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METHODS OF ASSESSING EFFICIENCY OF BONUS-MALUS SYSTEMS

Abstract. In civil responsibility transportation insurance the insurer assesses risk, inflicted by drivers, on the basis of observable risk factors such as e.g. driver's sex and age, vehicle characteristics. However, there are risk factors unobservable directly, accounting for substantial differences of risk among drivers.

Additional piece of information about driver for the insurer is the number of claims for a given year i.e. the insured claim record.

The role of the bonus-malus systems is to verify premium height through assigning the insured to appropriate tariff class on the basis of his claim record.

The objective of this paper is to evaluate the methods of the assessment of the effectivity of the bonus-malus system.

Key words: efficiency, Markov chains, bonus-malus systems.

1. INTRODUCTION

The insurer's main task is to adjust the premium amount appropriately to the level of risk represented by drivers. The risk is understood here as the insurer's expected loss, which depends on the number and amount of losses.

Due to the fact that the insurer is not able to observe certain risk factors, they are forced to estimate the future number and amount of losses on the basis of data from the past.

The number of losses declared in particular years came to be known as the loss history in the insurance business.

A characteristic feature of Motor Third Party Liability Insurance is a system of premium increases and reductions for loss-free driving, which is aimed at verification of the premium on the basis of the insured's loss history (Hossack 1983).

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Accepting certain assumptions the system of premium increases and reductions can be modeled by means of Markov chains (Lemaire 1995).

Since different bonus-malus systems are applicable, they can be, first of all, modeled differently and, secondly, it becomes necessary to compare the effectiveness of systems.

The goal of this article is to present a model of the premium increases and reductions system for loss-free driving in Motor TPL preserving Markov chains and to determine effectiveness measures of bonus-malus systems.

2. MODEL OF THE PREMIUM INCREASES AND REDUCTIONS SYSTEM

The following assumptions have been accepted for modeling the premium increases and reductions system in Motor TPL Insurance by means of Markov chains:

1. A fixed group of drivers (the insureds) divided into risk classes called tariff classes on the basis of *a priori* characteristics is called a portfolio.

2. The number of tariff classes is finite and amounts to r. $R = \{1, 2, ..., r\}$ will denote a set of tariff class numbers. Let us accept that class $j = 1$ is burdened with the highest premium increases and class $j = r$ with the biggest reductions.

3. The insured's classification in class i in a given year is dependent upon the class, in which they were classified in the previous year and the number of losses caused in the previous year. It could be added that drivers without a loss history will be classified in the starting class.

4. The number of losses in a given year for any driver in a given class is random variable K with its probability distribution being known and constant over time. The amount of losses caused by an individual driver is random variable Y. Variables K and Y are independent variables. Random variable X is the total value of losses declared within any one time period, that is, during one year.

5. Premium b_i, $i = 1, ..., r$ is attributable to each i-th class.

Let us note that the expected loss for a driver random chosen from a definite class amounts to:

$$EX = EK \cdot EY. \tag{1}$$

With such assumptions the sequence of random variables $\{X_n\}_{n \in N}$ is such that for each $i_0, i_1, ..., i_{n-1} \in R$ and $n \in N$ occurs

$$P(X_n = j | X_0 = i_0, X_1 = i_1, ..., X_{n-1} = i) = P(X_n = j | X_{n-1} = i) \tag{2}$$

is a finite Markov chain with the state space $R = \{1, 2, ..., r\}$ (Fisz 1958).

The probability matrix of transition of a finite homogeneous Markov chain with the state space $R = \{1, 2, ..., r\}$ is the following matrix:

$$\mathbf{M} = \begin{bmatrix} p_{11} \cdots p_{1r} \\ \vdots \\ p_{r1} \cdots p_{rr} \end{bmatrix}, \tag{3}$$

where p_{ij} is a probability of the chain transition from state i to j in one step

and

$$\sum_{j \in R} p_{ij} = 1, \quad p_{ij} \geqslant 0 \quad \text{for} \quad i,\ j \in R.$$

Row i-th of matrix $\mathbf{M}$ denotes probabilities of the insured's finding themselves in the next period in each of tariff classes if the insured is in class i in a given period.

