A C T A U N I V E R S I T A T I S L O D Z I E N S I S
FOLIA OECONOMICA 285, 2013

Grażyna Trzpiot[*]

PROPERTIES OF TRANSFORMATION QUANTILE REGRESSION MODEL

Abstract. We present in this paper a few important direction on research using quantile regression. We start from some motivation for this method of regression. Secondly we present some main areas of application this method. Finally we wanted to point out transformation of the main model. This model, introduced by Powell (1991) and further analyzed by Chamberlain (1994) and Buchinsky (1995), specifies the conditional quantiles of the Box-Cox transformation of the variable under appraisal as a linear function of the covariates. It provides, within a simple set-up, the needed flexibility, as both the transformation parameter and the coefficients of the linear function are allowed to vary freely at each point of the distribution. The Box-Cox quantile regression, which has the linear and log-linear models as particular cases, will provide, therefore, a direct answer to the question of the appropriate transformation to be used.
Key words: quantile regression, quantile regression model, Box-Cox transformation.

I. QUANTILE REGRESSION – MOTIVATION

From standard regression to quantile regression

Regression is used to quantify the relationship between a response variable and some covariates. Standard regression has been one of the most important statistical methods for applied research for many decades. More complicated models, such as polynomial regression models, may also be used to model different relationship.

From conditional skew distributions to quantile regression

Fig. 1(a) displays weight against age for a sample of 4011 US girls (Cole,1988). The intuitively reasonable notion of a relationship between weight and age is further supported by Fig. 1(b) which presents several smoothed quantile regression curves. These suggest that the associated conditional distributions are skew to the right.

Two questions of interest are: first: what is a typical weight profile as a function of age second: what is a typical weight profile as a function of age for overweight and underweight people?

[*] Prof., University of Economics in Katowice.

A sensible answer to the first question is not provided by standard mean regression, as the mean at any specific year is pulled downwards. Hence, the median curve is a more appropriate curve to display. This median curve corresponds to the middle quantile regression curve displayed in Fig. 1(b). If it is thought that girls whose weights lie on or above the 97% curve for the population are overweight, then the appropriate curve to display is that based on quantile regression with p = 0,97. Similarly, the p = 0,03 quantile regression curve displays the relationship of the weight of underweight girls with age.

II. APPLICATIONS OF QUANTILE REGRESSION

In this section we present some typical applications of quantile regression to medical reference charts, survival analysis, financial research, economics research and the detection of heteroscedasticity

2.1. Applications to reference charts in medicine

In medicine, reference (or centile) charts provide a collection of useful quantiles. These are widely used in preliminary medical diagnosis to identify unusual subjects in the sense that the value of some particular measurement lies in one or other tail of the appropriate reference distribution. The need for quantile curves rather than a simple reference range arises when the measurement (and hence the reference range) is strongly dependent on a covariate such as age, as Cole and Green (1992) and Royston and Altman (1994) have discussed. The chosen quantiles are usually a symmetric subset of {0,03; 0,05;0,1;0, 25;0,5;0,75;0, 9;0, 95;0, 97}. An example of a reference chart is shown in Fig. 1, Hahn (1995) with the *Y*-variable being weight and the *X*-variable being age. How can these quantile regression curves be obtained?

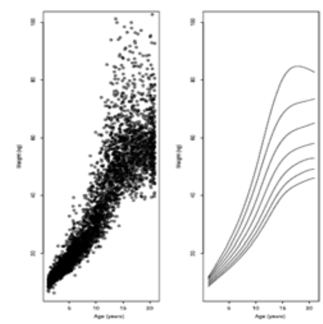

Figure 1. Weight against age for a sample of 4011 US girls
Source: own work.

An obvious approach is to use a known conditional distribution $F(y|x)$ to fit the underlying conditional distribution. The 100 θ% quantile curve corresponds to $q_\theta(x) = F^{-1}(\theta|x)$. Now, if the distribution is normal, then estimating the 100 θ% quantile curve is straightforward. If, however, the distribution is skew, as is more usual, then often a transformation to normality is applied. A typical transformation is the Box–Cox transformation to which we shall return, see Cole (1988), Altman (1990) and Royston and Wright (2000).

