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USING CONTROL CHARTS TO DETECT SMALL PROCESS SHIFTS

Abstract

The selection of proper SPC charts is essential to effective statistical process control implementation and use. It is important to use best chart for the given situation and need. Using Shewhart quality control charts to detecting small process shift is not effective. This paper shows that the cumulative-sum control charts (CUSUM) and Exponentially Weighted Moving Average control charts (EWMA) are appropriate to detect these shifts.

Key words: quality, cumulative-sum control charts, CUSUM, Exponentially Weighted Moving Average control charts, EWMA.

I. INTRODUCTION

The term „quality'' is defined as any factor that enhanced the value of a product in the eyes of the customer. In order to produce a product that meets customer requirements, it is of utmost importance to have a process operating on target. Quality control has become a key part of every manufacturing environment.

The most implemented to achieve process control are often referred to as statistical process control (SPC). By far the most implemented SPC control charts are the Shewhart-type charts. However, Shewhart-type charts are incapable of detecting small, incremental process shifts. In Shewhart control charts, all emphasis is placed on the last sample point plotted. Small, but increasing shifts take a long time to show up on a chart. For example, if, due to machine wear, a process slowly "slides" out of control to produce results above target specifications, this plot would show a steadily increasing (or decreasing) cumulative sum of deviations from specification. We can use runs tests to increase the sensitivity, but they create more false alarms.

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In the automated manufacturing environment the small shifts are more likely to occur. If one is interested in a small, sustained shift in a process, other types of control charts may be preferred, for example the cumulative-sum (CUSUM) control charts and an Exponentially Weighted Moving Average (EWMA).

In this article, we show both of these control charts.

II. THE CUSUM CONTROL CHART FOR MONITORING THE PROCESS MEAN

CUSUM chart uses all historical up to the present sample point. The charts display cumulative sums of the deviations of measurements, or subgroup means, from a target value. If μ_0 is the target from the process mean, $\overline{x}_j$ is the average of the j^{th} sample, then the cumulative-sum control chart is formed by plotting the quantity:

$$C_i = \sum_{j=1}^{i} (\overline{X}_j - \mu_0). \tag{1}$$

So we are adding up how far we were from the process mean each time. If the mean has shifted up, we are likely to be above the mean each time and that will accumulate to a signal. Another method is to keep track of each side of the mean separately.

Let x_i be the i^{th} observation on the process. If the process is in control then $x_i \sim N(\mu_0, \sigma)$. Assume σ is known or can be estimated. Accumulate derivations from the target μ_0 above the target with one statistic is C^+.

Accumulate derivations from the target μ_0 below the target with another statistic is C^-. C^+ and C^- are one-sided upper and lower cusums, respectively.

The statistics are computed as follows:

$$C_i^+ = \max\{0, x_i - (\mu_0 + K) + C_{i-1}^+\}, \tag{2}$$

$$C_i^- = \max\{0, (\mu_0 + K) - x_i + C_{i-1}^-). \tag{3}$$

Starting values are $C_0^+ = C_0^- = 0$. K is the reference value (or allowance or slack value). If either statistic exceeds a decision interval H (often taken as a $H = 5\sigma$), the process is considered to be out of control.

If we are above the mean for a few subgroups, the plus side accumulates. Once we go below the mean for a subgroup: the plus side goes to zero, the minus side starts to accumulate. Notice that we have now the mean plus k standard deviation. The value of k fine tunes the CUSUM chart.

K is often chosen halfway between the target μ_0 and the out-of-control value of the mean μ_1 that we are interesting in detecting quickly. When shift is expressed in standard deviation units as $\mu_1 = \mu_0 + \delta\sigma$, then K is

$$K = \frac{\delta}{2}\sigma = \frac{|\mu_1 - \mu_0|}{2}. \tag{4}$$

If the adjustment has to be made to the process, may be helpful to estimate the process mean following the shift. The estimate can be computed from:

$$\hat{\mu} = \begin{cases} \mu_0 + K + \dfrac{C_i^+}{N^+}, & C_i^+ > H \\ \mu_0 - K - \dfrac{C_i^-}{N^-}, & C_i^- > H \end{cases}. \tag{5}$$

CUSUM 'V-masks' are used to detect shifts in either direction from the target mean and give a simple way of applying decision rules to segments of data.

The dimensions of the V-mask can by specified using two distinct sets of two parameters:

– θ, defined as half of the angle formed by the V-mask arms, and d, the distance between the origin and the vertex, as shown in Figure 1. This parameterization is used by Montgomery (1991).