Probabilities of a transition between classes depend on principles governing a transition between classes characteristic of a system and familiarity with the distribution of losses in a given tariff class.

Let $F_k(i) = j$ denote that a driver passes from class i to class j, when they have caused k accidents during one year, with $F: R \to R$, $R = \{1, 2, ..., r\}$, $(i, j \in R,\ k = 0, 1, 2, ...)$. Function F defined in such way is called the transformation function, whereas transition principles can be written in the form of k binary matrices $\mathbf{F}_k = [f_{ij}^{(k)}]$ where

$$f_{ij}^{(k)} = \begin{cases} 1 & \text{for} \quad F_k(i) = j \\ 0 & \text{for} \quad F_k(i) \neq j \end{cases} \tag{4}$$

$i,\ j \in R,\ k = 0, 1, 2, ...$

If the loss distribution and the transformation function are known, it is possible to estimate the probability matrix of the transition of $\mathbf{M} = [p_{ij}]$ finite homogeneous Markov chain, which can be a model for the analysed system of premium increases and reductions.

If the number of losses is a random variable with Poisson distribution (Domański 2000), then the probability that a driver will cause k-losses during any single time period is expressed by formula:

$$p_k(\lambda) = \frac{\lambda^k (e^{-\lambda})}{k!}, \quad k = 0, 1, ... \tag{5}$$

On the other hand, if the number of losses is a random variable with mixed Poisson distribution, which means that λ is a random variable with distribution density of $g(\lambda)$, then the probability of causing k-losses in a given year amounts to:

$$p_k(\lambda) = \int_0^\infty p_k(\lambda) g(\lambda) d\lambda. \tag{6}$$

Formula (5) is applicable to homogeneous portfolios, whereas formula (6) finds application to heterogeneous portfolios, with λ most frequently having the gamma distribution (Hossack 1983).

In such case probabilities of a transition from class i to class j in any time period amount to:

$$p_k(\lambda) = \sum_{k=0}^{\infty} p_k(\lambda) f_{ij}^{(k)} \quad i, j \in R \tag{7}$$

or in the matrix notation:

$$\mathbf{M} = \sum_{k=0}^{\infty} p_k(\lambda) \mathbf{F}_k, \tag{8}$$

where $\mathbf{M}$ is a stochastic matrix and it is a matrix of transition probabilities of the analysed model (Lemaire 1995).

Since different bonus-malus systems can be found in practice, it becomes necessary to evaluate these systems. One of such methods is the measurement of systems efficiency.

Two different efficiency measures will be presented in this work.

3. GENERAL EFFICIENCY

Assuming that the process of insureds transition between classes is a uniform finite Markov chain with transition probability matrix $\mathbf{M}$, it is possible to estimate asymptotic probabilities of belonging to particular classes.

Let $\mathbf{W}(\lambda) = [w_1(\lambda), \ldots w_r(\lambda)]$ be a vector, whose elements are probabilities of classifying an insurance policy in i-th class, with

$$\sum_{i=0}^{r} w_i(\lambda) = 1. \tag{9}$$

Accepting the above assumptions vector $\mathbf{W}(\lambda)$ can be estimated as the left-sided characteristic vector of transition matrix $\mathbf{M}$ corresponding to characteristic value 1.

If vector $\mathbf{b} = (b_1, ..., b_r)$ is the vector of premiums, where b_i – a premium in class i, then the asymptotic mean for a single period after reaching a stationary state by the system amounts to:

$$B(\lambda) = \sum_{i=1}^{r} w_i(\lambda) \cdot b_i \tag{10}$$

and does not depend on the start class.

The function

$$\eta(\lambda) = \frac{B'(\lambda)}{B(\lambda)} \cdot \lambda \tag{11}$$

is called the general efficiency of a system.

