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BOOTSTRAP CONFIDENCE INTERVALS FOR POPULATION MEAN IN THE CASE OF ASYMMETRIC DISTRIBUTIONS OF RANDOM VARIABLES

ABSTRACT. In the paper we present some chosen bootstrap methods of interval estimation of the population expectation for asymmetric distribution. We consider the standard bootstrap method, percentile method and *t*-bootstrap method. These methods can be used to estimate the expected value of asymmetric distribution for, both, small and large sample sizes. The analysis of the properties of bootstrap methods of interval estimation is performed by means of a simulation experiment.

Key words: bootstrap, confidence interval, asymmetric population.

I. INTRODUCTION

The population mean of random variable of asymmetric distribution can be estimated using information about real or estimated value of asymmetry distribution coefficient. The methods of this type are presented, among others, in: Rousson V., Choi E. (2003); Zhou X. H., Dinh P. (2004).

Other methods of interval estimation of population mean to be used in such situation are bootstrap methods. They belong to the group of simulation methods and their characteristic feature is approximating parameter of unknown random variable distribution on the basis of many values which are generated under specified distributions.

There are different bootstrap methods of interval confidence parameter estimator, for example, the standard bootstrap method (cp.: DiCiccio T., Efron B. (1996)), the percentile method and t-bootstrap method (cp.: Domański Cz., Pruska K. (2000)). These methods can be used to estimate population mean in asymmetric populations for, both, small and large sample sizes. The analysis of the properties of bootstrap interval estimation methods is made by means of a simulation experiment.

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II. CHOSEN METHODS OF INTERVAL ESTIMATION OF POPULATION MEAN

Let assume that we investigate a population with regard to random variable X with unknown distribution with population mean μ. Let $X_1,...,X_n$ be a non-complex sample drawn from this population, and $x_1,...,x_n$ be the realization of this sample.

To derive the bootstrap evaluation of parameter μ we generate N ($N \geq 1000$) values $x_1^*, x_2^*, ..., x_n^*$ from bootstrap distribution $P(X_B = x_k) = \frac{1}{n}$, for $k = 1, ..., n$. Values $x_1^*, x_2^*, ..., x_n^*$ are the realizations of bootstrap sample $X_1^*, X_2^*, ..., X_n^*$. For N replications we get N sequences $x_{1,i}^*, x_{2,i}^*, ..., x_{n,i}^*$, $i = 1,...,N$, which are bootstrap samples values.

Let $1-\alpha$ be an assumed confidence coefficient. Let us consider three chosen bootstrap methods of confidence interval for population mean:

I – standard method,

II – percentile method,

III – t-bootstrap method.

Applying the standard method of bootstrap interval estimation of population mean, for every sequence $x_{1,i}^*, x_{2,i}^*, ..., x_{n,i}^*$ we determine:

$$\overline{x}_{i,B} = \frac{1}{n}\sum_{k=1}^{n} x_{k,i}^* \quad i = 1, ..., N, \tag{1}$$

$$\overline{\overline{x}}_B = \frac{1}{N}\sum_{i=1}^{N} \overline{x}_{i,B}, \tag{2}$$

and

$$s_B = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\overline{x}_{i,B} - \overline{\overline{x}}_B\right)^2}. \tag{3}$$

The confidence interval is given by (cp.: Koronacki J., Michniczuk J., (2006)):

$$P\left(\overline{\overline{x}}_B - u_{1-\frac{\alpha}{2}} s_B < \mu < \overline{\overline{x}}_B + u_{1-\frac{\alpha}{2}} s_B\right) \approx 1-\alpha,, \tag{4}$$

where $\Phi\left(u_{1-\frac{\alpha}{2}}\right) = 1-\frac{\alpha}{2}$, and Φ is the cumulative distribution function (cdf) of the normal standardized distribution.

