_t) \cdot x_{t-1}$. Where x_t denotes the current size of a firm, x_{t-1} the size that it had in the preceding period, and ε_t is a random variable, the same for all the companies operating within one industry, expressing the proportionate change in the firm's size from time $t - 1$ to time t [Sutton, 1997].

This law, taken literally, simply implies that the current size of a firm has no influence on its pace of growth. Thus, proportionate changes in the sizes of two firms operating within one industry are expected to be equal and independent of their initial size. Given that the changes to firm size over time take a multiplicative form in which continued and prolonged growth occurs only on a limited scale, the firm size distribution becomes skewed towards the small firm taking in effect the observed lognormal form [Kalecki, 1945].

Ijiri and Simon and the relationship between past and future growth

Gibrat's *law of proportionate effect* was further refined by Herbert Simon and Yuji Ijiri. In a series of papers, the two authors tested its applicability to the American economy over the period spanning from the 1950s to the 1970s

[e.g. Ijiri and Simon, 1971]. However, unlike Robert Gibrat, they state that the rate of growth of the firm, apart from being in its nature stochastic, is also correlated with the historical growth rates. In algebraic terms, the model of the firm growth proposed by the two authors can be summarised as follows: let the size of firm i at the end of period t, $t \in \mathbb{N}$, be given by: $S_{it} = r_{it} \cdot S_{i(t-1)}$; where r_{it} is the rate of growth of the firm consisting of two factors, i.e. its own growth factor p_{it} and an industry-wide growth factor $\bar{p}_t$; then, the rate of growth of the firm i can be decomposed into: $r_{it} = p_{it} \cdot \bar{p}_t$; hence the size of the firm i evolves according to the formula: $S_{it} = p_{it} \cdot \bar{p}_t \cdot S_{i(t-1)}$ [Ijiri and Simon, 1967]. Therefore, the growth of any particular firm differs from the industry-wide growth rate only inasmuch as its own unique growth factor allows it to. According to Herbert Simon and Yuji Ijiri, the firm growth factor depends on the past rates of growth of firm i and random factor ε_{it}, which is identically distributed for every firm in the same industry. Hence, the firm growth factor takes the form given by: $p_{it} = \varepsilon_{it} \cdot p^{\alpha}_{i(t-1)}$. Where power α is a constant representing the past behaviour of the firm growth factor p_{it}. And with $\alpha \in [0, 1)$, it is clearly visible that a firm with a history of low growth is expected to grow slower than the rest of the industry in the future. Furthermore, the growth rate of the firm is expected to converge to the industry average over the time following any event triggering an abnormal change in its size [Ijiri and Simon, 1967].

As a result of such a set-up, firms with more recent growth history grow faster than those that managed to grow in size at an earlier point in time. This observation is backed by the assumption stating that a growing firm obtains "an impetus for growth" via spotting and capitalising on an opportunity, like introduction of a new production technology, new marketing or management techniques. Having obtained such an innovation, the firm starts to outperform the growth rates of its industry peers, and what is more, the increased rate of growth becomes a self-propelling mechanism [Ijiri and Simon, 1967]. In addition, large firms are expected to grow proportionately more rapidly than their smaller counterparts, and all the firms should be able to grow only as much as the available opportunities allow them to. All this leads to the commonly observed firm size distributions. Nonetheless, even though the rate of growth does initially depend on some kind of a profitable opportunity being taken advantage of, as time passes the growth of the firm will have less and less in common with its efficiency but more and more with the stochastic growth process [Simon, 1991].

Boyan Jovanovic and the model of a learning organisation

Yet another model of stochastic growth of the firm was proposed by Boyan Jovanovic [1982]. In this particular model it is assumed that there exists a cohort of firms entering a small industry characterised by constant input costs and the number of firms large enough to make every firm a price-taker. Each and single firm, however, has a different cost function, which is not known to its

incumbents upfront. Instead, as the situation develops firms become aware of their relative cost efficiency.

