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ESTIMATION OF BIAS AND VARIANCE OF SAMPLE MEDIAN BY JACKKNIFE AND BOOTSTRAP METHODS

Abstract. In the paper the estimation of sample median bias and variance by jackknife and bootstrap methods are considered. Monte Carlo analysis of properties of estimators is presented (mean of bias and mean of variance for some groups of experiments). Sensitivity of distribution of sample median to changes of the sample size is investigated.

Key words: jackknife method, bootstrap method, Monte Carlo methods.

1. INTRODUCTION

The jackknife and bootstrap methods are the data-resampling methods which are applied in statistical analysis (see: Efron, Tibshirani 1993; Shao, Tu 1996). They are used for the estimation of bias and variance of different estimators. They can be applied for the construction of confidence sets (intervals) and statistical tests, too.

In this paper the application of jackknife and bootstrap methods to the estimation of bias and variance of sample median are considered.

2. ESTIMATION OF BIAS AND VARIANCE OF ESTIMATOR BY JACKKNIFE METHOD

We assume that we investigate a population with respect to random variable X. Let $X_1, ..., X_n$ be simple sample drawn from the population and $x_1, ..., x_n$ – realization of this sample.

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Let $T_n = T_n(X_1, ..., X_n)$ be an estimator of parameter θ of X's distribution and let $T_{n-1,i}$ be an estimator of θ determined on the basis of $X_1, ..., X_{i-1}, X_{i+1}, ..., X_n$ in the analogous way as T_n. It means that

$$T_{n-1,i} = T_{n-1}(X_1, ..., X_{i-1}, X_{i+1}, ..., X_n) \quad (1)$$

The jackknife estimator of bias of T_n, that is $\mathrm{E}(T_n - \theta)$, is defined by the formula (see: Shao, Tu 1996)

$$b_{JACK}(T_n) = (n-1)(\overline{T}_n - T_n) \quad (2)$$

where

$$\overline{T}_n = \frac{1}{n}\sum_{i=1}^{n} T_{n-1,i} \quad (3)$$

The jackknife estimator of θ is of the following form:

$$T_{JACK} = T_n - b_{JACK} = nT_n - (n-1)\overline{T}_n \quad (4)$$

The jackknife estimator of variance of T_n is defined as (see: Shao, Tu 1996):

$$v_{JACK}(T_n) = \frac{n-1}{n}\sum_{i=1}^{n}(T_{n-1,i} - \overline{T}_n)^2 \quad (5)$$

There are also considered jackknife procedures in which some sample elements are deleted.

3. ESTIMATION OF BIAS AND VARIANCE OF ESTIMATOR BY BOOTSTRAP METHOD

We assume that X is an investigated population variable, the sequence $X_1, ..., X_n$ is simple sample drawn from the population and $x_1, ..., x_n$ is the realization of this sample.

Let $T_n = T_n(X_1, ..., X_n)$ be an estimator of parameter θ of X's distribution and let X_B be the random variable for which probability distribution function has the following form:

$$\mathrm{P}(X_B = x_i) = \frac{1}{n} \quad \text{for} \quad i = 1, ..., n \quad (6)$$

We generate n-element sequences of pseudorandom numbers from distribution (6). Let N be a number of these sequences. They are called realizations of the bootstrap sample $X^*_{1k}, ..., X^*_{nk}$ and denoted by $x^*_{1k}, ..., x^*_{nk}$, where $k = 1, ..., N$. The variable X^*_{lk}, $l = 1, ..., n$ and $k = 1, ..., N$, has distribution given by formula (6).