In bootstrap estimation based on percentiles, for every sequence $x_{1,i}^*, x_{2,i}^*, \ldots, x_{n,i}^*$, $i = 1, \ldots, N$, we compute value $\overline{x}_{i,B}$ which is an evaluation of parameter μ on the basis of the i-th bootstrap sample. In this way we receive a sequence of values $\left(\overline{x}_{i,B}\right)_{i=1,\ldots,N}$ On the basis of this sequence we determine the percentiles $\overline{x}_{i,B}^{\left(\alpha/2\right)}, \overline{x}_{i,B}^{\left(1-\alpha/2\right)}$ of the order $\frac{\alpha}{2}$ and $1-\frac{\alpha}{2}$, respectively. The confidence interval for the mean μ is the following (cp.: Domański Cz., Pruska K., (2000)):

$$P\left(\overline{x}_{i,B}^{\left(\alpha/2\right)} < \mu < \overline{x}_{i,B}^{\left(1-\alpha/2\right)}\right) \approx 1-\alpha \tag{5}$$

The next considered bootstrap method is the *t*-bootstrap method. The statistic of the following form is used:

$$t_B^* = \frac{\overline{x}_B - \overline{x}}{\hat{D}\left(\overline{x}_B\right)}, \tag{6}$$

where: $\overline{x}$ is the arithmetic mean for sample $X_1, \ldots, X_n$,

$\overline{x}_B$ is the arithmetic mean for fixed bootstrap sample $X_1^*, X_2^*, \ldots, X_n^*$,

$\hat{D}\left(\overline{x}_B\right)$ is the estimator of the standard deviation of mean for fixed bootstrap sample $X_1^*, X_2^*, \ldots, X_n^*$.

For *N* bootstrap samples we received a sequence of values $\left(t_i^*\right)_{i=1,\ldots N}$, where $t_{i,B}^* = \frac{\overline{x}_{i,B} - \overline{x}}{\hat{D}\left(\overline{x}_{i,B}\right)}$ and we determine percentiles $t_B^{*\left(\alpha/2\right)}, t_B^{*\left(1-\alpha/2\right)}$ of the order $\frac{\alpha}{2}$ and $1-\frac{\alpha}{2}$, respectively. In that case confidence interval for the mean μ is the following (Domański Cz., Pruska K., (2000)):

$$P\left(\overline{x}-t_B^{\bullet(1-\alpha/2)}\hat{D}(\overline{x})<\mu<\overline{x}+t_B^{\bullet(\alpha/2)}\hat{D}(\overline{x})\right)\approx 1-\alpha, \tag{7}$$

where $\hat{D}(\overline{x})$ is the estimator of standard deviation of mean of random variable X, for the sample $X_1,...,X_n$.

III. THE STUDY ON EFFICIENCY OF CHOSEN BOOTSTRAP ESTIMATION METHODS OF CONFIDENCE INTERVAL FOR THE MEAN OF POPULATION

To make the analysis of bootstrap estimation methods of population mean, two groups of experiments were carried out.

In the first group of experiments the population were generated using gamma distribution with the following density function:

$$f(x)=\begin{cases}\dfrac{1}{\lambda^p\Gamma(p)}x^{p-1}\exp\left(-\dfrac{x}{\lambda}\right) & \text{for } x>0,\\ 0 & \text{for } x\le 0.\end{cases}$$

The following values of parameters were used: $\lambda = 0{,}5; 1; 1{,}5; 2; 2{,}5; 3$ and $p=0{,}2; 0{,}4; 0{,}6; 0{,}8; 1{,}0; 1{,}2; 1{,}4; 2{,}0; 4{,}0; 6{,}0; 8{,}0; 10{,}0; 15{,}0; 20{,}0$. The asymmetry of distribution is positive and its size depends on value of parameter p.

In the second group of experiments the population was generated using beta distribution with the following density function:

$$f(x)=\begin{cases}\dfrac{1}{B(p,q)}x^{p-1}(1-x)^{q-1} & \text{for } 0<x<1\\ 0 & \text{for } x<0\vee x>1\end{cases}$$

For the beta distribution the following values of parameters were used: $p=2; 3; 4; 5; 6$, $q=0{,}2; 0{,}4; 0{,}6; 0{,}8; 1{,}0; 1{,}2; 1{,}4; 2{,}0; 2{,}4; 2{,}8$. The parameters were chosen in such a way that it is possible to consider populations of different sizes of negative asymmetry.