Given this, the firm faces a profit maximisation problem according to the rules of neoclassical microeconomics. Yet, the assumptions of Jovanovic's function randomly allocate the varying cost efficiency across companies. This, in turn, positions some of them at a disadvantage at all levels of output. Furthermore, firms which have learnt that they are the cost-inefficient ones are not only unable to grow, but actually face the decision of whether to exit the industry altogether. As the less profitable firms exit, the average profitability of the industry increases.

Moreover, as, according to this particular model, firms within one industry can have different development prospects, size and age do matter when it comes to the determination of a firm's growth path. The younger, and by definition smaller, firms tend to grow faster than their older and larger counterparts as they learn about their efficiency. At the same time, the younger and smaller the firm is, the higher its growth variability is.

In conclusion, according to Boyan Jovanovic, the growth of the firms, even though it does resemble a stochastic process, is in fact a process of "noisy selection", whereby each firm has to learn about its distinctive efficiency level as it grows [Jovanovic, 1982]. Every firm, having acquired such knowledge, will either be able to grow according to its own pace determined by the unique level of efficiency it possesses, or will decide to withdraw from the market being unable to cope with the conditions it faces. At the same time, firm profitability is viewed as its main growth factor, and cost inefficiency as the main boundary to the growth of the Jovanovic firm.

Edith Penrose and managerial limitations to firm growth

In parallel to the more formalised approaches to firm growth rooted in Robert Gibrat's book, a more narrative stream of research into the nature of the growth of the firm has also evolved.

Basing on neoclassical microeconomics, Edith Penrose [1966] assumes that growth and the search for profit are one and the same phenomenon. Yet she rejects the neoclassical model of size adaptation and growth as merely a by-process of changes in the firm's size. Instead, she assumes that there is no optimal size of the firm, and that the size of the firm is nothing else but a by-product of organisational growth. Furthermore, every firm is defined by the productive opportunities, which the entrepreneur is capable of spotting and taking advantage of. It is also assumed that every such opportunity will be taken by the firm, and thus the firm will be able to grow via exploitation of the opportunities available to it [Penrose, 1966].

Moreover, unlike the model of firm growth advocated by Herbert Simon and Yuji Ijiri, but consistent with Robert Gibrat's views, the firm's past growth is believed to have no influence over its future growth rates whatsoever. This is based on the thesis that past growth has served its purpose of

capitalising on an opportunity and, therefore, any new growth is subject to the emergence of another expansion possibility [Penrose, 1955]. In addition, every attempt to capitalise on the available growth opportunities is limited by both the external environment of the firm and the resources, and managerial capabilities in particular, being at its disposal. Managerial limits become the prime consideration when it comes to the growth process of the firm as the capacity of the managerial staff determines its growth process twofold. Firstly, by planning how much the firm should grow, its management limits its growth. Secondly, the capacity of the management team determines the number of people who can be hired and successfully integrated within the firm. Thus, the managerial limits to firm growth prevent it from growing based on both the current human resources, as well as on the ability to cater for new joiners. Nonetheless, it is also the very same managerial resources of the firm that are viewed as the prime engine of its growth. According to Edith Penrose, after each expansion stage, the managerial staff will be concerned with absorbing the growth that the firm has undergone. Once, however, the past growth has been fully absorbed by the firm and its other resources have been accordingly adjusted, some of the managerial team can once again be released. As a result, the managers will once again become interested in seeking further expansion opportunities [Penrose, 1966].

Nelson and Winter and the evolutionary theory of firm growth

Edith Penrose's contribution was a direct result of her critique of the then-emerging biological theories of the firm [Penrose, 1952]. The biological theories of the firm, assuming that firms grow and develop just like living organisms, laid foundations for the evolutionary theory of the firm. The evolutionary theory of the firm was first unified and presented as a stand-alone body of economic knowledge by Nelson and Winter [1982].