The bootstrap estimator of θ is of the form

$$T_{BOOT} = \frac{1}{N} \sum_{k=1}^{N} T^*_{n,k} \tag{7}$$

where

$$T^*_{n,k} = T_n(X^*_{1k}, ..., X^*_{nk}) \tag{8}$$

The bootstrap estimator of bias of T_n is the following:

$$b_{BOOT}(T_n) = \frac{1}{N} \sum_{k=1}^{N} T^*_{n,k} - T_n \tag{9}$$

and the bootstrap estimator of variance of T_n is the form

$$v_{BOOT} = \frac{1}{N} \sum_{k=1}^{N} \left(T^*_{n,k} - \frac{1}{N} \sum_{l=1}^{N} T^*_{n,l} \right)^2 \tag{10}$$

We can consider bootstrap sample whose size is not equal n.

4. ESTIMATORS OF MEDIAN

The median (Me) is a parameter of distribution of random variable. We can estimate this parameter by different methods. The sample median is used for it. We can apply jackknife or bootstrap methods, too.

Let n is the size of population sample. In this paper the sample median (Me_n) is defined as the statistic whose value is observation being on the position with number $(n + 1)/2$ for odd n in nondecreasing sequence of observations, or the average of two observations with numbers $n/2$ and $n/2 + 1$ for even n in nondecreasing sequence of observations (the first variant). The statistic whose value is observation on the position with number $[n/2]$ in nondecreasing sequence of observations is another variant of sample median (the second variant).

The estimators of population median are obtained from formulas (4) and (7), too. Then we take $T_n = Me_n$.

For each estimator the values of its bias and variance are very important. The bias of sample median can be estimated on the basis of formulas (2) or

(9). We can consider different ways of estimation or approximation of sample median variance. If we repeat experiments in which we estimate the median we can determine the variance for many obtained estimates of median. We may apply formulas (5) and (10) for estimation of sample median variance. In this way we obtain jackknife and bootstrap estimators of sample median variance.

For approximation of sample median variance we can use the theorem which says that the asymptotical distribution of sample median is normally $N\left(Me, \frac{1}{2\sqrt{n}f(Me)}\right)$, where f is density function of investigated variable (the variable is continuous).

5. MONTE CARLO ANALYSIS OF PROPERTIES OF SAMPLE MEDIAN

In order to investigate the properties of sample median Monte Carlo experiments were conducted. The algorithm of carrying out these experiments was of as follows:

1) we generate n ($n = 20, 21, 40, 41$) values from fixed distribution among distributions which are given in Tab. 1;

Table 1

Parameters of distributions used in Monte Carlo experiments

Population distribution[a]	Expectation	Variance	Standard deviation	Median
N(0; 1)	0	1	1.0000	0.0000
χ_3^2	3	6	2.4495	2.3660
χ_5^2	5	10	3.1623	4.3515
χ_7^2	7	14	3.7417	6.3458
$\frac{3}{4}$ N(10; 2) + $\frac{1}{4}\chi_3^2$	8.25	13.69	3.7000	9.1810[b]

[a] Symbol N(μ, σ) denotes the normal distribution with expectation μ and standard deviation σ, symbol χ_k^2 – the chi-square distribution with k degrees of freedom and symbol $\frac{3}{4}$ N(10; 2) + $\frac{1}{4}\chi_3^2$ – the mixture of distributions N(10; 2) and χ_3^2 with weghts 3/4 and 1/4. [b] Median estimate is obtained on the basis of 1001 values generated from distribution $\frac{3}{4}$ N(10; 2) + $\frac{1}{4}\chi_3^2$.

Source: own preparation.

2) we estimate median Me on the basis of obtained n-element sequence of pseudo values. It means we calculate the value of estimator Me_n (sample median). This value will be denoted by me_n. We apply two ways for calculations: the classical definition of median (the first variant) and the definition of $[n/2]$-th order statistic (the second variant);

3) we estimate the bias and variance of sample median by jackknife and bootstrap method. In the bootstrap estimation we apply 1000 repetitions of bootstrap sample drawing;

4) we repeat steps from 1) to 3) one thousand times and we obtain 1000 estmates of median on the basis of sample, by jackknife methods and by bootstrap method, 1000 estimates of bias of sample median and 1000 estimates of variance of sample median for jackknife method and bootstrap method. Next, we calculate the mean and standard deviation for:

