## A C T A U N I V E R S I T A T I S L O D Z I E N S I S FOLIA OECONOMICA 286, 2013

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# EXPERIMENTAL DESIGN IN EVALUATING VAR FORECASTS

**Abstract.** Following a dynamic development of VaR estimation methods from 90s, in recent literature much attention has been paid to testing procedures designed to evaluate quality of VaR models. There has been a wide-ranging discussion on both – statistical properties and empirical application of the two most popular tests, which are Kupiec test from 1995 that considers the ratio of VaR exceedances and Christoffersen autocorrelation test from 1998. We focused on autocorrelation property and compared Christoffersen test to Ljung Box test of 1978 and to the proposition of Engle and Mangianelli from 2004. The goal of the paper was to explore the design of experiments in the context of evaluating power of autocorrelation tests. We presented and contrasted simulation experiments proposed in the literature, indicated their design influence on the results and proposed a new scheme for power evaluating in autocorrelation tests.

**Key words:** VaR, experimental design, Monte Carlo, power of the test, correlation test, Kupiec test, Markov test, Ljung Box test, dynamic quantile test.

#### I. INTRODUCTION

Over last two decades, the concept of value at risk (VaR) has become very popular in risk valuation, especially in financial market. The idea of this measure is to give, in one number, information on the volume of risk that, in a given time interval, with a given probability will not be exceeded. Since the inception of VaR there has been a dynamic development in the area of model estimation and, on the other hand, statistical tests have been proposed to verify the quality of VaR forecasts in the light of historical process trajectories. These developments, in turn, created the need to assess properties of alternative testing procedures.

Many alternative approaches have been proposed, referring to the design of experiments used in VaR tests evaluation. The way of conducting simulation studies in this context is broadly discussed in the literature. Moreover, the design of such experiments strongly influences practical conclusions referring to specific tests.

The aim of this paper was to provide an overview of simulation experiments proposed in the literature for evaluating VaR tests and to present own proposition of an experimental design. For illustrative purposes, the proposed

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experiment was used to provide an analysis of properties of Kupiec [1995] and Markov [1998] tests, as most often used for evaluating VaR forecasts. These popular tests were compared to Ljung Box test of 1978 and a dynamic quantile test proposed by Engle and Mangianelli in 2004.

#### II. VAR BACKTESTING PROCEDURES

The statistical inference referring to VaR models evaluation is based on the stochastic process of VaR exceptions:

$$I_{t+1} = \begin{cases} 1, & r_{t+1} < VaR_t(p) \\ 0, & r_{t+1} \ge VaR_t(p) \end{cases}$$
 (1)

where p – given tolerance level,  $r_t$  – value of the rate of return at time t,  $VaR_t(p)$  – value of the VaR forecast from moment t.

The Kupiec test form 1995 is designed to assess the unconditional coverage property i.e. to verify whether the unconditional probability of exceeding VaR matches the assumed tolerance level p. To check this property the empirical rate of exceptions is utilized. The null hypothesis  $H_0: \pi_1 = p$  is tested by the following statistic:

$$LR_{uc} = -2\log\frac{p^{t_1}(1-p)^{t_0}}{\hat{\pi}_1^{t_1}(1-\hat{\pi}_1)^{t_0}} \sim {}_{as}\chi_{(1)}^2,$$
 (2)

where  $\pi_1$  – unconditional probability of exceeding VaR forecast,  $t_0$  – number of non-exceptions,  $t_1$  – a number of exceptions,  $\hat{\pi}_1 = \frac{t_1}{t_0 + t_1}$ .

A dynamically developing group of tests, used to evaluate VaR forecasts, are tests of serial correlation in VaR failures. The idea behind these tests is to detect clusters of exceptions. They check therefore, whether the conditional probability of VaR failure is constant in time and equal to the assumed tolerance level (conditional coverage property). In Ljung-Box test of 1978, the null is formulated in terms of correlation coefficients between VaR exceptions,  $H_0: \gamma_k = 0, \ k = 1...K$ , and the test statistic has a following form:

$$LB = t(t+2)\sum_{k=1}^{K} \frac{\hat{\gamma}_k^2}{t-k} \sim {}_{as}\chi_{(K)}^2,$$
 (3)

where t-number of observations,  $\gamma_k$ - correlation coefficient of order k between VaR exceptions [Berkowitz, Christoffersen, Pelletier 2011].