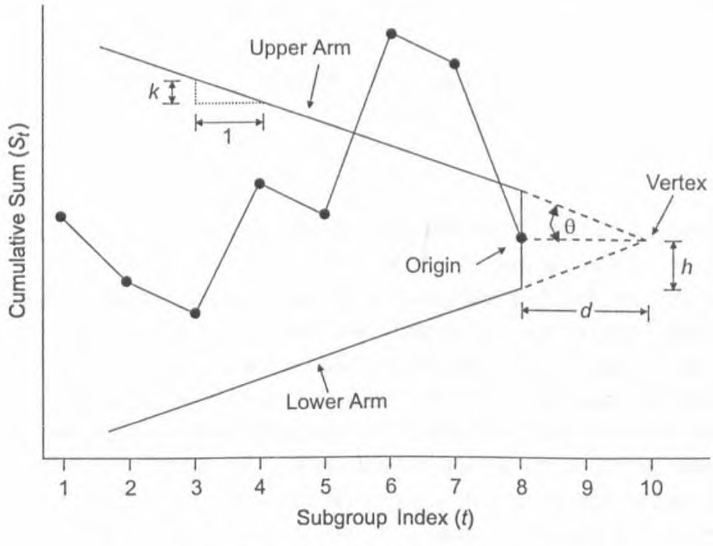

Figure 1. V-Mask parameters

– h, the vertical distance between the origin and the upper (or lower) V-mask arm, and k, the rise (drop) in the lower (upper) arm corresponding to an interval of one subgroup unit on the horizontal axis. You can specify the definition of interval with the INTERVAL = option. This parameterization is used by Lucas (1976).

In this article, we use the first parameterization.

The two parameterizations are related by the equations:

$$\theta = \arctan(k/a), \tag{6}$$

$$d = h/k. \tag{7}$$

where the aspect ratio a is the number of units on the vertical axis corresponding to one unit on the horizontal axis.

The V-mask is specified in terms of error probabilities: α (type I error) and β (type II error). If we provide α and β, h and k can be computed using the formulas:

$$h = |\delta|^{-1}\log((1-\beta)/(\alpha/2)), \tag{8}$$

$$k = |\delta|/2. \tag{9}$$

If we provide α but not β, h and k can be computed using the following formulas:

$$h = -|\delta|^{-1}\log(\alpha/2), \tag{10}$$

$$k = |\delta|/2. \tag{11}$$

In that case the error probability α is divided by two because two-sided deviations from the target mean are detected.

The origin of the V-mask is located at the most recently plotted point. As additional data are collected and the cumulative sum sequence is updated, the origin is relocated at the newest point. A shift or out-of-control signaled at time t if one or more of the point plotted up to time t cross an arm of the V-mask. An upward shift is signaled by point(s) crossing the lower arm, a downward shift is signaled by point(s) crossing the upper arm. The time at which the shift occurred corresponds to the time at which a distinct change is observed in the slope of the plotted points.

III. THE EXPONENTIALLY WEIGHTED MOVING AVERAGE CONTROL CHART MONITORING THE PROCESS MEAN

The Exponentially Weighted Moving Average (EWMA) is defined as:

$$z_i = \lambda x_i + (1-\lambda)z_{i-1}, \tag{12}$$

where

$$0 < \lambda \leqslant 1 \text{ is a constant,}$$

$$z_0 = \mu_0 \text{ (sometimes } z_0 = \overline{x}).$$

The control limits for the EWMA control chart are:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]}, \tag{13}$$

$$CL = \mu_0, \tag{14}$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]}. \tag{15}$$

where L is the width of the control limits.
As i gets larger, the term $[1-(1-\lambda)^{2i}]$ approaches infinity. So the control limits settle down to

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}}, \tag{16}$$

$$CL = \mu_0, \tag{17}$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}}. \tag{18}$$

EWMA is sometimes called a *geometric moving average*, since the weights of past observations are declining as in a geometric series. The choice of λ determines the decline of the weights. Small values provide more smoothing and better ability to see small changes. If $\lambda \to 0$, then the most recent observation receives a small weight, whereas the weight attached to previous observations only slightly declines with the age of the observations. In general, $0.05 \leqslant \lambda \leqslant 0.25$ works well in practice. $L = 3$ works reasonably well, especially with the larger value of λ. L between 2.6 and 2.8 is useful when $\lambda \leqslant 0.1$.

IV. AN EXAMPLE

Consider the following simulated manufacturing process involving a drill press, where we may reasonably estimate the process to be centered around 4 mm. Currently, this process is being monitored by obtaining rational subgroups of size 4 at regular intervals, and that these selected parts are measured using an acceptable measuring system.