– 1000 estimates of sample median: $(\overline{me}, s_{me})$,
– 1000 jackknife estimates of median: $(\overline{me}_{JACK}, s_{me_{JACK}})$,
– 1000 bootstrap estimates of median: $(\overline{me}_{BOOT}, s_{me_{BOOT}})$,
– 1000 jackknife estimates of variance of sample median: $(\overline{v}_{JACK}, s_{v_{JACK}})$,
– 1000 bootstrap estimates of variance of sample median: $(\overline{v}_{BOOT}, s_{v_{BOOT}})$.

Moreover, we calculate

$$\overline{b}_{me} = \frac{1}{1000} \sum_{i=1}^{1000} |me_{n,i} - Me| \tag{11}$$

$$\overline{b}_{JACK} = \frac{1}{1000} \sum_{i=1}^{1000} me_{JACK,i} \tag{12}$$

$$\overline{b}_{JACK} = \frac{1}{1000} \sum_{i=1}^{1000} |me_{BOOT,i} - me_n| \tag{13}$$

$$\overline{b}_{BOOTMe} = \frac{1}{1000} \sum_{i=1}^{1000} |me_{BOOT,i} - Me| \tag{14}$$

where:

$me_{n,i}$ – the value of sample median for i-th repetition,
$me_{JACK,I}$ – the jackknife estimate of population median for i-th repetition,
$me_{BOOT,i}$ – the bootstrap estimate of population median for i-th repetition.

The values (11)–(14) were calculated for two variants of definition of median.

The results of experiments are presented in Tab. 2–4. We can note that the estimates of bias mean and variance mean for the sample median are very different in many cases when we use jackknife method and the sample

Table 2

Results of Monte Carlo experiments for median estimation of some probability distributions and for sample size: $n = 20$ i $n = 21$

Distribution of population	Mean (stand. deviation) for estimates of suitable parameters	Variant of estimator: first		second	
		size of sample			
		$n = 20$	$n = 21$	$n = 20$	$n = 21$
1	2	3	4	5	6
N(0; 1)	$\overline{me}$	0.0118	–0.0043	–0.0524	–0.1207
	(s_{me})	(0.2820)	(0.2670)	(0.2918)	(0.2669)
	$\overline{me}_{JACK}$	0.0118	0.0097	1.2605	–1.2294
	$(s_{me_{JACK}})$	(0.2820)	(0.9558)	(1.3497)	(1.1364)
	$\overline{me}_{BOOT}$	0.0115	–0.0027	–0.0510	–0.1173
	$(s_{me_{BOOT}})$	(0.2588)	(0.2431)	(0.2605)	(0.2436)
	$\overline{b}_{me}$	0.2284	0.2125	0.2391	0.2333
	$\overline{b}_{JACK}$	0.0000	0.0000	–1.3129	1.1087
	$\overline{b}_{BOOT}$	0.0533	0.0622	0.0693	0.0629
	$\overline{b}_{BOOTME}$	0.2081	0.1951	0.2147	0.2160
	$\overline{v}_{JACK}$	0.1476	0.0936	0.1419	0.1283
	$(s_{v_{JACK}})$	(0.2903)	(0.1382)	(0.2649)	(0.2575)
	$\overline{v}_{BOOT}$	0.0883	0.0850	0.0958	0.0851
	$(s_{v_{BOOT}})$	(0.0518)	(0.0522)	(0.0561)	(0.0525)
χ^2_3	$\overline{me}$	2.4560	2.4389	2.3218	2.1852
	(s_{me})	(0.5980)	(0.5803)	(0.5877)	(0.5395)
	$\overline{me}_{JACK}$	2.4560	2.4119	4.9284	–0.2308
	$(s_{me_{JACK}})$	(0.5980)	(2.0290)	(2.9825)	(2.4369)
	$\overline{me}_{BOOT}$	2.5044	2.4800	2.3719	2.2374
	$(s_{me_{BOOT}})$	(0.5540)	(0.5316)	(0.5345)	(0.4934)
	$\overline{b}_{me}$	0.4704	0.4645	0.4645	0.4676
	$\overline{b}_{JACK}$	0.0000	0.0270	–2.6066	2.4161
	$\overline{b}_{BOOT}$	0.1179	0.1397	0.1443	0.1351
	$\overline{b}_{BOOTME}$	0.4460	0.4291	0.4241	0.4185
	$\overline{v}_{JACK}$	0.6573	0.4667	0.6079	0.6235
	$(s_{v_{JACK}})$	(1.3078)	(0.6858)	(1.3453)	(1.2725)
	$\overline{v}_{BOOT}$	0.4149	0.4240	0.4180	0.3676
	$(s_{v_{BOOT}})$	(0.2913)	(0.3105)	(0.2993)	(0.2687)