In 1998, in order to test for serial correlation, Markov test was proposed [Christoffersen], in which the assumption is used that the process (1) forms a part of a Markov chain. The null hypothesis, formulated in terms of conditional probabilities of a single-step transition,  $H_0: \pi_{01} = \pi_{11}$ , is verified by the statistic

$$LR_{ind} = -2\log \frac{\hat{\pi}_{1}^{t_{1}}(1-\hat{\pi}_{1})^{t_{0}}}{\hat{\pi}_{01}^{t_{01}}(1-\hat{\pi}_{01})^{t_{00}}\hat{\pi}_{11}^{t_{11}}(1-\hat{\pi}_{11})^{t_{10}}} \sim {}_{as}\chi_{(1)}^{2},$$

$$\tag{4}$$

where  $\pi_{ij}$  -probability of reaching the state j at time t+1 on condition that at time t the process was at the state i,  $t_{ij}$  - number of transitions form the state i to

the state 
$$j, \ \hat{\pi}_{01} = \frac{t_{01}}{t_0}, \ \hat{\pi}_{11} = \frac{t_{11}}{t_1}$$
 .

In the dynamic quantile test (DQ test), Engle and Mangianelli [2004] suggested using a stochastic process of the form  $Hit_t = I_t - p$ . The test is based on the regression  $Hit_t = X\beta + \varepsilon$ , where X – matrix of m exogenous variables, particularly a constant, lags of  $I_t$  and VaR forecasts for time t. The testing procedure is based on checking statistical significance of the above regression, i.e. hypothesis  $H_0: \beta = 0$  is verified by the statistic

$$DQ = \frac{\hat{\beta}X'X\hat{\beta}}{p(1-p)} \sim {}_{as}\chi^{2}_{(m)}$$
 (5)

where  $\hat{\beta}$  – MNK estimate of the vector of parameters in regression of the variable  $Hit_t$ .

#### III. THE DESIGN OF SIMULATION EXPERIMENTS

The null hypothesis in Kupiec test assumes the unconditional probability of VaR exception at the fixed level p. The size properties are assessed through generating T independent Bernoulli trials with the probability of success in a single trial set to p, which equals the assumed tolerance level. In the presented study the tolerance was set to a 5% level. A standard procedure to obtain

<sup>&</sup>lt;sup>1</sup> In this paper 5 lags of the variable  $I_t$  were included.

required VaR regressors in DQ test is to generate VaR estimates from GARCH(1,1)-normal process with innovations independent of the variable  $I_t$ , which ensures statistical insignificance of the regression. Such procedure gives the sequence of outcomes of the process  $\{I_t\}$  that stays in line with the hypothesis of the unconditional probability of VaR exception at level p as well as with the hypothesis of lack of serial correlation in exceptions. The model can be thus used to evaluate the size properties of correlation tests as well.

A simple technique of designing an experiment to evaluate the power of Kupiec test is to generate the process of VaR exceptions from the Bernoulli model with exception probabilities different than p. A procedure that may be used to assess the impact that serial correlation has on the power of Kupiec test is generating the trajectory of return process from GARCH(1,1)-normal model and getting VaR estimates from a homoscedastic model. Through a variance manipulation it can be ensured that the rate of exceptions is different than p. In this paper, we conducted an experiment in 8 variants, assuming the Bernoulli model for VaR exceptions with probabilities  $\{0,07,0,08,0,09,0,10,0,04,0,03,0,02,0,01\}$ .

In the literature, many methods of designing experiments to assess power properties of correlation tests are discussed. Some authors suggest that data on return process may be obtained form a normal distribution with a constant variance or from a t distribution with a constant number of degrees of freedom. Than, to match the alternative hypothesis, VaR forecasts should be obtained from models with higher or lower variance, from different distributions, from heteroscedastic models or from the historical simulation model [Lopez 1999].