The efficiency defined in such way is the elasticity of average premium $B(\lambda)$ in relation to the level of risk λ. Hence, it allows to estimate the degree, according to which drivers with a varying risk level are assessed by the system. In the ideal state $\eta(\lambda) = 1$. As a rule, however, changes in premiums are smaller than in the loss ratio.

The efficiency defined in such way has two shortcomings. Firstly, the stationary state of a process cannot be achieved due, for instance, to economic changes and, secondly, the efficiency assesses all drivers taken together.

4. EFFICIENCY DEPENDENT UPON THE START GROUP

Let $\mathbf{V}(\lambda) = [v_1(\lambda), ..., v_r(\lambda)]$ be a vector, whose elements are expected premiums of a driver starting from class i discounted for the beginning of insurance period. If a driver starts from class i, then the discounted payments amount to $v_i(\lambda)$.

The function

$$\mu_i(\lambda) = \frac{v_i'(\lambda)}{v_i(\lambda)} \cdot \lambda \tag{12}$$

is called efficiency dependent upon the start class i.

The stream of discounted payments amounts to:

$$v_i(\lambda) = b_i + q \sum_{k=0}^{\infty} p_k(\lambda) \cdot v_{F_k(i)}(\lambda), \quad i = 1, ..., r, \tag{13}$$

where:

b_i – denotes a premium in class i for one year,

q – discount factor,

$F_k(i) = j$ – transformation function.

Equation (13) has exactly one solution (the proof of this theorem can be found in the work Lemaire 1995).

The efficiency dependent upon the start group allows to assess how quickly drivers come to be classified in classes corresponding to the risk level represented by them. Class 1 is an optimal start group maximising the efficiency.

5. APPLICATIONS

We will present now the system of premium increases and reductions in Motor TPL Insurance of two insurance companies operating in the Polish insurance market (Tables 1 and 2).

The matrices of transition probabilities based on an assumption of average loss ratios equal to $\lambda = 0.3$ in the portfolio will be estimated for these companies.

Table 1. Premium increases and decreases in Motor TPL Insurance of A insurer

Class	% of basic premium	Number of losses			
		0	1	2	3+
1	160	2	1	1	1
2	130	3	1	1	1
3	100	4	2	1	1
4	90	5	2	1	1
5	80	6	3	1	1
6	70	7	3	2	1
7	60	8	5	3	1
8	50	9	5	4	1
9	50	10	6	5	1
10	50	11	8	5	1
11	50	12	9	6	2
12	50	13	9	6	2
13	40	13	10	7	3

Source: premium tariffs of Motor Insurance.

Note: 0 – zero losses declared during a year, 1 – one loss declared during a year, 2 – two losses declared during a year, 3+ – three or more losses declared during a year.

Binary transformation matrices $\mathbf{F}_k(i) = j$ in the A insurer's system of premium increases and reductions take the following form:

$$\mathbf{F}_0 = \begin{bmatrix} 0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \qquad \mathbf{F}_1 = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0 \end{bmatrix}$$

$$\mathbf{F}_2 = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0 \end{bmatrix} \qquad \mathbf{F}_{3+} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

The transition probability matrix for Motor TPL Insurance of A insurer on the basis of Table 1:

$$\mathbf{M} = \begin{bmatrix}
p_{1+} & p_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_{1+} & 0 & p_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_{2+} & p_1 & 0 & p_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_{2+} & p_1 & 0 & 0 & p_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_{3+} & p_2 & p_1 & 0 & 0 & p_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_{3+} & 0 & p_2 & p_1 & 0 & 0 & p_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
p_{3+} & 0 & p_2 & 0 & p_1 & 0 & 0 & p_0 & 0 & 0 & 0 & 0 & 0 \\
p_{3+} & 0 & 0 & p_2 & p_1 & 0 & 0 & 0 & p_0 & 0 & 0 & 0 & 0 \\
p_{3+} & 0 & 0 & 0 & p_2 & p_1 & 0 & 0 & 0 & p_0 & 0 & 0 & 0 \\
p_{3+} & 0 & 0 & 0 & p_2 & 0 & 0 & p_1 & 0 & 0 & p_0 & 0 & 0 \\
0 & p_{3+} & 0 & 0 & 0 & p_2 & 0 & 0 & p_1 & 0 & 0 & p_0 & 0 \\
0 & p_{3+} & 0 & 0 & 0 & p_2 & 0 & 0 & p_1 & 0 & 0 & 0 & p_0 \\
0 & 0 & p_{3+} & 0 & 0 & 0 & p_2 & 0 & 0 & p_1 & 0 & 0 & p_0
\end{bmatrix}$$