2.2. Applications to survival analysis

Applications to survival analysis include studying the effect of a specific covariate on the survival time of an individual. A given covariate may have a different effect on low, medium and high risk individuals. These effects can be understood by considering several quantile functions of survival time; see Koenker and Geling (2001) for details. Fig. 2 presents three quantile regression curves with p = 0,1; 0,5; 0,9 based on the 184 survival times of patients with covariate age between 12 and 64 years from the Stanford heart transplant survey (Crowley and Hu,1977); see Yang (1999) for further details about censored median regression.

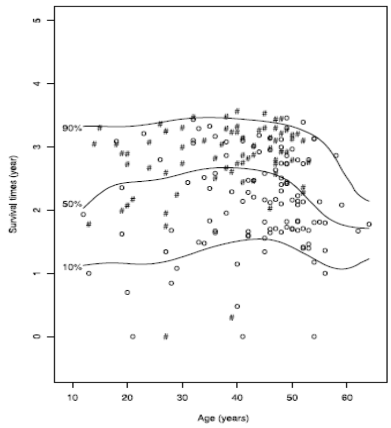

Figure 2. Survival times of patients with covariate age between 12 and 64 years

Source: own work.

Cox's proportional hazard model is often used for survival analysis. Alternatively, the accelerated failure time approach that models the logarithm of the survival time as a function of covariates can be employed.

The basic model posits survival times T_i, i=1, . . . , n, that may be censored and that depend on covariates $\boldsymbol{x}_i$. In the absence of censoring, it is natural to consider the pairs $\{T_i, \boldsymbol{x}_i\}^n_{i=1}$ as a multivariate independently and identically distributed sample. If the *i*th observation has been censored, then we observe Y_i for T_i. The 'log'-transformation of T_i provides the usual accelerated failure time model, which regresses the logarithm of T_i linearly on $\boldsymbol{x}_i$, i.e .

$$\log(T_i) = \boldsymbol{x}^T_{\mathrm{i}}\boldsymbol{\beta} + \varepsilon_i,$$

where ε_i, i=1, . . . , n, are independently and identically distributed with an unknown distribution function. The mean of ε_i is not assumed to be zero because we observe Y_i instead of T_i in the case of censoring and so the intercept term is not included in the vector $\boldsymbol{\beta}$. Because of this, mean regression analysis is not a good estimation technique for the accelerated failure time approach. However, the quantile regression technique that models the quantiles of the survival time or a monotone transform thereof, as a function of the covariates and the intercept is appropriate (see Yang (1999).

2.3. Applications in financial research

Financial regulations usually require banks to report their daily risk measures called value at risk (VaR). VaR models are the most commonly used measure of market risk in the financial industry (Lauridsen,2000). Let Y be the financial return, so that the y satisfying $P(Y \leq y) = p$ for a given low value of p is the VaR. The variable *Y* may depend on covariates **x** such as exchange rates. Clearly, VaR estimation relates to extreme quantile estimation through estimating the tail of financial return. The distribution of financial return could also be illustrated by several quantiles.

For example, the common approach to estimating the distribution of one-period return in financial models is to forecast the volatility and then to make a Gaussian assumption (see Hull and White (1998)). Market returns, however, are frequently found to have more kurtosis than a normal distribution. A general discussion of using quantile regression for return-based analysis was given by Bassett and Chen (2001).

2.4. Applications in economics research

Quantile regression is useful in the study of consumptive markets as the influence of a covariate may be very different for individuals who belong to high, medium and low consumption groups. Similarly, changes in interest rates may have a different inference on the share prices of companies which belong to high, medium and low profits groups.

In particular, quantile regression is now regarded as a standard analysis tool for wage and income studies in labour economics; see, for example, Buchinsky

(1995). It is also important to study how incomes are distributed among the members of a population, e.g. to determine tax strategies or for implementing social policies.

Other applications include modeling household electricity demand over time in terms of weather characteristics. The low quantile curves correspond to background use, where as possibly the high quantile curves reflect high use during active periods of the day particularly due to air conditioning; see Hendricks and Koenker (1992).