The two authors claim that organisations operate according to some routines, i.e. behavioural algorithms based on past experience. Whenever a decision has to be made these behavioural algorithms activate and guide the firm through the decision making process. What is more, the knowledge at the firm's disposal is limited, and the firm struggles from the bounded rationality of its members. All this makes different firms tackle similar problems differently based on their past experience and perceptions regarding the future outcomes of their decisions. Over time successful firms emerge and the unsuccessful ones cease to exist just as it occurs in biology. Furthermore, as the external environment of the firm is viewed as dynamic, what also matters is the adaptability of the firm and the flexibility of its routines. Thus, both the inherent ways of doing things and the ability to alter them to cater for the changing environment, determine whether a firm can survive and grow or whether it is bound to shrink and, subsequently, withdraw [Nelson and Winter, 1982].

Corollary

Concluding our review of the most seminal theories of the growth of the firm to date, there are two broadly perceived schools of thought within the analysed field. The first, rooted in Robert Gibrat's work, advocates a more or less stochastic pattern of firm growth: Robert Gibrat himself proving the total randomness of firm growth, Ijiri and Simon introducing a random growth factor and linking all new growth to past growth, and finally Boyan Jovanovic advocating for a random cost-efficiency allocation across firms. The second stream of research is linked to the resources at the firm's disposal, be it in the form of managerial capabilities, like in the case of Edith Penrose, or routines, as in the case of Nelson and Winter. This research school holds that it is these very resources that are differentiators, drivers of, but also limits to, firm growth.

A model of firm growth

Over the last thirty years, that is post the 1982 works of Jovanovic and Nelson and Winter, the academic community has been mostly concerned with either testing the applicability of the theories discussed herein to the real world [see for instance Evans, 1987], or with revisiting the assumptions behind those theories in order to generate a closer match between the theoretical models and the empirical observations [see for instance Kwaśnicki, 2000, Growiec et al., 2008, or Lockett et al., 2011]. At the same time, despite the fact that the stochastic and the resource-based schools offer opposing explanations to the nature of firm growth, very little progress has been made in developing alternative approaches to the matter. This, is turn, highlights the need for an alternative model of firm growth.

Model set-up

The proposed model tries to integrate the insights of both schools, i.e. the stochastic and the resource-based one. Organisational resources, and in particular their ability to cater for new growth, are central to our analysis. We suggest that, at different stages of their development, firms are able to internalise varying levels of staff, and that the relationship between the firm's age and its size is not necessarily linear. We view the search for profit as the key driver of the firm's activity, yet its incumbents as boundedly rational agents with limited foresight abilities. In addition, we assume, following the business administration literature, that each and single firm is capable of undergoing a five-stage life cycle, subject to its external and internal environments.

Therefore, our analysis is based on five key assumptions as follows:

1. the firm in question is a profit-maximising one,
2. the size of the firm is measured by its headcount, which is directly related to its overall profitability, i.e. the larger the firm is in terms of headcount, the more profit it generates for its owners,

3. the firm has a short planning horizon and a short memory, i.e. only recent growth history matters and the firm can foresee only as far ahead as the next period,
4. supply of labour is unlimited and its cost does not change with the firm's size, while the supply of capital potentially increases as the firm grows,
5. the firm is, however, limited in its size by its organisational resources, i.e. at any given time its physical, human and organisational capital allow it to absorb only a limited number of people.

Finally, let us define γ_i as a function of the organisational resources of firm i, i.e. its ability to absorb only a given number of people, and let γ_i be unique to each and single firm and be potentially variable in time.

Organisational resources and firm size in time

Let us note that at different stages of its development the firm in question is characterised by diverse organisational resources and, what is more, by corresponding changes in its size. At different stages of the firm's life its decision makers have different goals and risk attitudes, the firm's resources such as routines, organisational culture, physical and human capital evolve and its members are willing to accept different scales of activity.