where:

p_k – probability of causing k losses during a year,

p_{k+} – probability of causing k or more losses during a year.

If we accept that distribution of the number of losses in a portfolio is Poisson distribution with the average loss ratio $\lambda = 0.3$, then $p_0 = 0.74082$, $p_1 = 0.22225$, $p_2 = 0.03334$, $p_{3+} = 0.0036$.

The transition probability matrix for A insurer takes the following form:

$$\mathbf{M} = \begin{bmatrix}
0.259 & 0.741 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.259 & 0 & 0.741 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.033 & 0.222 & 0 & 0.741 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.033 & 0.222 & 0 & 0 & 0.741 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.004 & 0.033 & 0.222 & 0 & 0 & 0.741 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.004 & 0 & 0.033 & 0.222 & 0 & 0 & 0.741 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.004 & 0 & 0.033 & 0 & 0 & 0 & 0 & 0.741 & 0 & 0 & 0 & 0 & 0 \\
0.004 & 0 & 0 & 0.033 & 0.222 & 0 & 0 & 0 & 0.741 & 0 & 0 & 0 & 0 \\
0.004 & 0 & 0 & 0 & 0.033 & 0.222 & 0 & 0 & 0 & 0.741 & 0 & 0 & 0 \\
0.004 & 0 & 0 & 0 & 0.033 & 0 & 0 & 0.222 & 0 & 0 & 0.741 & 0 & 0 \\
0 & 0.004 & 0 & 0 & 0 & 0.033 & 0 & 0 & 0.222 & 0 & 0 & 0.741 & 0 \\
0 & 0.004 & 0 & 0 & 0 & 0.033 & 0 & 0 & 0.222 & 0 & 0 & 0 & 0.741 \\
0 & 0 & 0.004 & 0 & 0 & 0 & 0.033 & 0 & 0 & 0.222 & 0 & 0 & 0.741
\end{bmatrix}$$

Table 2. Premium increases and reductions in Motor TPL Insurance of W insurer

Class	% of basic premium	Number of losses			
		0	1	2	3+
1	200	4	1	1	1
2	150	4	1	1	1
3	125	4	1	1	1
4	100	5	2	1	1
5	90	6	3	2	1
6	80	7	4	3	1
7	70	8	5	3	1
8	60	9	6	4	2
9	50	10	7	5	3
10	50	11	8	6	4
11	40	11	9	7	5

Source: Motor Insurance tariffs

Binary matrices of $\mathbf{F}_k(i) = j$ transformation in W insurer's system of premium increases and reductions on the basis of Table 2 have the form:

$$\mathbf{F}_0 = \begin{bmatrix} 0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \qquad \mathbf{F}_1 = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0 \end{bmatrix}$$

$$\mathbf{F}_2 = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0 \end{bmatrix} \qquad \mathbf{F}_{3+} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0 \end{bmatrix}$$