Table 2 (contd.)

1	2	3	4	5	6
χ^2_5	$\overline{me}$	4.3798	4.3685	4.2049	4.0462
	(s_{me})	(0.8058)	(0.7801)	(0.7990)	(0.7488)
	$\overline{me}_{JACK}$	4.3798	4.3042	7.8038	0.9772
	$(s_{me_{JACK}})$	(0.8058)	(2.7135)	(3.7358)	(3.1206)
	$\overline{me}_{BOOT}$	4.4324	4.4121	4.2533	4.0827
	$(s_{me_{BOOT}})$	(0.7468)	(0.7178)	(0.7272)	(0.6821)
	$\overline{b}_{me}$	0.6346	0.6170	0.6547	0.6587
	$\overline{b}_{JACK}$	0.0000	0.0643	–3.5989	3.0691
	$\overline{b}_{BOOT}$	0.1604	0.1834	0.1914	0.1837
	$\overline{b}_{BOOTME}$	0.5824	0.5700	0.5818	0.5969
	$\overline{v}_{JACK}$	1.1217	0.7704	1.0546	0.9971
	$(s_{v_{JACK}})$	(2.4214)	(1.1399)	(2.0843)	(2.2104)
	$\overline{v}_{BOOT}$	0.7406	0.7466	0.7582	0.6683
	$(s_{v_{BOOT}})$	(0.4745)	(0.4043)	(0.4924)	(0.4218)
χ^2_7	$\overline{me}$	6.3857	6.3463	6.1788	5.9435
	(s_{me})	(0.9603)	(0.9407)	(0.9686)	(0.9044)
	$\overline{me}_{JACK}$	6.3857	6.1062	10.5914	2.1077
	$(s_{me_{JACK}})$	(0.9603)	(3.5698)	(4.7260)	(3.9339)
	$\overline{me}_{BOOT}$	6.4385	6.4136	6.2217	6.0010
	$(s_{me_{BOOT}})$	(0.8850)	(0.8562)	(0.8687)	(0.8186)
	$\overline{b}_{me}$	0.7657	0.7535	0.7944	0.8134
	$\overline{b}_{JACK}$	0.0000	0.2401	–4.4127	3.8358
	$\overline{b}_{BOOT}$	0.1935	0.2437	0.2335	0.2143
	$\overline{b}_{BOOTME}$	0.7092	0.6743	0.7109	0.7273
	$\overline{v}_{JACK}$	1.5757	1.2816	1.6427	1.5698
	$(s_{v_{JACK}})$	(3.0402)	(1.8519)	(3.4286)	(3.0904)
	$\overline{v}_{BOOT}$	1.0842	1.1760	1.1194	1.0643
	$(s_{v_{BOOT}})$	(0.6902)	(0.7509)	(0.7149)	(0.6634)
	$\overline{me}$	9.0734	9.1248	8.8878	8.7442
	(s_{me})	(0.8780)	(0.8816)	(0.9397)	(0.9790)
	$\overline{me}_{JACK}$	9.0734	9.3389	13.5362	5.1191
	$(s_{me_{JACK}})$	(0.8780)	(3.0402)	(5.0646)	(4.4158)
	$\overline{me}_{BOOT}$	8.9551	9.0146	8.7420	8.5966
	$(s_{me_{BOOT}})$	(0.8730)	(0.8758)	(0.9306)	(0.9823)

Table 2 (contd.)