From the generated populations, samples of sizes 20, 30, 50, 70 and 100 were chosen. For the fix confidence coefficients, confidence intervals for the population mean were computed using the bootstrap methods mentioned earlier. Every estimation procedure was repeated 10000 times. In the experiment, arithmetic means of the lengths of obtained intervals were computed. Moreover, confidence coefficient was estimated as proportion of intervals which cover real parameter of population's distribution. Obtained results for chosen sample sizes and for confidence coefficient 0,95 are presented in tables 1–6.

Table 1

Lengths of confidence intervals and estimated confidence coefficients for chosen bootstrap estimation methods for group 1 of experiments and for sample size 30

Distribution's parameters		Lengths of confidence intervals			Estimated confidence coefficient		
λ	p	standard method	percentile method	t- bootstrap method	standard method	percentile method	t- bootstrap method
0,5	0,2	0,144	0,141	0,263	0,850	0,862	0,942
	0,4	0,205	0,203	0,279	0,891	0,901	0,945
	0,6	0,261	0,259	0,331	0,895	0,903	0,942
	0,8	0,308	0,306	0,375	0,907	0,911	0,945
	1,0	0,341	0,339	0,405	0,916	0,918	0,946
	1,2	0,378	0,376	0,440	0,915	0,918	0,947
	2	0,482	0,480	0,543	0,923	0,924	0,945
	4	0,695	0,693	0,764	0,935	0,935	0,950
	6	0,849	0,847	0,923	0,930	0,929	0,947
	15	1,338	1,336	1,440	0,933	0,934	0,947
1,00	0,2	0,288	0,282	0,525	0,852	0,865	0,941
	0,4	0,421	0,416	0,589	0,879	0,887	0,939
	0,6	0,520	0,516	0,658	0,901	0,907	0,945
	0,8	0,620	0,616	0,756	0,907	0,910	0,942
	1,0	0,686	0,682	0,814	0,915	0,918	0,945
	1,2	0,753	0,749	0,882	0,915	0,919	0,949
	2	0,971	0,968	1,095	0,922	0,925	0,949
	4	1,373	1,370	1,505	0,932	0,932	0,951
	6	1,709	1,705	1,860	0,932	0,932	0,947
	15	2,711	2,708	2,921	0,933	0,932	0,947
1,50	0,2	0,428	0,420	0,766	0,849	0,860	0,939
	0,4	0,622	0,616	0,853	0,882	0,890	0,943
	0,6	0,781	0,775	0,987	0,901	0,908	0,948
	0,8	0,915	0,909	1,114	0,909	0,914	0,946
	1,0	1,015	1,009	1,204	0,913	0,915	0,944
	1,2	1,121	1,114	1,304	0,913	0,915	0,941
	2	1,455	1,450	1,639	0,925	0,925	0,947
	4	2,078	2,073	2,282	0,924	0,924	0,944
	6	2,552	2,547	2,774	0,935	0,934	0,950
	15	4,062	4,056	4,374	0,936	0,934	0,950

Source: Own's calculations.

Table 2

Lengths of confidence intervals and estimated confidence coefficients for chosen bootstrap estimation methods for group 1 of experiments and for sample size 50