The transition probability matrix for W insurer's Motor TPL Insurance has the form:

$$\mathbf{M} = \begin{bmatrix} p_{1+} & 0 & 0 & p_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{1+} & 0 & 0 & p_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{1+} & 0 & 0 & p_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{2+} & p_1 & 0 & 0 & p_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{3+} & p_2 & p_1 & 0 & 0 & p_0 & 0 & 0 & 0 & 0 & 0 \\ p_{3+} & 0 & p_2 & p_1 & 0 & 0 & p_0 & 0 & 0 & 0 & 0 \\ p_{3+} & 0 & p_2 & 0 & p_1 & 0 & 0 & p_0 & 0 & 0 & 0 \\ 0 & p_{3+} & 0 & p_2 & 0 & p_1 & 0 & 0 & p_0 & 0 & 0 \\ 0 & 0 & p_{3+} & 0 & p_2 & 0 & p_1 & 0 & 0 & p_0 & 0 \\ 0 & 0 & 0 & p_{3+} & 0 & p_2 & 0 & p_1 & 0 & 0 & p_0 \\ 0 & 0 & 0 & 0 & p_{3+} & 0 & p_2 & 0 & p_1 & 0 & p_0 \end{bmatrix}$$

If we accept that the distribution of the number of losses in a portfolio is Poisson distribution with average loss ratio equal to $\lambda = 0.3$, then the transition probability matrix for W insurer looks as follows:

$$\mathbf{M} = \begin{bmatrix} 0.259 & 0 & 0 & 0.741 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.259 & 0 & 0 & 0.741 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.259 & 0 & 0 & 0.741 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.033 & 0.222 & 0 & 0 & 0.741 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.004 & 0.033 & 0.222 & 0 & 0 & 0.741 & 0 & 0 & 0 & 0 & 0 \\ 0.004 & 0 & 0.033 & 0.222 & 0 & 0 & 0.741 & 0 & 0 & 0 & 0 \\ 0.004 & 0 & 0.033 & 0 & 0.222 & 0 & 0 & 0.741 & 0 & 0 & 0 \\ 0 & 0.004 & 0 & 0.033 & 0 & 0.222 & 0 & 0 & 0.741 & 0 & 0 \\ 0 & 0 & 0.004 & 0 & 0.033 & 0 & 0.222 & 0 & 0 & 0.741 & 0 \\ 0 & 0 & 0 & 0.004 & 0 & 0.033 & 0 & 0.222 & 0 & 0 & 0.741 \\ 0 & 0 & 0 & 0 & 0.004 & 0 & 0.033 & 0 & 0.222 & 0 & 0.741 \end{bmatrix}$$

If λ can accept any free value then elements of matrix **M** will be the functions of variable λ. General efficiency and efficiency dependent upon the start class for A and W insurance companies will be estimated accepting an assumption that distribution of the number of losses is Poisson distribution and distribution of 6% interest rate. The results are shown in graphic form in Figures 1, 2 and 3.

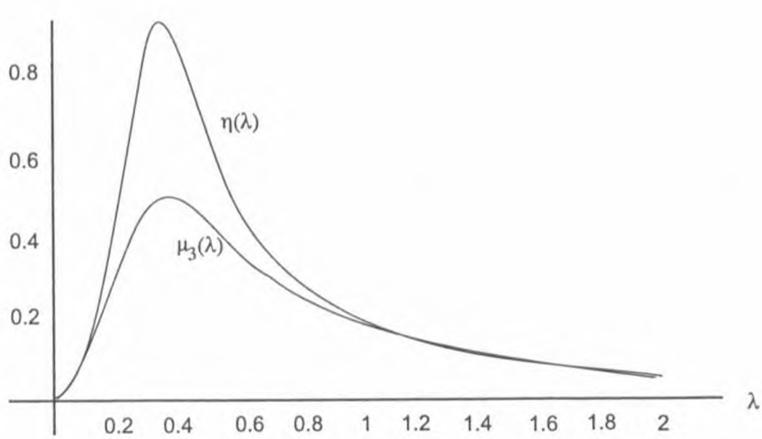

Fig. 1. General efficiency and efficiency dependent upon the start group for A insurer