2.5. Applications to detecting heteroscedasticity

Recognizing heteroscedasticity is an important task for the data analyst. Quantile plots can provide a useful descriptive tool. These plots not only help to detect heteroscedasticity but also provide an impression of the location,spr ead and shape of the conditional distribution of Y given $\boldsymbol{X} = \boldsymbol{x}$.

Quantile regression can be used to assess departures from the assumptions of the model $Y = \boldsymbol{x}^T \boldsymbol{\beta} + \varepsilon$. If the distribution of ε does not depend on the value of the covariate $\boldsymbol{X}$, all regression quantiles will be parallel. For example, the seven quantile curves for the US girls data in Fig. 1 are clearly not parallel, indicating heteroscedasticity

III. ESTIMATION METHODS AND ALGORITHMS

We will now present estimation methods and algorithms for quantile regression.

3.1. The parametric quantile regression model

To quantify the relationship between a response variable Y and covariates **x**, we often assume that E[Y|**X**=**x**] can be modeled by a simple linear combination $\mathbf{x}^T\boldsymbol{\beta}$. Similarly, the basic quantile regression model specifies the linear dependence of the conditional quantiles of Y on **x**.

Consider the following regression model (Trzpiot, 2009b)

$$y_i = g(x_i) + e_i \tag{1}$$

where the dependent variable $y = (y_1, y_2, \ldots\ldots y_n)$ and independent $x = (x_1, x_2, \ldots\ldots x_n)$ where $y \in \mathrm{R}$ and $x \in \mathrm{R}^p$, $g(\cdot)$ is real valued and unknown. We are interested in estimating the regression function $g(\cdot)$ given x_i. In the parametric framework of the linear regression model when $g(x_i) = \beta(\theta)x_i$ the quantile regression was proposed as a solution of

$$\min_{\beta \in R^p} \frac{1}{n} \sum_{i=1}^{n} \rho_\theta (y_i - x_i \beta) \tag{2}$$

where $\rho_\theta(z) = |\theta - I(z < 0)| \cdot |z|$, I is the indicator function [1].

The conditional quantile τ of y_i given x_i, by monotonicity of quantile function,

$$Q(\theta|x) = g(x) + D^{-1}(\theta|x) \equiv g_\tau(x) \tag{3}$$

where $D^{-1}(\tau|x)$ is conditional θ^{th} quantile of error term ε_i and $Q(\theta|x) \equiv \inf\{\lambda : \ P(y_i \le \lambda|x) \ge \theta\}$. In equation (3) g(x) and $D^{-1}(\theta|x)$ are not identified separately. However $g_\theta(x)$, the conditional θ^{th} quantile can be identified, then the equation(1) can be rewritten as

$$y_i = g_\theta(x_i) + v_i \tag{4}$$

where

$v_i = \varepsilon_i - D^{-1}(\theta|x)$ and v_i is a new error term which has a zero conditional quantile.

Given $(y_i; x_i)$, the quantile model can be estimated by regression quantiles, which are defined by the minimization problem,

$$\beta^*(\theta) = \min_{b \in R} \left\{ \sum_{y_i \ge x_i b} w_i (\theta |y_i - x_i b|) + \sum_{y_i < x_i b} w_i (1-\theta) |y_t - x_t b| \right\} \tag{5}$$

where the weights w_i are introduced to account for different variability of x_i and the different number of observations at each x_i.

There is no explicit solution for the regression coefficients under this parametric quantile regression model since the check function is not differentiable at the origin. However, using recent advances in interior point methods for solving linear programming problems discussed by Portnoy and Koenker (1997),this

[1] I[A] = 1 if A is true, I[A] = 0 otherwise.

minimization can be performed by using the algorithm that was provided by Koenker and D'Orey (1987).

3.2. The Box–Cox transformation quantile model

Let *y* denote response variable and ***x*** a vector of *k* covariates representing industry attributes. For θ in (0,1), the θ^{th} quantile of the conditional distribution of y given x, is defined as

$$Q_\theta(y|x) = \inf\{y|F(y|x) \geq \theta\}$$

where $F(\cdot\,|x)$ denotes the conditional distribution function.