Distribution's parameters		Lengths of confidence intervals			Estimated confidence coefficient		
λ	p	standard method	percentile method	t- bootstrap method	standard method	percentile method	t- bootstrap method
0,5	0,2	0,116	0,115	0,166	0,888	0,898	0,946
	0,4	0,168	0,166	0,203	0,906	0,911	0,945
	0,6	0,210	0,209	0,243	0,920	0,922	0,948
	0,8	0,238	0,237	0,268	0,924	0,928	0,949
	1,0	0,271	0,270	0,300	0,927	0,928	0,945
	1,2	0,294	0,294	0,322	0,926	0,929	0,947
	2	0,386	0,385	0,415	0,935	0,936	0,951
	4	0,550	0,549	0,581	0,936	0,939	0,950
	6	0,665	0,665	0,699	0,938	0,936	0,949
	15	1,058	1,056	1,102	0,942	0,941	0,951
1,00	0,2	0,227	0,224	0,318	0,888	0,897	0,944
	0,4	0,332	0,330	0,405	0,908	0,915	0,946
	0,6	0,415	0,413	0,480	0,918	0,920	0,946
	0,8	0,480	0,478	0,542	0,928	0,931	0,948
	1,0	0,539	0,538	0,597	0,926	0,930	0,951
	1,2	0,593	0,592	0,652	0,932	0,933	0,951
	2	0,766	0,765	0,823	0,932	0,932	0,946
	4	1,089	1,087	1,149	0,935	0,934	0,947
	6	1,332	1,330	1,398	0,941	0,939	0,951
	15	2,113	2,111	2,204	0,938	0,938	0,947
1,50	0,2	0,346	0,342	0,493	0,878	0,890	0,944
	0,4	0,509	0,506	0,611	0,904	0,909	0,947
	0,6	0,624	0,621	0,721	0,918	0,922	0,947
	0,8	0,718	0,716	0,808	0,927	0,929	0,950
	1,0	0,761	0,759	0,812	0,923	0,925	0,946
	1,2	0,808	0,805	0,875	0,935	0,938	0,952
	2	0,977	0,975	1,026	0,938	0,940	0,948
	4	0,943	0,943	1,440	0,943	0,943	0,952
	6	1,698	1,696	1,757	0,942	0,942	0,948
	15	2,704	2,703	2,786	0,939	0,940	0,948

Source: Own's calculations.

Table 3

Lengths of confidence intervals and estimated confidence coefficients for chosen bootstrap estimation methods for group 1 of experiments and for sample size 100

Distribution's parameters		Lengths of confidence intervals			Estimated confidence coefficient		
λ	p	standard method	percentile method	t- bootstrap method	standard method	percentile method	t- bootstrap method
0,5	0,2	0,084	0,085	0,100	0,915	0,922	0,949
	0,4	0,121	0,120	0,133	0,925	0,926	0,946
	0,6	0,149	0,148	0,162	0,932	0,933	0,947
	0,8	0,172	0,171	0,182	0,934	0,938	0,951
	1,0	0,192	0,192	0,203	0,936	0,938	0,948
	1,2	0,213	0,212	0,222	0,942	0,942	0,949
	2	0,227	0,277	0,287	0,940	0,940	0,946
	4	0,386	0,386	0,396	0,945	0,945	0,950
	6	0,478	0,477	0,489	0,944	0,943	0,949
	15	0,750	0,750	0,763	0,943	0,943	0,945
1,00	0,2	0,171	0,170	0,205	0,912	0,919	0,947
	0,4	0,243	0,242	0,270	0,920	0,925	0,945
	0,6	0,297	0,296	0,320	0,933	0,935	0,951
	0,8	0,348	0,347	0,370	0,936	0,938	0,948
	1,0	0,390	0,389	0,410	0,938	0,938	0,950
	1,2	0,425	0,425	0,445	0,940	0,941	0,952
	2	0,546	0,545	0,565	0,945	0,945	0,951
	4	0,772	0,771	0,793	0,945	0,945	0,949
	6	0,961	0,960	0,984	0,944	0,944	0,949
	15	1,503	1,502	1,533	0,945	0,945	0,952
1,50	0,2	0,259	0,258	0,311	0,909	0,917	0,946
	0,4	0,362	0,360	0,400	0,929	0,932	0,949
	0,6	0,449	0,447	0,485	0,928	0,929	0,943
	0,8	0,523	0,522	0,555	0,937	0,939	0,950
	1,0	0,575	0,573	0,605	0,933	0,935	0,944
	1,2	0,631	0,630	0,661	0,940	0,941	0,951
	2	0,816	0,814	0,844	0,942	0,943	0,949
	4	1,167	1,166	1,200	0,945	0,945	0,950
	6	1,436	1,435	1,471	0,942	0,941	0,949
	15	2,260	2,258	2,307	0,948	0,947	0,950

Source: Own's calculations.