Such a wide variety of models used in experimental design in the context of evaluating power of VaR tests stimulated the discussion in the literature that has led to some general postulates. According to them, the data generating process for the rate of return should satisfy some widely recognized facts about financial processes. Firstly, it should exhibit a great variation of the process from one point in time to the next. Moreover a persistence of the process, which results in clusters of low and high volatility, should be assumed. The above two postulates are satisfied by the family of GARCH models, which are therefore used to generate the trajectory of the return process. Then a variety of models may be used to get VaR estimates, so that the resulting VaR exceptions process would match the alternative hypothesis. In the simplest case, a homoscedastic VaR model is assumed. A more advanced approach is to include variation of the variance in the VaR model, assuming a false scheme for generating variance values. For instance variance values may come from a weighted moving average model or from the historical simulation model. Some authors also consider models that systematically under report variance. Typical levels of under

reporting range from 5% to 25% [Campbell 2005]. In a number of studies it is assumed that the true data generating process for the rate of return is a GARCH process and, following market practice, VaR estimates are obtained by the historical simulation method [Christoffersen 2004, Berkowitz, Christoffersen and Pelletier 2011].

Historical simulation used to generate VaR estimates in a simulation experiment does not, however, ensure a clear-cut interpretation of the results of the study. The technique of historical simulation, not being a theoretical model which parameters can be controlled, does not satisfy basic assumptions of Monte Carlo experiments and can only be treated as an estimation method. A presumption that, with the use of this method a model that matches the alternative hypothesis will be obtained is not equivalent to assuming a theoretical model of an experiment. Moreover, such a presumption is equivalent to a negative evaluation of historical simulation as a method that implies serial correlation in a resulting process.

In the presented study, following stylized facts about financial time series, we assumed that the model used to assess the power properties of correlation tests should be generated in a way that guarantees volatility clustering. The return series was obtained from GARCH(1,1)-normal model with parameters  $\omega = 0.000001$ ,  $\alpha = 0.14$  and  $\beta = 0.85$ . Serial correlation in VaR exceptions was ensured by generating VaR estimates from the model that incorrectly described the mechanism behind volatility clustering. It was also assumed that the rate of VaR exceedances should be close to the assumed 5% in order to reduce the influence of unconditional coverage property on evaluation of correlation tests properties. In this paper we suggested an experiment where VaR forecasts are generated in 12 variants with the use of GARCH models with given parameters (Table 1). In 8 models of the experiment the parameters  $\alpha$  and  $\beta$ were modified in such a way that their sum was at a fixed level implying a constant unconditional variance of the process. In 4 of them the persistence of the process was increased by assuming higher values of  $\beta$  parameter and in subsequent 4 variants the procedure was reversed. In the last 4 models a higher value of  $\omega$  parameter was assumed, which gave models closer to a homoscedastic assumption. In order to ensure a constant unconditional variance the sum  $\alpha + \beta$  was modified, leaving the relation between  $\alpha$  and  $\beta$ unchanged. VaR estimates implied by the null model and the 12 models following alternative hypothesis in correlation tests are presented in Figures 1 to 13. The graphical presentation shows that the highest power should be observed in models 4, 8 and 12 as there is the largest discrepancy between the true and alternative model. Rejection rates in all experiments were calculated over 10000 Monte Carlo trials for sample sizes T = 250, 500,...,1500.

