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THE INDIVIDUAL TAXPAYER UTILITY FUNCTION WITH TAX OPTIMIZATION AND FISCAL FRAUD ENVIRONMENT

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Abstract

In this paper I examine a taxpayer utility function determined by the extended set of variables – i.e. consumption, labor and tax-evasion propensity. This constitutes the main framework for the analysis of taxpayer's decision making process under assumption that in the economy there exist two main reduction methods: a) access to tax optimization techniques, which may decrease effective tax burden and are fully compliant with binding laws, but generate transactional costs and 2) possibility of fiscal fraud – in particular tax evasion, as the alternative method of reducing tax due, which has no direct transactional costs, but involves tax litigation risk.

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Introduction

One of the pivotal problems of both contemporary public economics and economic policy is the optimal taxation issue in relation to income, estate and consumption expenses. This covers various research pursuits, among others: optimal income tax schemes, setting effective tax incentives and exemptions, defining institutional framework to protect government revenue sources from fraudulent behavior or economic models describing taxpayer's decision making process towards tax authorities.

J. A. Mirrlees is deemed one of the founders of contemporary optimal taxation theory. In his pioneer article (Mirrlees, 1971, p. 176) he conceived a general mathematical model for maximization of social welfare for conceptual society consisting of individuals maximizing their individual utility functions, determined by time spent for work (denoted as labor – which generates some taxable income) and by the level of consumption.

Further articles led to extensions of Mirrleesian model. Among others, Sadka (1976) proved that for an optimum of welfare state function with consideration of individual taxpayers' utility functions under fiscal target constraints (i.e. achieving a set level of income to be collected for the central budget) induced the marginal income tax rate at the level higher than 0%, but lower than 100%.

Cremer, Pestieau and Rochet (2001) analyzed the effectiveness of direct taxation in comparison with indirect taxation subject to heterogeneous structure of individuals' income capability. In one of the more recent results (Simula and Trannoy, 2010, p. 172)

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Mirrleesianassumption regarding closed economy environment was repealed and the effects of top-earners migration on optimal taxation strategies were thoroughly analyzed.

Optimal non-linear taxation models were also examined subject to tax evasion phenomena[2], the most interesting of which cover the psychological and sociological determinants of fraudulent behavior in relation to tax authorities (Dell'Anno, 2009, p. 989-990).

Notwithstanding the existence of robust papers on tax fraud effects on determination of optimal tax schemes (both globally and for individual taxpayers), little thought was spared for the sole formal construction of individual utility function, which would (provided still the most general form possible) cover not only standard determinants such as consumption, labor and wage, but also institutional environment in which a taxpayer operates, in particular the access to the tax advisory services (and some of its unique features, such as tax optimization – which may effectively reduce tax levied on taxpayer's income) and the existence of tax fraud possibilities (which also reduces effective tax rate, but simultaneously generates risk of litigation).

This paper consists of four parts. Firstly, I outline the necessary assumptions for formal definition of the model to be used throughout the text. In the second part, there is an analysis of an individual taxpayer decision making problem in particular – is it economically rational to use tax advisory services and/or tax evasion methods to optimize effective tax rate for income? If so, to which extent should these techniques be facilitated? In the third part, the individual propensity for tax evasion is examined more profoundly and some characteristics are set. Finally, I pose conclusions emerging from the research with indications for further study.

The model

The maximization of the individual taxpayer utility usually involves a standard two-variable function of labor and consumption, broadly explored in literature (Ebert, 1992, p. 50). In this section the function will be expanded with additional variable to be facilitated throughout the analysis.

