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GARCH MODELS OF TIME SERIES ON DAM**

Abstract. In this paper an analysis of the time series on the Day Ahead Market (DAM) of the Polish Power Exchange is presented. In this analysis Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models are used to describe the time series of rates of return of price of electric energy on DAM. This analysis is based on the data from July 2002 to June 2004.

Key words: Polish Power Exchange, Day Ahead Market, Balance Market, Autoregressive Conditional Heteroscedasticity, Generalized Autoregressive Conditional Heteroscedasticity, Maximum Likelihood Method, Akaike's information criterion, Schwarz's consistent criterion, Hannan–Quinn's consistent criterion, Rissanen's stochastic complexity criteria.

1. INTRODUCTION

The Day Ahead Market (DAM) was the first market, which was established on the Polish Power Exchange. This whole-day market consists of the twenty-four separate, independent markets where participants can freely buy and sell electricity. The breakthrough in the development of the Polish Power Exchange was made 1st July 2000, when the first transaction was completed on the DAM. Advantage of the Exchange is that all the participants of market can buy and sell electric energy, independently whether there are producers or receivers of electric energy.

Since 1[st] July 2002 Balance Market (BM) – technical market, which looks after balance on Polish energy market, has introduced additional price: Price Accounting Deviations of sale PADs and Price Accounting

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Deviations of purchase PADp. These prices should helpin expectation future demand for the electric energy on whole-day and futures market.

2. METHODOLOGY

A lot of the empirical results show that the time series of rates of return aren't dependent only on the firs moment of the data:

– the volatility of rates of return is characterizes with volatility clustering, it is cause of heteroscedasticity and the growing of variance of terror term,

– the rates of return have the leptokurtic distribution and the fat-tailed, the distribution of the returns data have the substantially heavier tails than a normal distribution,

– the volatility of rates of return is inverse correlation with the volatility of their variance – leverage effects,

– the long memory processes in the series of variance, the squares returns data are characterizes with the significant autocorrelation coefficients.

R. F. Engle (1982) introduced the Autoregressive Conditional Heteroscedasticity (ARCH) model, which incorporated into variance equation some of the stylized characteristics common to the second of moment of financial basset price information.

The ARCH(q) model is defined as

$$Z_t = \mu + \sqrt{h_t}\,\varepsilon_t \tag{1}$$

$$h_t = c_0 + \sum_{i=1}^{q} c_i Z_{t-i}^2 \tag{2}$$

where:

μ – mean of rates of return,

noise $\varepsilon_t \sim \mathrm{N}(0,1)$,

$Z_t = \ln\left(\frac{X_t}{X_{t-1}}\right)$ – logarithmic rates of return has conditional distribution $\mathrm{N}(0, h_t)$,

c_i – coefficient, $c_0, c_q > 0$, $c_i \geqslant 0$ $(i = 1, ..., q-1)$,

if $\sum_{i=1}^{q} c_i < 1$, then the time series Z_t is strict stationary,

h_t – conditional variance.

A more generalized version of ARCH, the Generalized Autoregressive Conditional Heteroscedasticity GARCH, was formulated by Engle's graduate student T. Bollerslev (1986). In comparison to the ARCH model, the GARCH model allows a potentially more complete representation of the dynamic nature of the process by which the conditional variance in financial market data may evolve.

The GARCH(p, q) model is defined as

$$Z_t = \mu + \sqrt{h_t}\,\varepsilon_t \tag{3}$$

$$h_t = c_0 + \sum_{i=1}^{q} c_i Z_{t-i}^2 + \sum_{i=1}^{p} b_i h_{t-1} \tag{4}$$

where c_0, c_q, $b_p > 0$ and otherwise coefficients are nonnegativees,

if $\sum_{i=0}^{q} c_i + \sum_{i=1}^{p} b_i < 1$, then the time series Z_t is strict stationary.

The process GARCH is characterizes with return to mean. The mean long-term variance of this process is defined as

$$V = \frac{c_0}{1 - \sum_{i=1}^{q} c_i - \sum_{i=1}^{p} b_i} \tag{5}$$

An effective method used to estimate the coefficients in ARCH(q) and GARCH(p, q) models is maximum likelihood method (ML). The coefficients are the results of maximum of a function

$$\ln L = -\frac{N}{2}\ln 2\pi - \frac{1}{2}\sum_{t=1}^{N}\ln h_t - \frac{1}{2}\sum_{t=1}^{N}\frac{Z_t^2}{h_t} \tag{6}$$

where $Z_1, \ldots, Z_N$ are empirical rates of return.