Hence, let us note that the organisational resources function γ_i is a function of the current stage of the firm's life cycle (x_n). Let us also view the firm's life cycle as comprised of five stages following Miller and Friesen [1984]:

1. birth stage, which in Figure 1 is represented by (x_1, x_2) and which might be viewed as the organisation stage of the firm,
2. growth stage, which is represented by (x_2, x_3) and which might be viewed as the first stage of the life cycle when the minimum efficiency scale has been reached,
3. maturity stage, which is represented by (x_3, x_4),
4. revival stage, which is represented by (x_4, x_5),
5. decline stage, which is represented by (x_5, x_6).

Where the lower bound determines the start of a life-cycle stage, and the upper bound its end, with x_1 representing the firm's establishment and x_6 representing the firm's death.

Let the function of organisational resources $\gamma(x_n)$ be:

a monotonically increasing one for every

$$x_1 < x_n < x_3 \left(\forall x_i, x_j \in (x_1, x_3) : x_i \prec x_j \Rightarrow \gamma(x_i) \prec \gamma(x_j) \right),$$

a monotonically decreasing one for every

$$x_3 < x_n < x_4 \left(\forall x_i, x_j \in (x_3, x_4) : x_i \prec x_j \Rightarrow \gamma(x_i) \succ \gamma(x_j) \right),$$

a monotonically increasing one for every

$$x_4 < x_n < x_5 \left(\forall x_i, x_j \in (x_4, x_5) : x_i \prec x_j \Rightarrow \gamma(x_i) \prec \gamma(x_j) \right),$$

a monotonically decreasing one for every

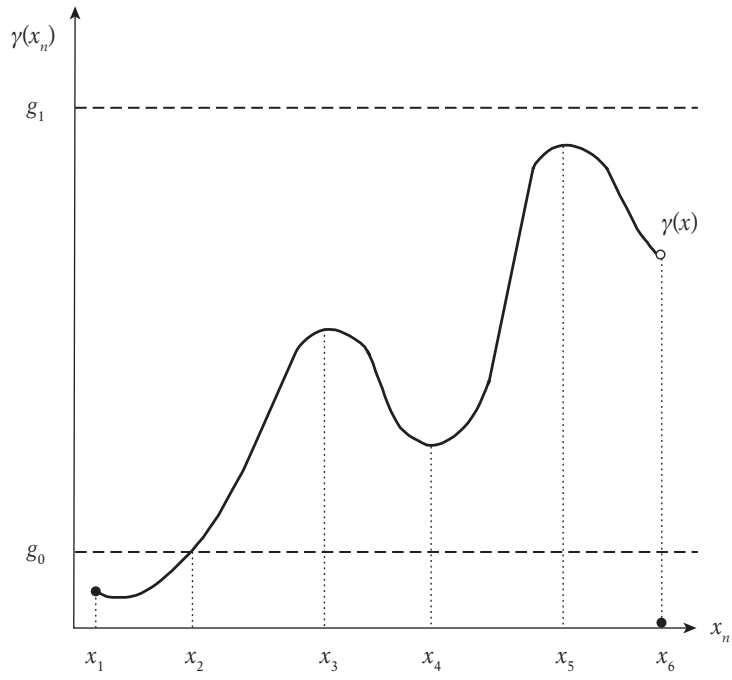
$$x_5 < x_n < x_6 \left(\forall x_i, x_j \in (x_5, x_6) : x_i \prec x_j \Rightarrow \gamma(x_i) \succ \gamma(x_j) \right),$$

Furthermore, let us assume that the firm can be larger in size at the peak of its revival stage than at the peak of its growth stage: $\gamma(x_5) \succ \gamma(x_3)$.

Moreover, let the optimal firm size at any given point in time be limited by its external environment, both from the up- and down-side. Let $g_0$be the smallest economically feasible size of a firm of the analysed type at a given point in time, determined by the minimum efficient scale requirements and the industry nature. Let g_1 be the largest economically feasible size of a firm of the analysed type at a given point in time, determined by the external environment of the firm.

All these assumptions, as depicted in Figure 1, allow us to trace the growth of the firm as a function of its organisational resources. Since its inception (x_1) the firm has to reach the minimum efficiency point (x_2) in order to become fully operational, and then via a growth phase (x_2, x_3) it can start moving towards its maturity stage (x_3, x_4), and possibly enter its revival (x_4, x_5) and decline (x_5, x_6) periods.