1	2	3	4	5	6
$\frac{3}{4}N(10;2)+\frac{1}{4}\chi_3^2$	$\overline{b}_{me}$	0.6562	0.6757	0.7199	0.7919
	$\overline{b}_{JACK}$	0.0000	-0.2141	-4.6483	3.6251
	$\overline{b}_{BOOT}$	0.2045	0.2244	0.2584	0.2615
	$\overline{b}_{BOOTME}$	0.6706	0.6831	0.7614	0.8487
	$\overline{v}_{JACK}$	1.2955	0.9816	2.0261	1.5588
	$(s_{v_{JACK}})$	(3.1103)	(1.6133)	(6.3460)	(4.0520)
	$\overline{v}_{BOOT}$	1.1772	1.1401	1.4534	1.4773
	$(s_{v_{BOOT}})$	(1.0929)	(1.0476)	(1.3540)	(1.3598)

Source: author's calculations.

Table 3

Results of Monte Carlo experiments for median estimation of some probability distributions and for sample size: $n = 40$ i $n = 41$

Distribution of population	Mean (stand. deviation) for estimates of suitable parameters	Variant of estimator			
		first		second	
		size of sample			
		$n = 40$	$n = 41$	$n = 40$	$n = 41$
1	2	3	4	5	6
N(0; 1)	$\overline{me}$	-0.0004	-0.0041	-0.0315	-0.0623
	(s_{me})	(0.1929)	(0.1982)	(0.1951)	(0.1973)
	$\overline{me}_{JACK}$	-0.0004	-0.0245	1.2972	-1.1981
	$(s_{me_{JACK}})$	(0.1929)	(0.9055)	(1.3663)	(1.1160)
	$\overline{me}_{BOOT}$	-0.0017	-0.0029	-0.0330	-0.0631
	$(s_{me_{BOOT}})$	(0.1825)	(0.1853)	(0.1823)	(0.1859)
	$\overline{b}_{me}$	0.1545	0.1574	0.1583	0.1645
	$\overline{b}_{JACK}$	0.0000	0.0204	-1.3287	1.1358
	$\overline{b}_{BOOT}$	0.0340	0.0387	0.0411	0.0387
	$\overline{b}_{BOOTME}$	0.1464	0.1466	0.1478	0.1562
	$\overline{v}_{JACK}$	0.0730	0.0509	0.0814	0.0641
	$(s_{v_{JACK}})$	(0.1475)	(0.0728)	(0.1827)	(0.1214)
	$\overline{v}_{BOOT}$	0.0435	0.0432	0.0455	0.0434
	$(s_{v_{BOOT}})$	(0.0238)	(0.0236)	(0.0250)	(0.0236)

Table 3 (contd.)