A lot of different model selection criteria are proposed in selecting an optimal ARCH model. The most of the standard class of these model selection procedures involve minimizing some loss function.

One of the most popular models is H. Akaike's (1973) information criterion, which takes the form

$$AIC = -2\ln L + 2k \quad (7)$$

where k is number of the coefficients.

G. Schwarz (1978) developed a consistent criterion based on Bayesan arguments

$$BIC = -2\ln L + 2k\ln N \quad (8)$$

where N is the sample size.

E. J. Hannan and B. G. Quinn (1979) proposed the consistent criterion for the order of an autoregressive Rusing the law of the iterated logarithm

$$HQ = -\ln L + 2k\ln(\ln N)) \quad (9)$$

J. J. Rissanen (1987) developed a model selection criterion, which is a sample approximation to a measure of stochastic complexity

$$RCL = -\ln L + \frac{k}{2}\ln N + \left(\frac{k}{2} + 1\right)\ln(k+2) \quad (10)$$

The Akaike's (7) and Schwarz's (8) criterions are most popular and very often used. H. Mitchell and M. McKenzie (2003) resumed and compared a lot of the used criteria. The results of their work, based on simulated data suggest, that HQ and RCL provide a superior level of performance for ARCH and GARCH process compared to the more commonly used criteria.

3. EMPIRICAL ANALYSIS

In this part of paper the results of estimation of ARCH and GARCH models are presented. To analysis the hourly logarithmic rates of return of price of electric energy on DAM were noted from 01.07.2002 to 30.06.2004 are used. The programs such as: EXCEL, GRETL and STATISTICA are used to calculate. The volatility of rates of return on DAM is characteristics with volatility clustering (Fig. 1).

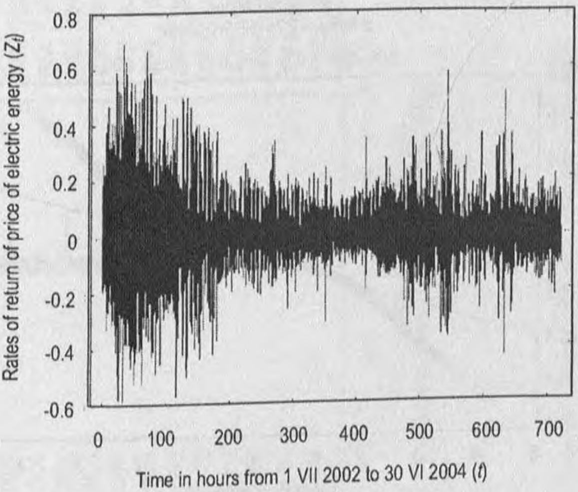

Fig. 1. Time series plot of rates of return of price of electric energy 1.07.02–30.06.04.

Source: author's own computations.

The rates of return have the leptokurtic distribution (Fig. 2) and fat-tailed (Fig. 3).

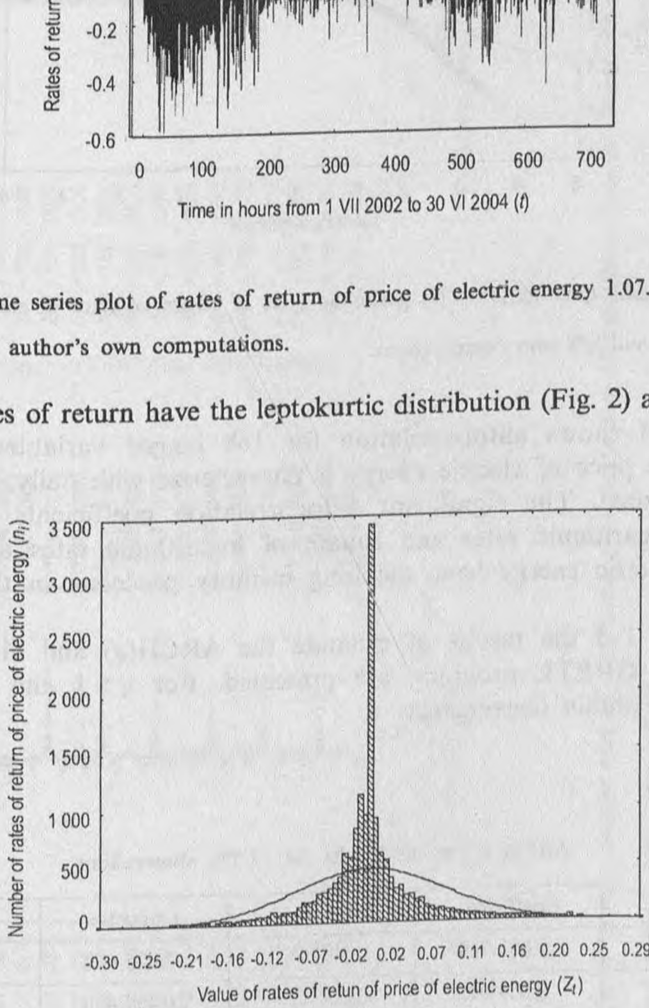

Fig. 2. Histogram of logarithmic rates of return of price of electric energy

Source: author's own computations.