Figure 1. Organisational resources and firm life cycle



Source: own visualisation

As at different stages of the firm's development its organisational resources function takes varying values, its size is also different at different stages of its development. In addition, given that the organisational resources function is unique to each and single firm, two firms being at the same stage of development and operating in the same industry, under exactly the same external conditions, might grow to different sizes.

Still, if it so that in certain stages of its life cycle the firm can be larger than in others, thus generating additional streams of profit from its increased size, why does the firm not strive to be at the x_5 state of its life cycle as soon as possible and for as long as possible?

Firm development as a Markov chain

The answer to the question of why the firm does not strive to enter its revival stage as soon as possible may be provided by noting that the organisational resources function is not known upfront to the firm's incumbents. On the contrary, the firm's incumbents have a limited ability to foresee what the organisational resources function will look like in the future. Only the current and the next stages of the firm's life cycle can be effortlessly assessed at any given point in time. Therefore, the decisions taken are boundedly rational.

All this can be conveniently presented using Markov chains. For the purposes of our analysis let us assume that the firm in question at discreet, albeit not equally distributed, times (t) has to decide on whether to stay where it currently is $(X(t_{n+1}) = X(t_n) = x_n)$, move to the next stage of its life cycle $(X(t_{n+1}) = x_{n+1})$, or exit $(X(t_{n+1}) = 0)$. Moreover, based on our initial assumption stating that firms have a short memory and on the basic assumption of the Markovian processes, we may say that the probability of deciding to move from the current stage of the firm's life cycle to the next one does not depend on organisational history, but only on the current stage of its life cycle, or in mathematical terms:

$$P\{X(t_{n+1}) = x_{n+1} \mid X(t_n) = x_n, X(t_{n-1}) = x_{n-1}, \ldots, X(t_1) = x_1\} = P\{X(t_{n+1}) = x_{n+1} \mid X(t_n) = x_n\},$$

where $t_1 < t_2 < \ldots < t_{n-1} < t_n < t_{n+1}$; and where X is a sequence of random variables X_i representing the outcome of the Markovian process at time t.

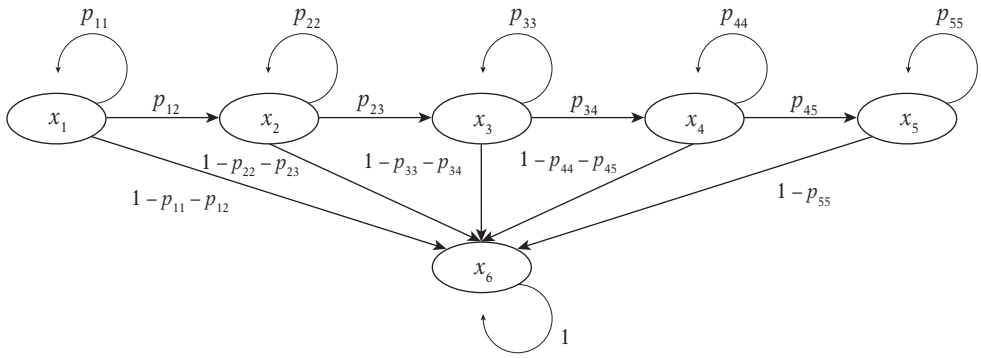
Furthermore, let us assume that the firm cannot re-enter a past stage of its life cycle and that in order to reach state x_n it has to first pass stages x_1 to x_{n-1}; still let us relax this assumption for the last stage x_6, i.e. death, which can be accessed from any other stage. Therefore, based on our assumptions, we can state the transition probability matrix in the following form:

$$P = \begin{bmatrix} p_{11} & p_{12} & 0 & 0 & 0 & (1-p_{11}-p_{12}) \\ 0 & p_{22} & p_{23} & 0 & 0 & (1-p_{22}-p_{23}) \\ 0 & 0 & p_{33} & p_{34} & 0 & (1-p_{33}-p_{34}) \\ 0 & 0 & 0 & p_{44} & p_{45} & (1-p_{44}-p_{45}) \\ 0 & 0 & 0 & 0 & p_{55} & (1-p_{55}) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where: $\forall_{i,j},\ p_{ij} \in [0,1]$ and $\forall_i, \sum_{j=1}^{n} p_{ij} = 1$.