1	2	3	4	5	6
χ_3^2	$\overline{me}$	2.4074	2.3748	2.3385	2.2507
	(s_{me})	(0.4299)	(0.4118)	(0.4234)	(0.3982)
	$\overline{me}_{JACK}$	2.4074	2.2573	4.9486	–0.1702
	$(s_{me_{JACK}})$	(0.4299)	(2.0322)	(2.7239)	(2.3739)
	$\overline{me}_{BOOT}$	2.4294	2.4038	2.3630	2.2766
	$(s_{me_{BOOT}})$	(0.4042)	(0.3805)	(0.3958)	(0.3681)
	$\overline{b}_{me}$	3.9385	3.9711	0.3393	0.3351
	$\overline{b}_{JACK}$	0.0000	0.1174	–2.6102	2.4209
	$\overline{b}_{BOOT}$	0.0750	0.0878	0.0860	0.0844
	$\overline{b}_{BOOTME}$	3.9164	3.9421	0.3167	0.3051
	$\overline{v}_{JACK}$	0.3723	0.2455	0.3118	0.2958
	$(s_{v_{JACK}})$	(0.9213)	(0.3945)	(0.6935)	(0.6739)
	$\overline{v}_{BOOT}$	0.2005	0.2040	0.2023	0.1883
	$(s_{v_{BOOT}})$	(0.1176)	(0.1226)	(0.1207)	(0.1111)
χ_5^2	$\overline{me}$	4.3810	4.3779	4.2883	4.2023
	(s_{me})	(0.5760)	(0.5710)	(0.5712)	(0.5541)
	$\overline{me}_{JACK}$	4.3810	4.2978	8.0887	0.7760
	$(s_{me_{JACK}})$	(0.5765)	(2.8303)	(3.8883)	(3.5856)
	$\overline{me}_{BOOT}$	4.4019	4.4016	4.3105	4.2244
	$(s_{me_{BOOT}})$	(0.5373)	(0.5294)	(0.5292)	(0.5139)
	$\overline{b}_{me}$	1.9659	1.9680	1.9224	1.8363
	$\overline{b}_{JACK}$	0.0000	0.0800	–3.8004	3.4262
	$\overline{b}_{BOOT}$	0.1025	0.1253	0.1209	0.1173
	$\overline{b}_{BOOTME}$	0.9447	1.9442	1.9445	1.8584
	$\overline{v}_{JACK}$	0.6942	0.4755	0.6490	0.6305
	$(s_{v_{JACK}})$	(1.7559)	(0.7206)	(1.3814)	(1.5139)
	$\overline{v}_{BOOT}$	0.3767	0.3908	0.3845	0.3707
	$(s_{v_{BOOT}})$	(0.2149)	(0.2264)	(0.2215)	(0.2137)
	$\overline{me}$	6.3273	6.3964	6.2150	6.1706
	(s_{me})	(0.6970)	(0.7013)	(0.6920)	(0.6940)
	$\overline{me}_{JACK}$	6.3273	6.5007	10.3856	1.7658
	$(s_{me_{JACK}})$	(0.6970)	(3.2457)	(4.4781)	(4.3365)
	$\overline{me}_{BOOT}$	6.3541	6.4101	6.2466	6.1985
	$(s_{me_{BOOT}})$	(0.6589)	(0.6507)	(0.6512)	(0.6355)

Table 3 (contd.)

1	2	3	4	5	6
χ_7^2	$\bar{b}_{me}$	0.5604	0.5594	3.8490	3.8046
	$\bar{b}_{JACK}$	0.0000	–0.1043	–4.1706	4.4048
	$\bar{b}_{BOOT}$	0.1194	0.1364	0.1367	0.1448
	$\bar{b}_{BOOTME}$	0.5285	0.5242	3.8806	3.8325
	$\bar{v}_{JACK}$	0.9720	0.6907	0.8294	0.9714
	$(s_{v_{JACK}})$	(2.2626)	(0.9280)	(2.0007)	(1.9145)
	$\bar{v}_{BOOT}$	0.5232	0.5500	0.5333	0.5285
	$(s_{v_{BOOT}})$	(0.3067)	(0.3083)	(0.3138)	(0.2990)
$\frac{3}{4}N(10; 2)+\frac{1}{4}\chi_3^2$	$\overline{me}$	9.1537	9.1823	9.0630	8.9908
	(s_{me})	(0.5947)	(0.5871)	(0.6158)	(0.6317)
	$\overline{me}_{JACK}$	9.1537	9.2693	12.9718	5.2532
	$(s_{me_{JACK}})$	(0.5947)	(3.1079)	(4.0141)	(4.6018)
	$\overline{me}_{BOOT}$	9.1162	9.1359	9.0221	8.9419
	$(s_{me_{BOOT}})$	(0.5759)	(0.5790)	(0.5926)	(0.6212)
	$\bar{b}_{me}$	0.4623	0.4540	0.4835	0.4823
	$\bar{b}_{JACK}$	0.0000	–0.0870	–3.9088	3.7376
	$\bar{b}_{BOOT}$	0.1059	0.1343	0.1215	0.1360
	$\bar{b}_{BOOTME}$	0.4483	0.4389	0.4697	0.4888
	$\bar{v}_{JACK}$	0.6434	0.5450	0.7190	0.8586
	$(s_{v_{JACK}})$	(1.5591)	(1.0601)	(1.7176)	(2.9242)
	$\bar{v}_{BOOT}$	0.4246	0.4595	0.4689	0.5271
	$(s_{v_{BOOT}})$	(0.3432)	(0.4430)	(0.3901)	(0.5208)