Let us have:

$$y = u\,(c, l, \varepsilon)\text{, where } u : \mathbb{R}^3 \ni D_u \to \mathbb{R} \text{ and } u \in C^2 \text{ within } D_u. \tag{1}$$

where: c = level of consumption achieved by the taxpayer
l = amount of time spent on labor
ε = tax-payer's individual tax-evasion propensity, $\varepsilon \in [0; 1]$

The abovementioned constitutes the individual taxpayer utility function with tax-evasion propensity determinant. Suppose that a given taxpayer is in disposal of overall time available at t – level. This must be shared between labor (l) and rest (r). Further in the text it is assumed that every unit of time spent on labor generates some nominal income at ω-level. Apart from this, taxpayer obtains additional incomes from numerous sources (for instance: social security support, donations etc.) which amount to the level of m. To render the

[2] One shall distinguish clearly tax evasion from tax avoidance. The first term denotes any unlawful and unethical operations which contributes to the effective reduction of tax burden (for instance, through hiding part of taxable income from tax authorities). The second term may not necessarily be illicit; it embraces all strategies (for instance – through tax optimization methods) which serve reducing tax due under all constraints emerging from institutional surrounding and commonly accepted codes of conduct.

calculations more manageable, it is assumed that m is subject to the taxation within the same scheme as is the labor-induced income.

The assumption that $u \in C^2$ enables the examination of partial derivatives signs. Priorly however, we shall introduce three axioms which stem from general economic theory:

1) all else equal, increase in consumption leads to increase in the level of total utility; continuous increase of achieved consumption, marginal utility with respect to consumption decreases (maintaining positive sign within its domain),
2) all else equal, increase in time value spent on labor generates decrease in total utility,
3) all else equal, increase in tax-evasion propensity level generates increase in total utility achieved; however, continuous increasing this factor summons marginal utility with respect to it decreasing and may potentially attain negative values (which consequently leads to decrease in total utility on exceeding some critical value ε.

Additionally, it is assumed later in the analysis that:

1) all income after taxation is spent by a taxpayer on consumption (commodities and services) which is aggregately shown in the level of actual consumption of a perfect commodity (c); the commodity is purchased at the fixed price p. One may find this assumption slightly unrealistic, though formally it is feasible to transform an entire bunch of goods acquired with different market prices as one through weighted mean,
2) every taxpayer has some individual propensity for tax evasion; this assumption is likely to rise numerous controversies, though the phenomenon of taxpayers actually perpetrating tax law to benefit from reduced or eliminated taxation remains undisputed. In the following model the taxpayer chooses the value of ε which denotes his individual attitude towards tax evasion. More specifically, it shows what part of taxable income the taxpayer hides from levying tax. Such conduct obviously increases the level of disposable income, while simultaneously increasing the risk of launching tax litigation or tax control against taxpayer. These procedures may result in issuing decisions with penal tax amounts to be paid[3],
3) all incomes are subject to a fixed tax rate at the level of τ.
4) every taxpayer has access to tax advisory services, through which he can implement beneficial and fully legal optimization strategy. This can reduce the effective tax rate levied upon income to the level of $\tau - \tau_s$. However, using such services generates some costs at the level of c_T,
5) the risk of issuing penal tax decisions and other fiscal consequences are denoted within the model as a decrease in amount of gross income dependent on ε and some factor $\varrho > 0$[4].

The optimization problem

The rational taxpayer struggles to maximize his individual utility function under budget constraint. Formally it is denoted as follows:

[3] In Poland, all incomes which have been hidden from taxation and found during tax control are subject to penal rate at the level of 75%. In the analysis we neglect some other forms of penalisation such as late payment penalties or sanctions resulting from fiscal penal codes.