Table 1 Parameters	of simulation	experiment used to as	sess nower prope	erties of correlation tests

	ω	α	β		ω	α	β		ω	α	β
Model 1	0.000001	0.105	0.885	Model 5	0.000001	0.19	0.8	Model 9	0.000005	0.133	0.817
Model 2	0.000001	0.07	0.92	Model 6	0.000001	0.24	0.75	Model 10	0.00001	0.126	0.774
Model 3	0.000001	0.035	0.955	Model 7	0.000001	0.29	0.7	Model 11	0.00005	0.07	0.43
Model 4	0.000001	0	0.99	Model 8	0.000001	0.34	0.65	Model 12	0.0001	0	0

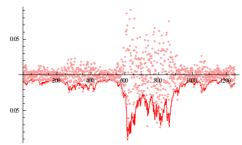


Fig. 1. VaR estimates in the true data generating process

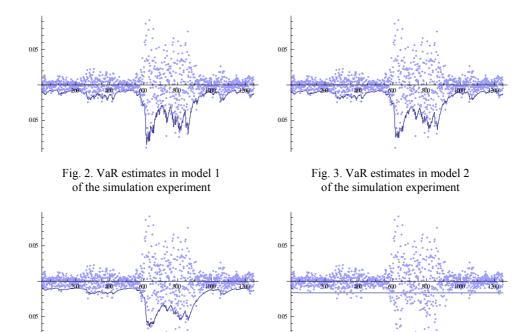


Fig. 4. VaR estimates in model 3 of the simulation experiment

Fig. 5. VaR estimates in model 4 of the simulation experiment

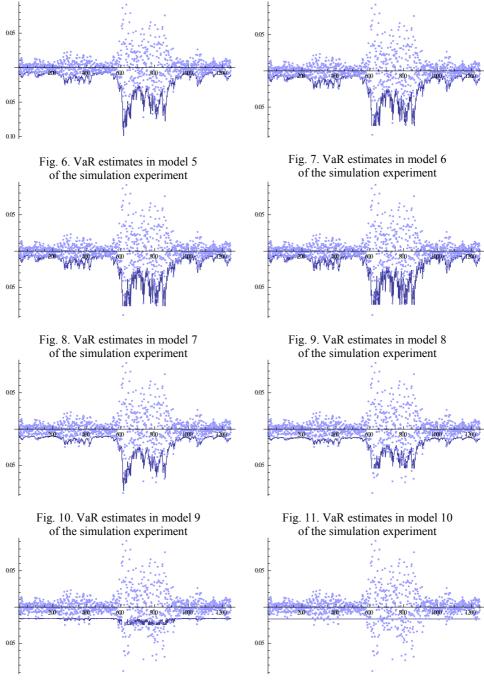


Fig. 12. VaR estimates in model 11 of the simulation experiment

Fig. 13. VaR estimates in model 12 of the simulation experiment

#### IV. SIMULATION RESULTS

For illustrative purposes, the proposed experiment was used to provide an analysis of properties of Kupiec [1995] and Markov [1998] tests, as most often used for evaluating VaR forecasts. These popular tests were compared to Ljung Box test of 1978 and a dynamic quantile test proposed by Engle and Mangianelli in 2004. The conducted experiment showed the largest difference between estimated size of the test and the assumed significance level for Markov test. Three other tests exhibited similar outcomes for similar series lengths, which were all lower than in case of Markov test. For series of over 500 observations a stable level of Kupiec, LB and DQ estimated test sizes was observed, close to the nominal value of 5%.

Table 2. Size of the VaR tests

	Series length						
	250	500	750	1000	1250	1500	
Test Kupca	0.0637	0.0597	0.0487	0.0573	0.0573	0.0573	
Test Markowa	0.0896	0.0950	0.0788	0.0913	0.0913	0.0913	
Test LB	0.0627	0.0593	0.0493	0.0513	0.0513	0.0513	
Test DQ	0.0700	0.0563	0.0550	0.0570	0.0570	0.0570	

Source: Author's calculations.

Table 3. Power of the Kupiec test

	Series length						
	250	500	750	1000	1250	1500	
Model 1	0.298	0.461	0.655	0.748	0.841	0.901	
Model 2	0.537	0.765	0.924	0.968	0.989	0.997	
Model 3	0.743	0.935	0.992	0.999	1.000	1.000	
Model 4	0.879	0.988	1.000	1.000	1.000	1.000	
Model 5	0.132	0.221	0.265	0.352	0.363	0.483	
Model 6	0.372	0.663	0.810	0.913	0.944	0.981	
Model 7	0.762	0.974	0.998	1.000	1.000	1.000	
Model 8	0.985	1.000	1.000	1.000	1.000	1.000	

Source: Author's calculations.