Table 4

Lengths of confidence intervals and estimated confidence coefficients for chosen bootstrap estimation methods for group 2 of experiments and for sample size 30

Distribution's parameters		Lengths of confidence intervals			Estimated confidence coefficient		
p	*q*	standard method	percentile method	*t*- bootstrap method	standard method	percentile method	*t*- bootstrap method
3,0	0,2	0,078	0,078	0,115	0,881	0,892	0,955
	0,4	0,105	0,104	0,127	0,907	0,916	0,954
	0,6	0,120	0,120	0,137	0,923	0,928	0,956
	0,8	0,130	0,129	0,144	0,927	0,930	0,957
	1,0	0,134	0,134	0,147	0,928	0,933	0,956
	1,2	0,139	0,138	0,150	0,936	0,939	0,959
	1,4	0,140	0,140	0,151	0,931	0,934	0,957
	2,0	0,140	0,140	0,150	0,934	0,938	0,957
	2,4	0,137	0,137	0,146	0,932	0,933	0,954
	2,8	0,133	0,133	0,142	0,934	0,936	0,956
4,0	0,2	0,061	0,062	0,094	0,876	0,888	0,954
	0,4	0,086	0,085	0,106	0,904	0,910	0,951
	0,6	0,099	0,099	0,115	0,924	0,929	0,957
	0,8	0,107	0,107	0,121	0,924	0,930	0,955
	1,0	0,113	0,112	0,124	0,928	0,932	0,957
	1,2	0,117	0,117	0,128	0,931	0,933	0,955
	1,4	0,120	0,120	0,130	0,937	0,939	0,960
	2,0	0,125	0,124	0,134	0,936	0,938	0,957
	2,4	0,124	0,123	0,132	0,933	0,935	0,954
	2,8	0,123	0,123	0,132	0,932	0,933	0,952
5,0	0,2	0,050	0,050	0,080	0,873	0,888	0,951
	0,4	0,070	0,070	0,089	0,902	0,909	0,951
	0,6	0,084	0,084	0,098	0,914	0,919	0,953
	0,8	0,091	0,091	0,103	0,919	0,923	0,953
	1,0	0,098	0,097	0,109	0,926	0,929	0,955
	1,2	0,104	0,104	0,115	0,930	0,934	0,955
	1,4	0,105	0,105	0,115	0,930	0,931	0,954
	2,0	0,112	0,111	0,120	0,937	0,938	0,955
	2,4	0,113	0,113	0,121	0,935	0,934	0,951
	2,8	0,113	0,113	0,121	0,936	0,938	0,955

Source: Own's calculations.

Table 5

Lengths of confidence intervals and estimated confidence coefficients for chosen bootstrap estimation methods for group 2 of experiments and for sample size 50