[4] ϱ is defined as the average rate of penalization during some fiscal period.

$$\begin{cases} \max\limits_{(c,l,\varepsilon)} u(c,l,\varepsilon) \\ pc = (1-\varepsilon)(\tau-\tau_s)(\omega l - c_T + m) - \varepsilon w l \varrho \end{cases} \quad (2)$$

To examine the solvability of the problem (2) it is convenient to use the Langrange multipliers method.
In the first step the Langrangian function is defined as:

$$L(l,c,\varepsilon) := u(c,l,\varepsilon) + \lambda[pc - (1-\varepsilon)(\tau-\tau_s)(\omega l - c_T + m) - \varepsilon\omega l\varrho] \quad (3)$$

where : λ = Lagrange multiplier.
L = Lagrangian function

The first-order conditions (FOC) of maximum existence for the function L require that all first-order partial derivatives of the function with respect to all variables l, c and ε and to the multiplier λ equal zero. Formally:

$$\frac{\partial L}{\partial c} = 0\,, \frac{\partial L}{\partial l} = 0\,, \frac{\partial L}{\partial \varepsilon} = 0\,, \frac{\partial L}{\partial \lambda} = 0 \quad (4)$$

Differentiating then yields:

$$\begin{cases} \frac{\partial u}{\partial c} + \lambda p = 0 \\ \frac{\partial u}{\partial l} - \lambda(1-\varepsilon)(\tau-\tau_s)\omega - \varepsilon\omega\varrho = 0 \\ \frac{\partial u}{\partial \varepsilon} + \lambda(\tau-\tau_s)(\omega l - c_T + m) - \omega l\varrho = 0 \\ pc - (1-\varepsilon)(\tau-\tau_s)(\omega l - c_T + m) - \varepsilon\omega l\varrho = 0 \end{cases} \quad (5)$$

The second-order condition for maximum is satisfied if the following inequalities are true [Tokarski, 2011, p. 36]:

$$\bigwedge_{i\epsilon\{3,4\}} (-1)^{i-1} m_i\, H(L) > 0 \quad (6)$$

where: m_i = i-th minor principle of H(L)
H(L) = the bordered Hessian of Lagrangian L

H(L) is represented as:

$$H(L) = \begin{bmatrix} 0 & p & 0 & 0 \\ p & \frac{\partial^2 L}{\partial c^2} & \frac{\partial^2 L}{\partial c \partial l} & \frac{\partial^2 L}{\partial c \partial \varepsilon} \\ 0 & \frac{\partial^2 L}{\partial l \partial c} & \frac{\partial^2 L}{\partial l^2} & \frac{\partial^2 L}{\partial l \partial \varepsilon} \\ 0 & \frac{\partial^2 L}{\partial \varepsilon \partial c} & \frac{\partial^2 L}{\partial \varepsilon \partial l} & \frac{\partial^2 L}{\partial \varepsilon^2} \end{bmatrix} \qquad (7)$$

The individual propensity for tax evasion

What determines the level of ε? Are there any conditions which would rationalize taxpayer's behavior leading to tax fraud, regardless of both ethical and moral objections? To answer these questions, one may examine the formula for the value of ε at the stationary point of Lagrangian L.

Firstly, it is necessary to facilitate first-order condition equations:

$$\begin{cases} \frac{\partial u}{\partial l} - \lambda(1-\varepsilon)(\tau - \tau_s)\omega - \varepsilon\omega\varrho = 0 \\ -\lambda = \frac{\partial u}{\partial c}\frac{1}{p} \end{cases} \qquad (8)$$

Proposition 1.

In the stationary point of lagrangian L(c,l,ε) the optimal value of ε is determined by the equation:

$$\varepsilon = \frac{\frac{\partial u}{\partial l} + \frac{\partial u}{\partial c}\frac{1}{p}\omega(\tau - \tau_s)}{\omega\varrho + \frac{\partial u}{\partial c}\frac{1}{p}\omega(\tau - \tau_s)} \geq 0 \qquad (9)$$

Proof.