Source: author's calculations.

size increases from 20 to 21 or from 40 to 41. In Tab. 4 there is also given the variance of normal distribution which is approximation of distribution of sample median (see asymptotical distribution of sample median in Sec. 4).

We can compare the obtained results. The estimates of bias mean for the sample median differ considerably with respect to the estimation method. We can observe similar results for estimates of variance mean for the sample median. Moreover, the estimates of variance of sample median are relatively big (especially for jackknife method).

Table 4

Results of Monte Carlo experiments for estimation of variance of median for some probability distributions

Population distribution	Estimation method	Estimate of variance of sample median for sample size		Estimate of variance of sample median[a] for sample size	
		$n = 20$	$n = 21$	$n = 40$	$n = 41$
N(0; 1)	without est.[b]	0.0785	0.0747	0.0393	0.0383
	mom. meth.[c]	0.0795	0.0713	0.0372	0.0393
	jackknife	0.1476	0.0939	0.0730	0.0509
	bootstrap	0.0883	0.0850	0.0435	0.0432
χ_3^2	without est.	0.3537	0.3368	0.1768	0.1725
	mom. meth.	0.3576	0.3368	0.1849	0.1696
	jackknife	0.6573	0.4667	0.3723	0.2455
	bootstrap	0.4149	0.4240	0.2005	0.2040
χ_5^2	without est.	0.6660	0.6342	0.3329	0.3249
	mom. meth.	0.6493	0.6035	0.3323	0.3261
	jackknife	1.1217	0.7704	0.6942	0.4755
	bootstrap	0.7406	0.7466	0.3767	0.3908
χ_7^2	without est.	0.9789	0.9321	0.4894	0.4775
	mom.meth.	0.9222	0.8849	0.4858	0.4918
	jackknife	1.5757	1.2816	0.9720	0.6907
	bootstrap	1.0842	1.1760	0.5232	0.5500
$\frac{3}{4}\,N(10; 2) + \frac{1}{4}\chi_3^2$	without est.	0.6044	0.5897	0.1826	0.1782
	mom. meth.	0.7709	0.7772	0.3537	0.3447
	jackknife	1.2955	0.9816	0.6434	0.5450
	bootstrap	1.1772	1.1401	0.4246	0.4595

[a] Sample median is calculated according to classical definition of median. [b] "Without est." denotes "without estimation". [c] "Mom. meth." denotes "moment method".

Source: author's calculations.

6. FINAL REMARKS

The jackknife and bootstrap methods are used for estimation of bias and variance of estimators. However, for the carried out experiments these methods did not give good results in case of sample median and sample size: 20, 21, 40, 41.

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ESTYMACJA OBCIĄŻENIA I WARIANCJI MEDIANY Z PRÓBY METODAMI *JACKKNIFE* I *BOOTSTRAP*

W pracy przedstawione są wyniki, przeprowadzonej przez autora, analizy Monte Carlo własności estymatorów typu *jackknife* i *bootstrap* mediany z próby z uwzględnieniem wpływu liczebności próby.