Also, the same relations are presented in Figure 2.

Figure 2. Firm life cycle transition graph



Source: own visualisation

Transition process dynamics

Even though the transition probabilities differ from firm to firm, based on our initial assumptions that growth in size is preferred as it leads to increased profit streams, and that all other states are preferred to death, we can state the following relations between the transition probabilities which should hold true for every single firm:

$$p_{11} < p_{12} \text{ as } \gamma(x_1) \prec \gamma(x_2),$$
$$p_{11} > 1-p_{11}-p_{12} \text{ as } \gamma(x_1) \succ \gamma(x_6),$$
$$p_{12} > 1-p_{11}-p_{12} \text{ as } \gamma(x_2) \succ \gamma(x_6),$$

$$p_{22} < p_{23} \text{ as } \gamma(x_2) \prec \gamma(x_3),$$
$$p_{22} > 1-p_{22}-p_{23} \text{ as } \gamma(x_2) \succ \gamma(x_6),$$
$$p_{23} > 1-p_{22}-p_{23} \text{ as } \gamma(x_3) \succ \gamma(x_6),$$

$$p_{33} > p_{34} \text{ as } \gamma(x_3) \succ \gamma(x_4),$$
$$p_{33} > 1 - p_{33} - p_{34} \text{ as } \gamma(x_3) \succ \gamma(x_6),$$
$$p_{34} > 1 - p_{33} - p_{34} \text{ as } \gamma(x_4) \succ \gamma(x_6),$$

$$p_{44} < p_{45} \text{ as } \gamma(x_4) \prec \gamma(x_5),$$
$$p_{44} > 1 - p_{44} - p_{45} \text{ as } \gamma(x_4) \succ \gamma(x_6),$$
$$p_{45} > 1 - p_{44} - p_{45} \text{ as } \gamma(x_5) \succ \gamma(x_6),$$

$$p_{55} > 1 - p_{55} \text{ as } \gamma(x_5) \succ \gamma(x_6).$$

What is clearly visible from the above set of inequalities is that there are only two stages, i.e. x_3 and x_5, which are likely to be satisfactory to the firm's incumbents. Therefore, the management of the firm is more inclined to settle for any of the two points. Thus, they will be working towards moving alongside their organisational resources function in ways promising that one of those two points can be attained and kept. Still, as the firm's incumbents are limited in their foresight ability, having arrived at point x_3 they will be apt to view it as the largest realistically attainable firm size. Hence, even though point x_3 represents just a local maximum, while point x_5 represents the global maximum of the organisational resources function, it will be more common for the former one to be perceived as the satisfactory growth target for the firm in question.

Furthermore, this behavioural pattern also explains why the firm size distribution is primarily dominated by the small firm, irrespective of the industry or location. The x_3 state, even though culminating the growth stage of the firm, may in reality still allow for a comparatively small size of the analysed firm. As a result, firms complacent with their size having reached the end of their growth stage may still have unrealised potential for size expansion, which they simply cannot see and do not pursue to attain.