The statistical model used in this paper specifies the θ^{th} conditional quantile of *y* given *x* as the inverse of the Box-Cox power transformation (Box and Cox, 1964) of an affine function of the covariates,

$$Q_\theta(y|x) = g(x'(\beta(\theta), \lambda(\theta)) \tag{6}$$

where

$$g(t,\lambda) = \begin{cases} (1+\lambda t)^{1/\lambda} & for\ \lambda \neq 0 \\ e^t & for\ \lambda = 0 \end{cases} \tag{7}$$

Model (6) is quite flexible since not only the coefficients β but also the whole transformation may change from quantile to quantile. Of course, the case where $\lambda = 1$ yields the linear model for the conditional quantiles.

By analogy with the linear model, the population quantile regression parameters may be defined as

$$\gamma_j(\theta,\bar{x}) = \partial Q_\theta(y|\bar{x})/\partial x_j = g_1(\bar{x}'\beta(\theta), \lambda(\theta))\beta_j(\theta)\, , j = 1,\ldots, k$$

where x denotes the vector of the regressors' sample means and $g_1(\cdot,\cdot) \equiv \partial g(t,\lambda)/\partial t$ The estimation of these regression quantiles for values of y in (0,1) constitutes the main aim of this study as they describe the relevancy of covariates at different points of response variable distribution.

3.3. Inference procedures for Box-Cox quantile regression model

The estimation of model (6) is based on an equivariance property of the quantile regression to monotonic transformations of the dependent variable and follows Chamberlain (1994).

Specifically, making z(λ) =$g^{-1}(y, \lambda)$ where $g^{-1}(\cdot,\cdot)$ is the Box-Cox transformation, the specification (1) implies that the quantiles of z are linear, i.e.

$$Q_\theta(z|x) = x'\beta(\theta)$$

Therefore, for given λ, $\beta(\theta)$ can be estimated by minimizing in β (Koenker and Bassett, 1978),

$$\frac{1}{n}\sum_{i=1}^{n}\rho_\theta(z_i - x_i'\beta) \tag{8}$$

with

$$\rho_\theta(u) = \begin{cases} \theta u & for\, u \geq 0 \\ (\theta-1)u & for\, u < 0 \end{cases}$$

Hence, for any given λ, model (1) can be estimated exactly in the same way as a standard linear quantile regression. Of course, the usual mean regression does not have this property unless λ = 1.

Denote by $\hat{\beta}(\theta,\lambda)$ a solution of model (8). Chamberlain (1994) suggested estimating $\lambda(\theta)$ by minimizing in λ:

$$\frac{1}{n}\sum_{i=1}^{n}\rho_\theta(y_i - g(x_i'(\hat{\beta}(\theta,\lambda),\lambda)) \tag{9}$$

Finally, $\beta(\theta)$ in model (1) is estimated by $\beta(\theta) = \beta(\theta, \lambda(\theta))$. We proceeded by solving model (8) for a grid of values of l and then choosing the pair ($\lambda,\beta(\theta)$) that yields the smallest value for model (9).

Under regularity conditions, it can be shown that the joint distribution of $\hat{\alpha}(\theta) \equiv (\hat{\beta}(\theta)', \hat{\lambda}(\theta))$ for m values of θ in (0,1),

$$\sqrt{n}(\hat{\alpha}(\theta_1)' - \alpha(\theta_1)',\ldots,\hat{\alpha}(\theta_m)' - \alpha(\theta_m)')'$$

will converge to a $m \times (k + 1)$-variate normal distribution, with 0 mean and covariance matrix whose j^{th} block is given by

$$V(\theta_j, \theta_l) = H(\theta_j)^{-1} L(\theta_j, \theta_l) H(\theta_l)^{-1'} \tag{10}$$

with

$$H(\theta) = A(\theta) E[f_{u(\theta)_i}(0) d(x_i, \alpha(\theta)) d_2(x_i, \alpha(\theta))'] \tag{11}$$

$$L(\theta_j, \theta_l) = (\min\{\theta_j, \theta_l\} - \theta_j \theta_l) A(\theta_j) E[d(x_i, \alpha(\theta_j)) d_2(x_i, \alpha(\theta_l))'] A(\theta_l)' \tag{12}$$