Distribution's parameters		Lengths of confidence intervals			Estimated confidence coefficient		
p	*q*	standard method	percentile method	*t*- bootstrap method	standard method	percentile method	*t*- bootstrap method
3,0	0,2	0,063	0,062	0,077	0,908	0,917	0,955
	0,4	0,083	0,083	0,093	0,927	0,930	0,957
	0,6	0,094	0,094	0,102	0,933	0,937	0,956
	0,8	0,101	0,100	0,107	0,934	0,937	0,955
	1,0	0,106	0,106	0,111	0,938	0,939	0,953
	1,2	0,108	0,108	0,113	0,941	0,943	0,956
	1,4	0,110	0,110	0,115	0,942	0,942	0,956
	2,0	0,110	0,110	0,114	0,940	0,941	0,951
	2,4	0,108	0,108	0,111	0,947	0,948	0,958
	2,8	0,104	0,104	0,108	0,944	0,943	0,953
4,0	0,2	0,049	0,048	0,062	0,904	0,914	0,953
	0,4	0,066	0,066	0,074	0,924	0,931	0,956
	0,6	0,077	0,077	0,084	0,931	0,934	0,952
	0,8	0,084	0,084	0,090	0,935	0,937	0,954
	1,0	0,089	0,089	0,094	0,933	0,936	0,952
	1,2	0,093	0,093	0,097	0,940	0,942	0,956
	1,4	0,095	0,095	0,100	0,943	0,945	0,957
	2,0	0,096	0,096	0,100	0,942	0,942	0,955
	2,4	0,097	0,097	0,101	0,940	0,940	0,951
	2,8	0,096	0,096	0,100	0,942	0,941	0,954
5,0	0,2	0,041	0,041	0,052	0,899	0,910	0,951
	0,4	0,056	0,056	0,063	0,920	0,924	0,950
	0,6	0,066	0,065	0,072	0,930	0,935	0,955
	0,8	0,072	0,072	0,078	0,929	0,932	0,949
	1,0	0,077	0,077	0,082	0,939	0,940	0,955
	1,2	0,081	0,081	0,085	0,940	0,943	0,955
	1,4	0,083	0,083	0,087	0,941	0,944	0,956
	2,0	0,087	0,087	0,091	0,941	0,942	0,954
	2,4	0,088	0,088	0,092	0,937	0,940	0,951
	2,8	0,088	0,088	0,091	0,939	0,940	0,954

Source: Own's calculations.

Table 6

Lengths of confidence intervals and estimated confidence coefficients for chosen bootstrap estimation methods for group 2 of experiments and for sample size 100

Distribution's parameters		Lengths of confidence intervals			Estimated confidence coefficient		
p	*q*	standard method	percentile method	*t*- bootstrap method	standard method	percentile method	*t*- bootstrap method
3,0	0,2	0,046	0,046	0,050	0,929	0,936	0,957
	0,4	0,059	0,059	0,062	0,938	0,942	0,952
	0,6	0,068	0,068	0,070	0,943	0,945	0,954
	0,8	0,072	0,072	0,074	0,940	0,941	0,950
	1,0	0,076	0,075	0,077	0,941	0,943	0,949
	1,2	0,077	0,076	0,078	0,942	0,944	0,949
	1,4	0,078	0,078	0,080	0,944	0,945	0,952
	2,0	0,078	0,078	0,079	0,945	0,946	0,951
	2,4	0,076	0,076	0,078	0,947	0,946	0,954
	2,8	0,075	0,075	0,076	0,944	0,944	0,951
4,0	0,2	0,035	0,035	0,039	0,924	0,929	0,954
	0,4	0,048	0,048	0,050	0,938	0,939	0,953
	0,6	0,055	0,055	0,058	0,938	0,941	0,951
	0,8	0,061	0,061	0,063	0,943	0,942	0,951
	1,0	0,063	0,063	0,065	0,942	0,943	0,951
	1,2	0,066	0,066	0,068	0,950	0,950	0,956
	1,4	0,068	0,068	0,069	0,944	0,946	0,954
	2,0	0,070	0,070	0,071	0,944	0,945	0,953
	2,4	0,069	0,069	0,071	0,948	0,948	0,954
	2,8	0,068	0,068	0,070	0,948	0,949	0,954
5,0	0,2	0,029	0,029	0,033	0,922	0,928	0,953
	0,4	0,039	0,039	0,041	0,935	0,938	0,949
	0,6	0,047	0,047	0,049	0,936	0,938	0,950
	0,8	0,051	0,051	0,053	0,944	0,945	0,955
	1,0	0,055	0,055	0,057	0,940	0,942	0,951
	1,2	0,058	0,058	0,059	0,942	0,943	0,951
	1,4	0,059	0,059	0,061	0,945	0,945	0,952
	2,0	0,062	0,062	0,064	0,947	0,948	0,952
	2,4	0,063	0,063	0,064	0,942	0,943	0,949
	2,8	0,063	0,063	0,064	0,946	0,945	0,950

Source: Own's calculations.