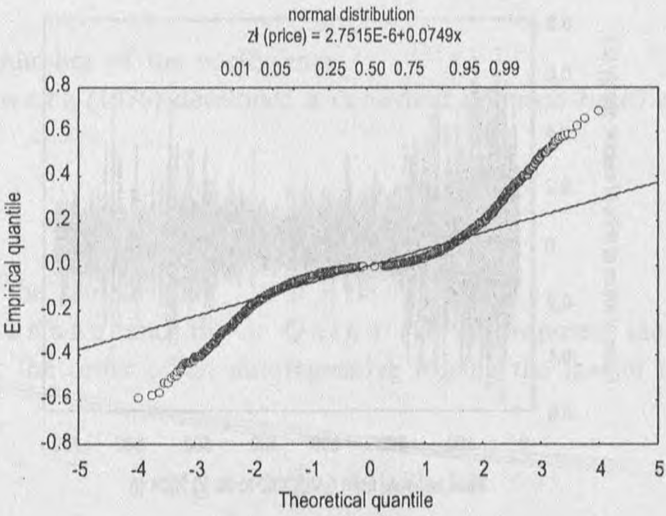

Fig. 3. Quantile-quantile plot of logarithmic rates of return of price of electric energy

Source: author's own computations.

Figure 4 shows autocorrelation for 168 lagged variables and their square. The price of electric energy is characterize with daily, weekly and yearly seasonal. The significant autocorrelation coefficients mean also, that the logarithmic rates and square of logarithmic rates of return of price of electric energy have the long memory processes in the series of variance.

In Tab. 1–3 the results of estimate the ARCH(q) and GARCH(p, q) models, by GRETL program, are presented. For $q > 1$ and $p > 2$ these models can obtain convergence.

Table 1

ARCH (1) model results for 17 276 observations

	Coefficient	Std. error	t-statistic	p-value
μ	–0.001 709	0.000 656	–2.605 600	0.009 179
c_0	0.005 092	0.000 193	26.409 400	<0.000 01
c_1	0.314 082	0.033 003	9.516 800	<0.000 01

Source: author's own computations.

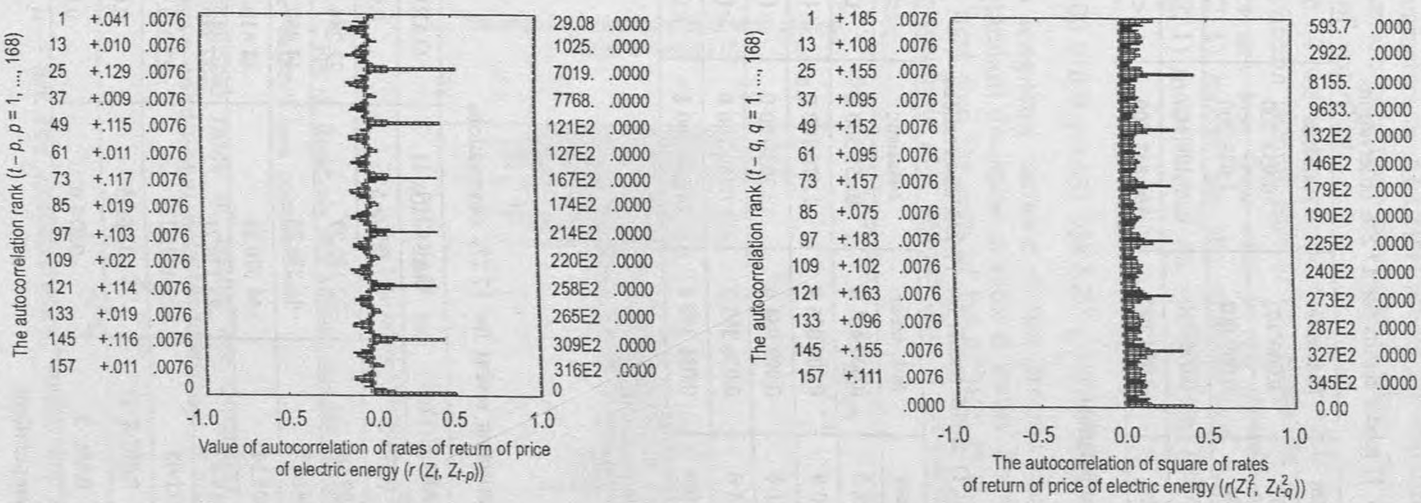

Fig. 4. Autocorrelation plot of logarithmic rates and square of logarithmic rates of return of price of electric energy

Source: author's own computations.