What is more, incumbents may have different risk preferences, own agendas and strategies in place. All this may lead to differences in the transition probabilities across firms, and even across time for the same firm. For instance, a risk averse owner-manager with a great wealth exposure to his firm and limited strategic planning capabilities may be totally reluctant to proceed to the next stage of the firm's development, settling at an attained local maximum of the organisational resources function. At the same time, a firm run by, for example, a salaried manager with limited wealth exposure, greater preference for risk and a clear growth strategy in place, may decide to enter the next stage of its life cycle even at a risk of a short-term decrease in size. This would be done in the hope that such a decision will eventually lead to its increased size. All in all, however, as a result of such a set-up, the industry landscape will still be dominated by the small firm, implying that the great majority of firm managers do not aim to achieve the largest attainable firm size, but rather prefer to stay at a satisfactory level for as long as possible. At the same time, the functions of organisational resources as well as the managerial traits may

indeed be viewed as randomly distributed across the whole population of firms supporting the findings of the stochastic theories of the growth of the firm.

Originality of the model and implications for further research

Although our model is relatively simple in terms of the adopted mathematical apparatus, it is the first model of its type to date. In previous papers by other authors, the properties of the Markov chains have indeed been used, yet mostly in the analysis of the dynamics of whole industries, business entry and exit decisions, or the mobility of firms across sectors [see for instance Hopenhayn, 1992, Ericson and Pakes, 1995 or Nehrebecka, 2011]. In addition, as almost all alternative microeconomic models of firm growth have been developed in isolation from the business administration literature, ours is likely to be the first formal model to incorporate the firm life-cycle hypothesis. Therefore, owing to the adoption of a Markov chain to describe the process through which firms grow to reach their observable sizes, our model is capable of providing novel and invaluable insights into the dynamics of the growth process at the firm, as opposed to industry, level.

Apart from the theoretical contribution to the theory of the firm, we aim to stimulate further empirical research into firm growth. Our proposals can be relatively easily checked in practice either through the adoption of the case-study method or through econometric analysis of a cohort of firms. Still, it is anticipated that the latter approach would yield a more quantitative insight into the transition probabilities between the different stages of the firm life cycle. Alternatively, if the predictions of the model are to be applied to whole industries rather than individual firms, the model could also undergo a calibration exercise. In such a case, however, it is envisaged that the transition probabilities would first need to be approximated based on real-life observations.

Conclusion

In this article, we have reviewed the most seminal works in the field of the growth of the firm to date. Based on their findings we have developed a new formal model illustrating why a boundedly rational agent may not let his firm grow in size.

The proposed model of firm growth shows that at any given time the firm is limited in its growth prospects by its optimum attainable size. What influences the optimal firm size is the external as well as the internal boundaries to firm growth. Throughout this paper we have focused on the latter, which are represented by the organisational resources function limiting the total number of people the firm can efficiently employ at any stage of its life cycle. The organisational resources function is viewed as a Markovian process, which, due to the firm's nearsightedness and preference for large size over small, makes its incumbents prefer its local maximum to the – uncertain and virtually unknown upfront – global maximum. Therefore, the firm's incumbents prefer

not to struggle with the internal boundaries to firm growth once the growth stage has been achieved, but rather to stay where they are. This, in turn, leads the firm to remain relatively small, which is by no means an exception to the rule; on the contrary, firms that do manage to successfully complete their revival stages are in fact exceptional in their ability and willingness to grow. What is more, even though in the long run every single firm will have to exit, as mathematically provable, before this happens firms are expected to go through a number of life-cycle stages. The exact number of stages that the firm in question will have completed before it exits is, however, determined on a case-by-case basis.

Although the developed model is relatively simple, we believe that it is its simplicity and the explanatory power that may be useful in both empirical testing and in future theoretical enquiries into the nature of the growth of the firm. Finally, even though it is true that the great majority of firms are small in size, our model is equally applicable to firms of all sizes operating in all industries and markets.

Bibliography

Chandler A.D., [1977], *The Visible Hand: The Managerial Revolution in American Business*, Cambridge, MA: The Belknap Press of Harvard University Press.

Coase R., [1937], *The Nature of the Firm*, „Economica", Vol. 4, No. 16, 386-405.

Ericson R., Pakes A., [1995], *Markov-Perfect Industry Dynamics: A Framework for Empirical Work*, „The Review of Economic Studies", Vol. 62, No. 1, 53-82.