The power of Kupiec test exceeded 70% for all sample sizes in experiments where the rate of exceedances was assumed at levels over 8% (models 3 and 4) or below 3% (models 7 and 8). For smaller differences, around 2 p.p., in rate of exceedances in comparison to the true data generating process (models 1 and 6), the sample size had large influence on the power of the test (Fig. 14-15). Power exceeding 70% was achieved for series of 1000 observations or longer.

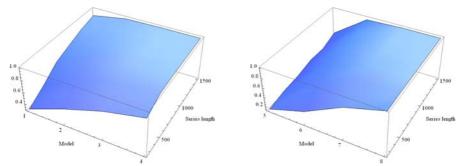


Fig. 14. Power of the Kupiec test in models 1-4 Fig. 15. Power of the Kupiec test in models 5-8

The power of Markov test estimated in the simulation study was at relatively low level in all variants of the experiment (Table 4). For models 4, 8 and 10, which exhibited largest discrepancies in comparison to the true model, in case of series shorter than 1000 observations, the power did not exceed 60% (Fig. 16-18).

	Series length							
	250	500	750	1000	1250	1500		
Model 1	0.108	0.101	0.094	0.091	0.096	0.104		
Model 2	0.145	0.132	0.139	0.137	0.150	0.169		
Model 3	0.213	0.214	0.242	0.257	0.299	0.332		
Model 4	0.393	0.499	0.591	0.654	0.707	0.763		
Model 5	0.074	0.067	0.066	0.069	0.093	0.113		
Model 6	0.072	0.065	0.071	0.099	0.144	0.173		
Model 7	0.072	0.072	0.085	0.154	0.214	0.256		
Model 8	0.076	0.081	0.129	0.233	0.307	0.361		
Model 9	0.119	0.098	0.092	0.095	0.102	0.104		
Model 10	0.152	0.126	0.141	0.149	0.149	0.155		
Model 11	0.246	0.307	0.389	0.442	0.489	0.539		
Model 12	0.388	0.497	0.590	0.654	0.708	0.763		

Table 4. Power of the Markov test

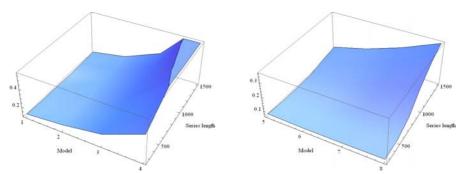


Fig. 16. Power of the Markov test in models 1-4 Fig. 17. Power of the Markov test in models 5-8

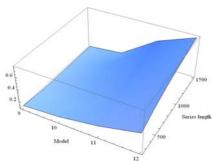


Fig. 18. Power of the Markov test in models 9-12

In experiments where parameters  $\beta$  (models 1-4) and  $\omega$  (models 9-12) were increased in order to obtain higher variance persistence or to move towards a homoscedastic model, LB test much outperformed Markov test. (Table 5). In models 4 and 12, for series of 500 observations the estimated power was close to 70%, while with extending the series the power approached 96% (Fig. 19-21).

All conducted experiments showed superiority of the dynamic quantile test of Engle and Mangianelli over other correlation tests in terms of statistical properties (Table 6). This was especially noticeable in the experiment where the value of  $\alpha$  parameter was increased, decreasing at the same time the variance persistence (models 5-8). For other tests the power estimated in experiment 8 did not exceed 40%, whereas in case of DQ test it reached the level of over 50% for the series of 500 observations and increased to over 95% for longer series. The comparison of simulation results for different sample sizes gave the observation that the power of DQ test grows rapidly with lengthening series over 250 observations (Fig. 22-24).