Table 2

GARCH (1,1) model results for 17 276 observations

	Coefficient	Std. error	t-statistic	p-value
μ	0.000 441	0.000 472	0.933 200	0.350 734
c_0	0.000 005	0.000 003	1.623 200	0.104 570
c_1	0.015 065	0.004 710	3.198 100	0.001 386
b_1	0.984 006	0.005 129	191.843 500	<0.000 01

Source: author's own computations.

Table 3

GARCH (2,1) model results for 17 276 observations

	Coefficient	Std. error	t-statistic	p-value
μ	0.000 302 5	0.000 461 6	0.655 300 0	0.512 266 0
c_0	0.000 010 9	0.000 006 9	1.573 100 0	0.115 714 0
c_1	0.031 981 8	0.009 984 6	3.203 100 0	0.001 362 0
b_1	0.194 275 0	0.026 399 3	7.359 100 0	<0.000 01
b_2	0.771 826 0	0.031 379 6	24.596 500 0	<0.000 01

Source: author's own computations.

Table 4

Model selection results for 17 276 observations

	ARCH(1)	GARCH(1, 1)	GARCH(2, 1)
Sum of coefficients	0.317 5	0.999 5	0.998 4
Log-likelihood	19 323.58	22 192.37	22 244.39
AIC	–38 643.17	–44 378.75	–44 480.77
BIC	–38 608.14	–44 326.21	–44 410.72
HQ	–19 314.47	–22 178.71	–22 226.16
RCL	–19 311.05	–22 173.71	–22 219.50
V	0.007 4	0.005 5	0.005 7
$\sqrt{V}$	0.086 2	0.074 0	0.075 4

Source: author's own computations.

All parameters in ARCH(1) model are significance. In GARCH models significance are only these coefficients, which are responsible for the lagged variables of volatility.

In Tab. 4 we compare these three models based on criterions, which were presentedin second part of this paper. In all models the sums of the coefficients are less then one, so all models are strict stationary. The GARCH(2, 1) model has the smallest loss function. We can write the GARCH(2, 1) model based on results from Tab. 3:

$$\hat{Z}_t = 0.000\,302\,5 + \sqrt{h_t}\,\varepsilon_t,$$

$$h_t = 0.000\,010\,9 + 0.031\,918\,8\,Z_{t-1}^2 + 0.194\,575\,0\,h_{t-1} + 0.771\,826\,0\,h_{t-2}.$$

The mean long-term variance of this process equals 0.0057, so the hourly residuals standard deviation of rates of return for this data set equals 7.54%.

In the next step the rests of GARCH(2, 1) model are analyzed. On the Fig. 5 the residuals plot against time is presented.

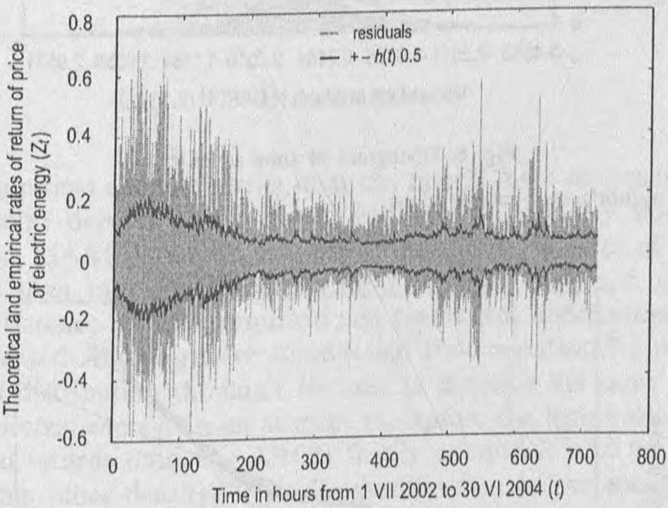

Fig. 5. Residuals plot against time the GARCH(2, 1) model

Source: author's own computations.

The empirical rates of return are described well by the generalized autoregressive conditional heteroscedasticity, if time series

$$\hat{\varepsilon}_t = \frac{Z_t - \hat{\mu}}{\sqrt{\hat{h}_t}} \sim \mathrm{N}(0, 1) \tag{11}$$

where $\hat{h}_t$, $\hat{\mu}$ – are the characteristics, which are estimated on base the Z_t process.