The standard bootstrap method and percentile method lead to the confidence intervals of almost the same lengths and similar values of confidence coefficient. The value of the estimated confidence coefficient was, however, lower than the fixed value, especially for populations with strong skewness. Asymmetry coefficients for considered populations can be found in Baszczyńska A., Pekasiewicz D. (2007a)).

The application of the t-bootstrap estimation method leads to the estimation of the considered populations mean with confidence coefficient approximately equal to the fixed value, but the received confidence intervals have bigger lengths.

Similar results were obtained for confidence coefficients 0.90 and 0.99.

IV. CONCLUSIONS

The simulation study described above was carried out to compare the three chosen bootstrap methods the population mean estimation in asymmetric populations, in the example of populations with the gamma and beta distribution. The results obtained allow to state that for populations considered the t-bootstrap method assures the estimation of population mean with confidence coefficient on the prefixed level. Other bootstrap methods lead to obtain confidence intervals covering the estimated population mean with confidence coefficient smaller than fixed level. For populations with strong asymmetry, the estimated confidence coefficient was much smaller than prefixed one, even in the case of large sample sizes. Thus, the t-bootstrap method has turned out the only one, among the considered bootstrap and nonbootstrap methods (cp. Baszczyńska A., Pekasiewicz D. (2007b)), to assure the estimation of population mean in asymmetric populations with the prefixed confidence coefficient.

The lengths of confidence intervals obtained with the t-bootstrap method were larger than lengths of intervals obtained with the standard bootstrap method and the percentile method. The smaller were the samples drawn from the population, the more important were the differences. The smallest lengths of confidence intervals were obtained using modifications of classical method. But the estimated confidence coefficient for this method, although not equal to prefixed one, was larger than for the standard bootstrap method and the percentile method (cp. Baszczyńska A., Pekasiewicz D. (2007b)).

REFERENCES

Baszczyńska A., Pekasiewicz D., (2007a), *Estymacja przedziałowa wartości oczekiwanej zmiennej losowej z wykorzystaniem współczynnika asymetrii*, Wiadomości Statystyczne, 7, 1–10.

Baszczyńska A., Pekasiewicz D., (2007b), *On Some Confidence Intervals for Population Mean in Case of Asymmetric Distributions of Random Variables*, Folia Oeconomica, in press.

DiCiccio T., Efron B., (1996), *Bootstrap Confidence Intervals*, Statistical Science, vol. 11, No 3, 189–228.

Domański Cz., Pruska K., (2000), *Nieklasyczne metody statystyczne*, Polskie Wydawnictwo Ekonomiczne, Warszawa.
Koronacki J., Michniczuk J., (2006), *Statystyka*, Wydawnictwa Naukowo-Techniczne, Warszawa.
Rousson V., Choi E., (2003), *On Descriptive Confidence Intervals for the Mean of Asymmertic Population*, Nonparametric Statistics, 15 (4–5), 553–563.
Zhou X., Dinh P., (2004), *Nonparametric Confidence Interval for the One- and Two-Sample Problems*, UW Biostatistics Working Paper Series, 233, University of Washington, The Berkeley Electronic Press, http://www.bepress.com/uwbiostat/paper233.

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BOOTSTRAPOWA ESTYMACJA PRZEDZIAŁOWA WARTOŚCI OCZEKIWANEJ ASYMETRYCZNYCH ROZKŁADÓW ZMIENNYCH LOSOWYCH

W pracy przedstawiono wybrane metody bootstrapowe estymacji przedziałowej wartości oczekiwanej populacji o rozkładzie asymetrycznym. Rozważano standardową metodę bootstrapowa, metodę percentyli oraz metodę *t*-bootstrapową. Metody te można stosować przy estymacji wartości oczekiwanej zmiennej losowej o rozkładzie asymetrycznym, zarówno dla małych jak i dużych prób. Analiza własności bootstrapowych metod estymacji przedziałowej przeprowadzona została metodami Monte Carlo.