Table 5. Power of the LB test

	Series length							
	250	500	750	1000	1250	1500		
Model 1	0.092	0.103	0.096	0.102	0.101	0.117		
Model 2	0.165	0.197	0.218	0.248	0.268	0.299		
Model 3	0.285	0.382	0.485	0.559	0.619	0.678		
Model 4	0.456	0.696	0.817	0.881	0.933	0.958		
Model 5	0.032	0.032	0.026	0.029	0.026	0.031		
Model 6	0.022	0.024	0.022	0.030	0.037	0.048		
Model 7	0.018	0.023	0.025	0.041	0.061	0.086		
Model 8	0.020	0.024	0.037	0.062	0.110	0.145		
Model 9	0.132	0.144	0.159	0.175	0.182	0.209		
Model 10	0.176	0.240	0.295	0.339	0.371	0.434		
Model 11	0.356	0.597	0.724	0.810	0.881	0.916		
Model 12	0.452	0.694	0.816	0.880	0.934	0.958		

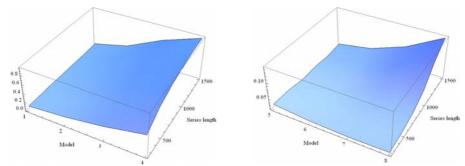


Fig. 19. Power of the LB test in models 1-4

Fig. 20. Power of the LB test in models 5-8

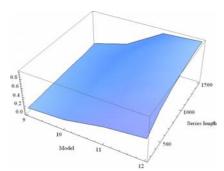


Fig. 21. Power of the LB test in models 9-12

Series length 250 500 750 1000 1250 1500 Model 1 0.078 0.073 0.076 0.089 0.070 0.071 Model 2 0.1100.134 0.1630.214 0.2660.320 Model 3 0.196 0.335 0.507 0.653 0.768 0.846 0.997 0.999 Model 4 0.373 0.873 0.9620.992 Model 5 0.0940.0950.099 0.1170.133 0.146Model 6 0.154 0.2080.261 0.336 0.396 0.457Model 7 0.240 0.785 0.373 0.501 0.623 0.711 Model 8 0.356 0.572 0.729 0.841 0.908 0.953 0.630 Model 9 0.084 0.208 0.361 0.508 0.718 Model 10 0.1460.479 0.711 0.8460.925 0.957 Model 11 0.324 0.8440.959 0.990 0.997 0.999 Model 12 0.326 0.836 0.944 0.985 0.996 0.998

Table 6. Power of the DQ test

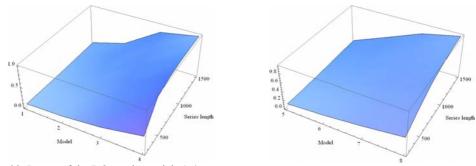


Fig. 22. Power of the DQ test in models 1-4

Fig. 23. Power of the DQ test in models 5-8

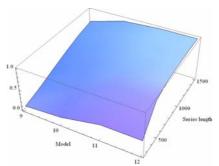


Fig. 24. Power of the DQ test in models 9-12

#### V. SUMMARY AND CONCLUSIONS

In the context of growing popularity of risk valuation methods based on VaR measure, there is a need to assess statistical properties of tests used to compare and evaluate alternative VaR models. Since the presence of clusters in VaR exceptions bears substantial risk in business management, a group of correlation tests develops very dynamically. In the literature a number of alternative approaches to designing experiments used to evaluate power properties of correlation tests are presented. In this paper we provided a review of such simulation experiments and presented a discussion on them in the context of the assumptions of Monte Carlo method. We also demonstrated own proposition of an experiment.

We suggested an experiment where it was assumed that the return series is generated by the GARCH-normal model. Then, to obtain VaR estimates controlled disturbances were introduced to the values of model parameters, which resulted in VaR exceptions series following the alternative hypothesis. In each experiment the unconditional variance of the process was fixed at a constant level while the parameters implying lower or higher persistence of the

variance and its fluctuations in time were modified. As an extreme case, a homoscedastic model was considered, which gave the largest discrepancy in comparison to the true data generating process.