The GARCH models describe well the real process Z_t, if the time series of residuals (11) have normal distribution.

Unfortunately the time series of residuals of GARCH(2, 1) model have the leptokurtic distribution (Fig. 6) and fat-tailed (Fig. 7).

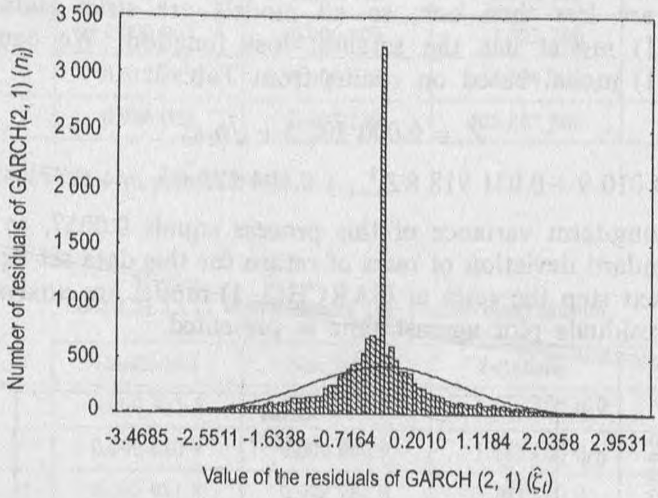

Fig. 6. Histogram of time series $\hat{\xi}_t$

Source: author's own computations.

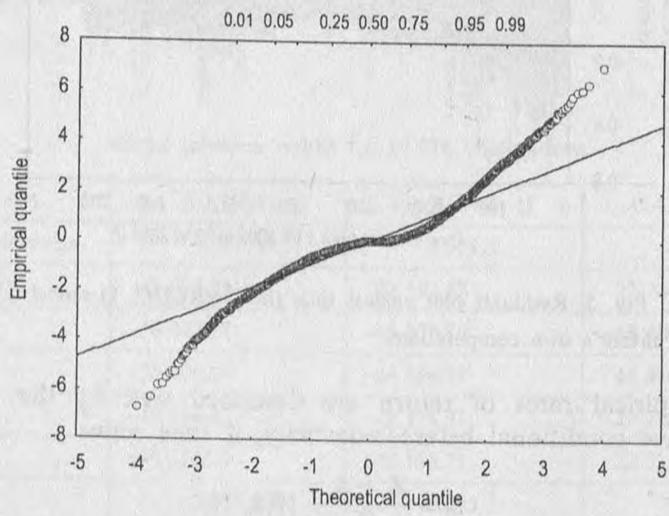

Fig. 7. Quantile-quantile plot of time series $\hat{\xi}_t$

Source: author's own computations.

The parameters of residuals series (Tab. 5) demonstrate the difference between normal distribution and the distribution of empirical residuals. The residuals have the right asymmetry and leptokurtic distribution. Standard deviation is close to one but mean of residuals equals –0.0067.

Table 5

Parameters of distribution of time series $\hat{\xi}_t$

Parameters	Values
Mean	–0.006 7
Median	–0.006 2
Mode	–
Standard deviation	1.000 3
Kurtosis	4.556 9
Skewness	0.540 0

Source: author's own computations.

4. CONCLUSION

This empirical exercise shows, that the hourly rates or return of price of electric energy depend on the lagged variables of volatility. Although that, the classical GARCH models aren't well described the rates of return, they are better than models, which establish the const variance at time. The sensible difference between empirical and theoretical distribution means that the Generalized Autoregressive Conditional Heteroscedasticity models based on normal distribution shouldn't be used to describe the rates of return of prices of electric energy. In an attempt to capture the leptokurtosis common to financial returns data, the ARCH family of models may be extended to assume some other density. Typically modification to the standard class of model GARCH involves replacing the standard normal density with some other assumed distribution for example *t*-density or the GED density.

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MODELE GARCH SZEREGÓW CZASOWYCH NA RDN

W pracy została przedstawiona analiza szeregów czasowych stóp zwrotu cen energii elektrycznej notowanych na rynku dnia następnego (RDN) Towarowej Giełdy Energii SA od lipca 2002 do czerwca 2004 r. za pomocą modeli GARCH. Celem pracy jest odpowiedź na pytanie, czy modele GARCH efektywnie opisują kształtowanie się cen energii elektrycznej na parkiecie polskiej giełdy energii i czy można je wykorzystywać do modelowania szeregów czasowych stóp zwrotu cen energii elektrycznej.