where $f_{u(\theta)}(\cdot|x)$ denotes the density of $u(\theta) \equiv y_i - g(x_i'\beta(\theta), \lambda(\theta))$ given $x, d(x_i, \alpha(\theta))' = (x_i' g_{1i} x_i' g_{2i}) \equiv (x_i' d_2(x_i, \alpha(\theta)))$, $g_{1i} \equiv g_1(x_i'\beta(\theta), \lambda(\theta))$, $g_{2i} \equiv g_2(x_i'\beta(\theta), \lambda(\theta))$

$$g_1(\cdot,\cdot) \equiv \partial g(t, \lambda) / \partial t \text{ and } g_2(\cdot,\cdot) \equiv \partial g(t, \lambda) / \partial t$$

with

$$A(\theta) = \begin{bmatrix} I_k & 0_k 0_k' & 0_k \\ 0_k' & \partial \beta'(\theta, \lambda) / \partial \lambda & 1 \end{bmatrix}$$

a $(k + 1) \times (2k + 1)$ matrix where

$$\frac{\partial \beta(\theta, \lambda)}{\partial \lambda} = -[E f_{u_i}(0) g_{1i} x_i x_i']^{-1} [E f_{u_i}(0) g_{2i} x_i]$$

A rigorous treatment of this derivation may be found in Powell (1991). Buchinsky (1995) develops the theory of the Box-Cox quantile regression for the case of discrete regressors where the estimation of $\hat{\alpha}(\theta)$ can be accomplished by minimum distance methods.

Interval inferences for the quantile regression parameters require the consistent estimation of the asymptotic covariance matrices (10). The critical feature of this method is the nonparametric estimation of $f_{u(\theta)_i}(\cdot|x)$ in (11) based on the histogram method of Siddiqui (1960). Alternatively to this type of estimator, one could have considered the bootstrap estimation of the asymptotic covariance matrix $V(\theta)$ as did Chamberlain (1994), for the linear model with independent errors, and Buchinsky (1994), also for the linear model but with general errors.

The theoretical basis for bootstrapping quantile regression estimators are provided in Hahn (1995) and Fitzenberger (1998). Monte Carlo comparisons in Koenker (1994) suggest that in i.i.d. situations the sparsity estimator fares better than does the bootstrap.

IV. QUANTILE REGRESSION FOR TIME SERIES

Most research in quantile regression has assumed that the observations of the response variable *Y* are conditionally independent. Recently, several researchers have discussed different methods for time series quantile regression modelling. For example,a method based on estimating the conditional distribution is given by Cai (2002),w hereas a method based on the check function is given by Gannoun *et al.* (2003). In the method of Cai (2002),the time series Y_i is assumed to be related to the time series X_i through the model

$$Y_i = \mu(X_i) + \sigma(X_i)\varepsilon_i$$

where $\mu(X_i)$ is the regression function and ε_i is the model error. The dependence of $\sigma(X_i)$ on X_i means that the model is heteroscedastic. The method first estimates the conditional distribution of Y_i given X_i and then estimates the condition quantile by the inverse of the conditional distribution function. In the method of Gannoun *et al.* (2003) for the estimation of the conditional quantile of a strictly stationary real-valued process Z given the present and past records, the quantile of *Z* is characterized as

$$q_\theta(x) = \arg\min_{\theta \in R}\{E[\rho_\theta(Z-\theta)|X = x]\}$$

4.1. Quantile regression as a risk measure

We should solve a problem of finding an minimum of coherent risk measures, which is equivalent to find a maximum of Choquet expected value using linear form of the utility function and a concave distortion function v_α.

When we write quantile regression problem in general case we have a problem of estimations a vector of unknowns parameters ***b***, for a sample of independent observations form a random variables $Y_1, Y_2, \ldots, Y_T$ according to rule:

$$\mathrm{P}(Y_t < y) = \mathrm{F}(y - x_t\boldsymbol{b}), \ t=1,\ldots T \qquad (12)$$

where $\{x_{t,}\ t = 1,\ldots T\}$ is a row in know matrices of observations (size $T \times K$) and distribution of F is unknown (Trzpiot 2007).