As a complementary study, the proposed experiment was used to provide an analysis of properties of Kupiec test and three correlation tests, namely Markov, Ljung Box and a dynamic quantile test of Engle and Mangianelli. The outcomes of the conducted experiment showed that Markov test of 1998, being the most popular tool for detecting serial correlation in VaR exceptions, exhibited the worst statistical properties, both in terms of size and power. According to the results of the study, Markov test was outperformed by traditionally applied in time series analysis Ljung Box test of 1978 and by DQ test of 2004. The lowest power of the tests was generally observed in models of the experiment where variance persistence was decreased in comparison to the true data generating process. In such situations DQ test gave better results than all other tests. Superiority of this test was also confirmed by the fast increase in the test power with lengthening the time series.

#### **LITERATURE**

- Berkowitz J., Christoffersen P., Pelletier D., Evaluating Value-at-Risk Models with Desk-Level Data "Management Science" 2011, 12(57), 2213-2227.
- Cheng W., Hung J., Skewness and leptokurtosis in GARCH-typed VaR estimation of petroleum and metal asset returns "Journal of Empirical Finance" 2010, 18, 160-173.
- Christoffersen P., Evaluating Interval Forecasts "International Economic Review" 1998, 39, 841-862.
- Christoffersen P., Pelletier D., *Backtesting Value-at-Risk: A Duration-Based Approach* "Journal of Financial Econometrics" 2004, 1(2), 84-108.
- Doman M., Doman R., Ekonometryczne modelowanie dynamiki polskiego rynku finansowego, Wydawnictwo Akademii Ekonomicznej w Poznaniu, Poznań 2004.
- Engle R., Manganelli S., CAViaR: Conditional Autoregressive Value at Risk by Regression Ouantiles "Journal of Business & Economic Statistics" 2004, 22, 367-381.
- Fiszeder P., Modele klasy GARCH w empirycznych badaniach finansowych, Wydawnictwo Naukowe Uniwersytetu Mikołaja Kopernika, Toruń 2009.
- Kupiec, P., Techniques for Verifying the Accuracy of Risk Measurement Models "Journal of Derivatives" 1995, 2, 174-184.
- Lopez J., Methods for Evaluating Value-at-Risk Estimates, FRBSF Economic Review 2, 1999, 3-17.
- McNeil A., Saladin T., *The Peaks over Thresholds Method for Estimating High Quantiles of Loss Distributions*, Proceedings of 28th International ASTIN Colloquium, 1997.
- Pipień M., *Wnioskowanie bayesowskie w ekonometrii finansowej*, Wydawnictwo Akademii Ekonomicznej w Krakowie, Kraków 2006.
- Sean D., A Review of Backtesting and Backtesting Procedures, Finance and Economics Discussion Series Divisions of Research & Statistics and Monetary Affairs Federal Reserve Board, Washington, D.C 2005.

#### Marta Małecka

#### PROJEKTOWANIE EKSPERYMENTÓW SYMULACYJNYCH W OCENIE PROGNOZ VAR

W ślad za dynamicznym rozwojem metod estymacji VaR, począwszy od lat dziewięćdziesiątych ubiegłego wieku, w literaturze pojawiła się obszerna dyskusja dotycząca możliwości testowania statystycznego w kontekście oceny modeli VaR. Z jednej strony powstało wiele prac odnoszących się do własności statystycznych dwóch najpopularniejszych testów – testu Kupca z 1995 roku, który bada udział przekroczeń VaR w szeregu i testu autokorelacji przekroczeń VaR Christoffersena z 1998 roku. Z drugiej strony istnieje bogata literatura dotycząca zastosowań rozważanych testów do empirycznych szeregów czasowych. W niniejszej pracy skoncentrowano się na analizie własności testów autokorelacji i porównano test Christoffersena do testów Ljunga Boxa z 1978 roku i testu Engla i Mangianelli'ego z 2004. Celem pracy było przedstawienie przeglądu eksperymentów symulacyjnych wykorzystywanych do badania mocy testów autokorelacji przekroczeń VaR w odniesieniu do założeń metody Monte Carlo oraz zaprezentowanie własnej propozycji eksperymentu.