With some transformations of (8) it yields:

$$\frac{\partial u}{\partial l} + \frac{\partial u}{\partial c}\frac{1}{p}\tau\omega - \tau_s\omega\frac{\partial u}{\partial c}\frac{1}{p} - \varepsilon\tau\omega\frac{\partial u}{\partial c}\frac{1}{p} + \varepsilon\tau_s\omega\frac{\partial u}{\partial c}\frac{1}{p} = \varepsilon\omega\varrho \qquad (10)$$

So :

$$\frac{\partial u}{\partial l} + \frac{\partial u}{\partial c}\frac{1}{p}\tau\omega - \tau_s\omega\frac{\partial u}{\partial c}\frac{1}{p} = \varepsilon(\tau\omega\frac{\partial u}{\partial c}\frac{1}{p} - \tau_s\omega\frac{\partial u}{\partial c}\frac{1}{p} + \omega\varrho) \qquad (11)$$

which finally leads to (9), completing the proof.

Having the formula for individual tax-evasion propensity, one may enquire about the conditions for its positive values.[5] This is denoted as:

Proposition 2.

$$\varepsilon > 0 \iff \left|MRS_{c,l}\right| < \left|\frac{\omega}{p}(\tau - \tau_s)\right| \tag{12}$$

where: MRS = marginal rate of substitution of consumption with respect to labor.

Proof.

For $\varepsilon > 0$ it suffices that both numerator and denominator are simultaneously positive or negative (with additional restriction, that denominator is other than 0). On the basis of the previous assumptions there is: $\frac{\partial u}{\partial c} > 0$ and $\frac{\partial u}{\partial l} < 0$ and other parameters are by definition positive, so the only condition which must be met is that the nominator is higher than 0. In other words:

$$\frac{\partial u}{\partial l} + \frac{\partial u}{\partial c}\frac{1}{p}\omega(\tau - \tau_s) > 0 \tag{13}$$

As $\frac{\partial u}{\partial l} < 0$, it is possible to use a more convenient notation with modulus:

$$\left|\frac{\partial u}{\partial l}\right| < \left|\frac{\partial u}{\partial c}\frac{1}{p}\omega(\tau - \tau_s)\right| \tag{14}$$

After dividing by $\frac{\partial u}{\partial c}$ one yields:

$$\frac{\left|\frac{\partial u}{\partial l}\right|}{\left|\frac{\partial u}{\partial c}\right|} < \left|\frac{1}{p}\omega(\tau - \tau_s)\right| \tag{15}$$

By dint of definition of MRS, it is finally:

$$\left|MRS_{c,l}\right| < \left|\frac{\omega}{p}(\tau - \tau_s)\right| \tag{15}$$

which completes the proof.

The yielded results can be interpreted as follows:

[5] Cases in which $\varepsilon > 0$ give insight into the phenomenon which can be described literally as fraud rationalization. Taxpayer's behavior involving some non-zero levels of tax evasion propensity may be economically relevant and rational, regardless of the moral and ethical objections. However, 2-way reduction of costs and risk connected with tax frauds (on the side of government by means of reducing financing of tax administration to ensure maximum compliance, on the site of taxpayers – by means of stability, trust and well-established business environment) increases social welfare in Pareto sense (Bayer and Sutter, 2009, p. 527).

1) tax-evasion becomes rational from taxpayer's point of view in all cases in which the marginal rate of substitution of consumption with respect to labor is less (as modulus) than real wage rate for the time unit spent on labor after taxation,
2) facilitating effective tax optimization (by means of tax advisory) becomes a relatively diminishing factor for individual tax-evasion propensity in as high degree, as lesser becomes the effective, optimized tax rate $\tau - \tau_s$.

Conclusions

The model presented in this paper was a short example of optimization problem solving for an individual taxpayer seeking opportunities to reduce his effective tax rate on income.

The highlighted problem of embracing in such analyses factors like tax frauds or tax advisory usage shall be extended and more thoroughly examined in further studies. For instance, the problems to be reviewed entail:

1) the behavior of taxpayer in the surrounding of different levels of tax advisory services available with prices highly at variance,
2) the maximization of taxpayer's utility function with access to transfers from tax havens,
3) the extension of analysis regarding government decisions in answer to taxpayers behavior and tendencies; this might involve mathematical apparatus for non-cooperative, zero-sum games.

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