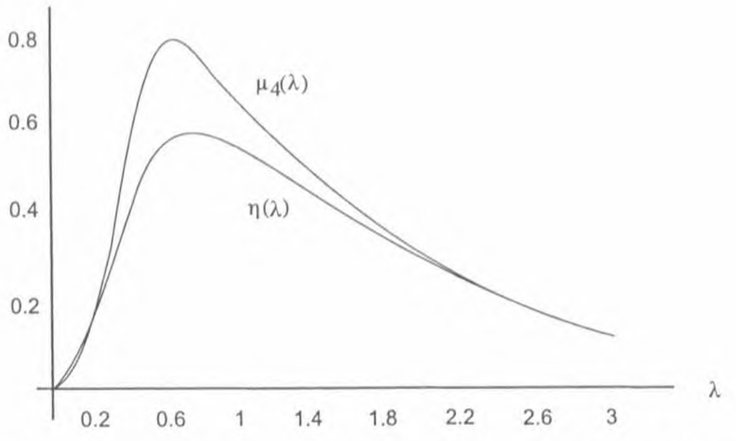

Fig. 2. General efficiency and efficiency dependent upon the start group for W insurer

In the case of A insurer general efficiency accepts much bigger values than efficiency dependent upon the start group, which points to a good evaluation of drivers by the system. Since efficiency $\mu_i(\lambda)$ has small values, the system evaluates drivers during quite a long time. In the case of W insurer, the evaluation if quick but not very precise.

Comparing general efficiencies alone:

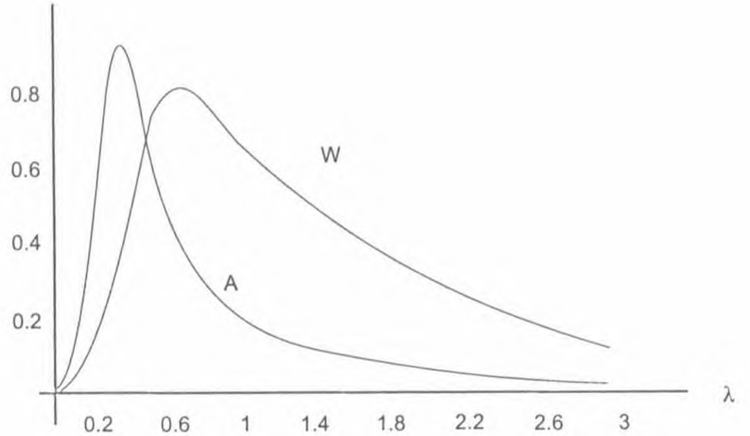

Fig. 3. General efficiency of A and W insurance companies

It can be stated on the basis of Figure 3 that A company assesses the risk better $\lambda < 0.5$, whereas W company $\lambda > 0.5$.

6. FINAL REMARKS

Wishing to classify drivers correctly insurers should expand the bonus-malus system. However, too expanded systems may not be of Markov chain type and then other efficiency measures should be sought. The faster and the more precisely a system evaluates the risk the more favorable are insurance terms both for insureds and insurers.

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METODY OCENY EFEKTYWNOŚCI SYSTMÓW *BONUS-MALUS*

(Streszczenie)

W ubezpieczeniach komunikacyjnych OC ubezpieczyciel szacuje ryzyko, jakie reprezentują kierowcy, na podstawie obserwowalnych czynników ryzyka, takich jak np.: płeć i wiek kierowcy, cechy pojazdu. Jednak istnieją czynniki ryzyka, bezpośrednio nieobserwowalne, istotnie różnicujące kierowców pod względem poziomu ryzyka.

Dodatkową informacją dla ubezpieczyciela o kierowcy jest liczba zgłoszonych w danym roku szkód, czyli przebieg szkodowości ubezpieczonego.

Zadaniem systemów *bonus-malus* jest weryfikacja składki poprzez przyporządkowanie ubezpieczonego do odpowiedniej klasy taryfowej, na podstawie przebiegu szkodowości ubezpieczonego.

Celem artykułu jest wskazanie metod oceny efektywności systemów *bonus-malus*.