Given $(y_t; x_t)$, for t = $1,\ldots T$, the quantile model can be estimated by regression quantiles, which are defined by the minimization problem:

$$\min_{\beta\in\mathbf{R}}\left\{\sum_{t\in\{t:\mathbf{y_t}\geq\beta\}}\alpha\left|\mathbf{y_t}-\beta\right|+\sum_{t\in\{t:\mathbf{y_t}<\beta\}}(1-\alpha)\left|\mathbf{y_t}-\beta\right|\right\}. \quad (13)$$

Writing as $\{x_{t,}\ t = 1,\ldots T\}$ sequence of K vectors (rows) of observation matrices, we assume, that $\{y_{t,}\ t = 1,\ldots T\}$ is a random sample of regression process: $u_t = y_t - x_t b$ having distribution F. Then regression quantile α, for $0 < \alpha < 1$ is done as a solution of a problem:

$$\min_{\beta\in\mathbf{R}}\left\{\sum_{t\in\{t:\mathbf{y_t}\geq\mathbf{x_t}\beta\}}\alpha\left|\mathbf{y_t}-\mathbf{x_t}\beta\right|+\sum_{t\in\{t:\mathbf{y_t}<\mathbf{x_t}\beta\}}(1-\alpha)\left|\mathbf{y_t}-\mathbf{x_t}\beta\right|\right\}. \quad (14)$$

If $K = 1$ and $x_t = 1$for all t, a problem (14) can reduce to problem (13). The smallest absolute error is then equales to median. The problem (14) always hale a solution, for a continuous distribution his solution is unique.

The problem of finding minimum can be reformulated as equivalent linear programming problem:

$$\min\{\alpha\mathbf{1}'\mathbf{r}^{+}+(1-\alpha)\mathbf{1}'\mathbf{r}^{-}\} \quad (15)$$

where

$$\mathrm{y}=\mathbf{X}\mathrm{b}+\mathbf{r}^{+}+\mathbf{r}^{-}$$
$$(\mathrm{b},\mathbf{r}^{+},\mathbf{r}^{-})\in\mathrm{R}^{K}\times\mathrm{R}_{+}^{2T}$$

where $\mathbf{1}$ is a unity vector of size T.

V. FINAL REMARKS

Quantile regression is emerging as a comprehensive approach to the statistical analysis of linear and non-linear response models, partly because classical linear theory is essentially a theory just for models of conditional expectations. We have illustrated that quantile regression has strong links to three very useful statistical concepts: regression, robustness and extreme value theory. We try to demonstrate that quantile regression is widely used in many important application areas, such as medicine and survival analysis, financial and economic statistics and environmental modeling.

REFERENCES

Altman, N. S. (1990) Kernel smoothing of data with correlated errors. *J. Am. Statist. Ass.*, 85, 749–759.

Bassett G.W., Chen H. (2001) Portfolio style: return-based attribution using quantile regression. *Emp. Econ.*, 26: 293–305.

Box G., Cox D. 1964. An analysis of transformations revisited. Journal of the Royal Statistical Society, Series B 26: 211–252.

Buchinsky M. (1995) Quantile regression, Box-Cox transformation model and the U.S. wage structure,1963– 1987. *J. Econometr.*, 65: 109–154.

Cai Z. (2002) Regression quantiles for time series. *Econometr. Theory*, 18, 169–192.

Chamberlain G. (1994). Quantile regression, censoring and the structure of wages. In Advances in Econometrics, Sims C (eds), Cambridge University Press: New York; 171–209.

Cole T. J. (1988). Fitting smoothed centile curves to reference data (with discussion). *J. R. Statist. Soc.* A, 151: 385–418.

Cole T. J. , Green P. J. (1992) Smoothing reference centile curves: the LMS method and penalized likelihood. *Statist. Med.*, 11: 1305–1319.

Crowley J., Hu M. (1977) Covariance analysis of heart transplant data. *J. Am. Statist. Ass.*, 72: 27–36.

Fitzenberger B. (1998). The moving blocks bootstrap and robust inference for linear least squares and quantile regressions. Journal of Econometrics 82: 235–287.