Eurostat, [2009], *European Business: Facts and Figures, 2009 edition*, Luxembourg.

Evans D.S., [1987], *Tests of Alternative Theories of Firm Growth*, „The Journal of Political Economy", Vol. 95, No. 4, 657-674.

Gibrat R., [1931], *Les inégalités économiques: applications aux inégalités des richesses, à la concentration des entreprises... d'une loi nouvelle, la loi de l'effet proportionnel*, Paris.

Growiec J., Pammolli F., Riccaboni M., Stanley H.E., [2008], *On the Size Distribution of Business Firms*, „Economics Letters", Vol. 98, 207-212.

Hopenhayn H.A., [1992], *Entry, Exit and Firm Dynamics in Long Run Equilibrium*, „Econometrica", Vol. 60, No. 5, 1127-50.

Ijiri Y., Simon H.A., [1967], *A Model of Business Firm Growth*, „Econometrica", Vol. 35, No. 2, 348-355.

Ijiri Y., Simon H.A., [1971], *Effects of Mergers and Acquisitions on Business Firm Concentration*, „The Journal of Political Economy", Vol. 79, No. 2, 314-322.

Jovanovic B., [1982], *Selection and the Evolution of Industry*, „Econometrica", Vol. 50, No. 3, 649-670.

Kalecki M., [1945], *On the Gibrat Distribution*, „Econometrica", Vol. 13, No. 2, 161-170.

Kwaśnicki W., [2000], *Ewolucyjny model rozwoju przemysłu – perspektywy badawcze i dydaktyczne*, „Ekonomista", No. 4, 541-569.

Lockett A., Wiklund J., Davidsson P., Girma S., [2011], *Organic and Acquisitive Growth: Re-examining, Testing and Extending Penrose's Growth Theory*, „Journal of Management Studies", Vol. 48, No. 1, 48-74.

Miller D., Friesen P.H., [1984], *A Longitudinal Study of the Corporate Life Cycle*, „Management Science", Vol. 30, No. 10, 1161-83.

Nehrebecka N., [2011], *Analysis of Polish Business Demography Using Markov Chains*, Working Paper No. 6 (46), University of Warsaw, Faculty of Economic Sciences.
Nelson R.R., Winter S.G., [1982], *An Evolutionary Theory of Economic Change*, Cambridge, MA: The Belknap Press of Harvard University Press.
Penrose E.T., [1966], *The Theory of the Growth of the Firm*, Oxford: Basil Blackwell.
Penrose E.T., [1952], *Biological Analogies in the Theory of the Firm*, „The American Economic Review", Vol. 42, No. 5, 804-819.
Penrose E.T., [1955], *Research on the Business Firm: Limits to the Growth and Size of Firms*, „The American Economic Review", Vol. 45, No. 2, 531-543.
Simon H.A., [1991], *Organizations and Markets*, „The Journal of Economic Perspectives", Vol. 5, No. 2, 25-44.
Sutton J., [1997], *Gibrat's Legacy*, „Journal of Economic Literature", Vol. 35, No. 1, 40-59.

A MODEL OF FIRM GROWTH

Summary

International statistics show that small firms are the dominant form of business enterprise today. Yet, despite ongoing research into the theory of the firm, there is still no common view on the mechanisms of firm growth. This article aims to stimulate further theoretical and empirical research into firm growth.

In the first part of the paper, the author reviews the most seminal theories of the growth of the firm to date, noting that there are two broadly perceived schools of thought within the analysed field. The first approach advocates a more or less stochastic pattern of firm growth. The second research school holds that the resources at the firm's disposal are the differentiators, drivers of, but also limits to, firm growth.

In the second part of the paper, based on the literature review and deduction, the author develops an alternative model of firm growth. Building on the properties of the Markovian processes, he shows that it may be because of the seemingly rational behaviour of firm incumbents that most firms do not grow in size beyond some satisfying level. The proposed model of firm growth is equally applicable to firms of all sizes operating in all industries and markets.

Keywords: firm, theory, growth, size, life cycle