Gannoun A., Saracco J., Yu K. (2003) Nonparametric prediction by conditional median and quantiles. *J. Statist. Planng Inf.*, to be published.

Hahn J. (1995). Bootstrapping quantile regression estimators. Econometric Theory 11: 105–121.

Hendricks,W. , Koenker,R. (1992) Hierarchical spline models for conditional quantiles and the demand for electricity. *J. Am. Statist. Ass.*, 93: 58–68.

Hull J., White A. (1998) Value at risk when daily changes in market variables are not normally distributed. *J. Deriv.*, 5: 9–19.

Koenker R. 1994. Confidence intervals for regression quantiles. In Asymptotic Statistics: Proceedings of the 5th Prague Symposium, Mandl P, Huskova M (eds), Physica-Verlag: Berlin.

Koenker R. and Geling R. (2001) Reappraising med fly longevity: a quantile regression survival analysis. *J. Am. Statist. Ass.*, 96: 458–468.

Koenker R., Machado J. (1999) Goodness of fit and related inference processes for quantile regression. *J. Am. Statist. Ass.*, 94: 1296–1309.

Koenker R., Park B.J. (1996). An interior point algorithm for nonlinear quantile regression. *J. Econometr.*,71: 265–283.

Koenker R. W., D'Orey V. (1987) Algorithm AS 229: Computing regression quantiles. *Appl. Statist.*, 36: 383–393.

Koenker R., Portnoy S., Ng P. (1992) *Nonparametric Estimation of Conditional Quantile Functions:*L1 *Statistical Analysis and Related Methods* (ed. Y. Dodge),pp . 217–229. Amsterdam: Elsevier.

Kottas A., Gelfand A. E. (2001) Bayesian semiparametric median regression model. *J. Am. Statist. Ass.*, 96: 1458–1468.

Lauridsen S. (2000) Estimation of value of risk by extreme value methods. *Extremes*, 3: 107–144.

Portnoy S. and Koenker R. (1997) TheGaussian hare and the Laplacian tortoise: computability of squared-error versus absolute-error estimators (with discussion). *Statist. Sci.*, 12: 279–300.

Powell J. (1991). Estimation of monotonic regression models under quantile restrictions. In Nonparametric and Semiparametric Methods in Econometrics and Statistics: Proceedings of the Fifth International Symposium on Economic Theory and Econometrics, Barnett W., Powell J., Tauchen G (eds), Cambridge University Press: New York; 357–384.

Royston P., Altman D.G. (1994) Regression using fractional polynomials of continuous covariates: parsimonious parametric modelling (with discussion). *Appl. Statist.*, 43: 429–467.

Royston P., Wright E. M. (2000) Goodness-of-fit statistics for age-specific reference intervals. *Statist. Med.*, 19: 2943–2962.

Taylor J. (1999) A quantile regression approach to estimating the distribution of multiperiod returns. *J. Deriv.*, 24: 64–78.

Trzpiot G. (2007) *Regresja kwantylowa a estymacja VaR*, Prace Naukowe AE Wrocław, 1176, Wrocław.

Trzpiot G. (2009). Application weighted VaR in capital allocation, Polish Journal of Environmental Studies, Vol 18, No. 5B: 203–208

Trzpiot G. (2009). Estimation methods for quantile regression, Studia Ekonomiczne 53: 81–90, Zeszyty Naukowe AE Katowice.

Yang S. (1999) Censored median regression using weighted empirical survival and hazard functions. *J. Am. Statist. Ass.*, 94: 137–145.

Grażyna Trzpiot

WŁASNOŚCI TRANSFORMACJI MODELU REGRESJI KWANTYLOWEJ

Przedstawiamy artykuł, w którym omawiamy modele regresji kwantylowej. Omawiamy motywacje dla stosowania klasycznego modelu, jak również główne kierunki zastosowań regresji kwantylowej. Następnie przechodzimy do transformacji podstawowego modelu. Ten model jest wprowadzony przez Powell'a (1991) a kolejno analizowany przez Chamberlain'a (1994) i Buchinsky'ego (1995), wprowadzono specyficzne warunkowe kwantyle znane jako transformacja *Box–Cox'a*. Omawiamy estymację modeli oraz testy istotności.