Table 1. Simulated data

Sample	Value1	Value2	Value3	Value4	Sample	Value1	Value2	Value3	Value4
1	4.00440	3.99801	3.99614	4.00066	37	4.00007	4.00076	4.00134	4.00069
2	3.99894	4.00075	3.99824	4.00109	38	3.99920	4.00029	4.00371	4.00275
3	4.00014	4.00299	3.99798	3.99931	39	3.99953	4.00028	4.00018	3.99894
4	3.99657	4.00176	4.00005	4.00461	40	3.99828	3.99908	3.99661	4.00002
5	3.99852	3.99847	4.00168	3.99988	41	4.00042	3.99568	3.99687	4.00171
6	4.00213	4.00043	4.00134	4.00101	42	3.99976	4.00109	4.00091	3.99941
7	3.99720	4.00532	3.99746	3.99595	43	4.00029	3.99986	3.99526	4.00086
8	3.99721	3.99954	4.00084	3.99839	44	3.99740	4.00022	3.99849	4.00037
9	3.99947	3.99755	4.00027	4.00106	45	4.00079	4.00051	3.99953	4.00531
10	3.99916	3.99571	4.00055	3.99831	46	4.00157	3.99647	4.00118	3.99800
11	4.00045	3.99841	4.00040	3.99719	47	4.00019	4.00107	4.00221	4.00230
12	4.00150	4.00032	4.00066	4.00155	48	3.99800	4.00167	4.00010	3.99773
13	3.99677	4.00163	3.99666	3.99852	49	3.99986	3.99674	4.00033	4.00171
14	3.99961	4.00006	4.00076	4.00377	50	4.00034	3.99869	4.00231	3.99934
15	3.99886	4.00015	3.99980	3.99895	51	4.00216	4.00214	3.99786	4.00440
16	3.99522	3.99782	4.00149	3.99911	52	4.00146	3.99904	4.00030	3.99701
17	3.99961	3.99908	4.00005	3.99775	53	4.00047	4.00137	4.00339	3.99660
18	4.00203	4.00116	4.00418	4.00195	54	4.00284	3.99999	4.00474	3.99611
19	4.00266	3.99901	4.00429	3.99920	55	4.00198	3.99978	4.00038	3.99922
20	4.00015	3.99713	4.00015	4.00223	56	4.00252	4.00253	3.99780	4.00290
21	3.99982	3.99926	3.99884	4.00138	57	4.00424	3.99793	4.00121	4.00122
22	4.00157	4.00062	4.00534	4.00146	58	3.99836	4.00105	4.00101	3.99857
23	4.00106	3.99866	4.00163	3.99854	59	4.00095	3.99863	4.00103	3.99724
24	4.00114	3.99961	3.99846	4.00136	60	3.99795	3.99775	3.99911	3.99923
25	3.99861	3.99841	4.00060	3.99901	61	4.00138	4.00325	3.99998	4.00351
26	3.99582	4.00007	4.00174	4.00039	62	3.99671	4.00081	3.99812	4.00230
27	4.00262	4.00234	4.00189	4.00002	63	4.00030	4.00272	3.99917	3.99783
28	4.00006	4.00126	4.00471	4.00147	64	3.99704	3.99863	3.99956	3.99517
29	3.99892	4.00224	3.99536	3.99835	65	4.00126	4.00284	3.99719	3.99556
30	3.99832	4.00247	3.99971	3.99737	66	3.99827	4.00116	4.00102	3.99879
31	3.99678	3.99876	4.00250	4.00128	67	4.00189	3.99994	3.99770	3.99859
32	4.00112	3.99869	4.00125	4.00310	68	4.00058	4.00151	3.99917	3.99881
33	3.99825	4.00166	4.00335	3.99694	69	4.00293	4.00038	3.99866	3.99813
34	4.00310	4.00035	4.00250	4.00028	70	3.99931	4.00464	3.99726	4.00149
35	3.99865	4.00056	4.00089	4.00138	71	4.00228	4.00170	4.00132	4.00094
36	4.00412	4.00056	4.00120	3.99871	72	3.99964	4.00007	4.00201	4.00162

Table 1. (contd.)

Sample	Value1	Value2	Value3	Value4	Sample	Value1	Value2	Value3	Value4
73	4.00141	4.00047	4.00237	3.99665	87	3.99910	4.00250	3.99787	3.99876
74	3.99961	3.99919	3.99945	4.00276	88	3.99833	3.99824	4.00461	3.99630
75	3.99898	3.99851	3.99835	3.99754	89	3.99707	4.00073	4.00068	3.99857
76	3.99776	3.99870	3.99620	3.99931	90	3.99765	4.00019	3.99820	3.99750
77	4.00026	4.00032	4.00039	4.00024	91	4.00030	3.99951	3.99732	3.99858
78	3.99924	3.99978	4.00098	3.99914	92	4.00023	3.99970	3.99917	3.99556
79	3.99885	3.99547	3.99773	3.99881	93	4.00000	3.99858	4.00072	3.99937
80	4.00074	3.99931	3.99654	4.00031	94	3.99300	4.00000	3.99700	4.00100
81	3.99769	4.00055	3.99751	3.99700	95	3.99300	4.00000	3.99900	4.00200
82	3.99920	4.00047	4.00021	3.99805	96	3.98900	4.00000	3.99000	4.00138
83	3.99949	4.00257	3.99840	4.00176	97	3.99680	4.00000	4.00100	3.99800
84	4.00049	4.00250	4.00121	3.99733	98	4.00026	3.99900	4.00010	4.00030
85	4.00252	3.99733	4.00058	4.00018	99	3.99871	4.00000	3.99864	3.99914
86	3.99996	4.00057	3.99770	4.00294	100	3.99903	3.99969	3.99721	3.99659

Firstly, we consider CUSUM control charts.
For $\alpha = \beta = 0.003$ and *detectedshift* $= \mu_0 \pm 1 \times \sigma = 4 \pm 0.002071$ ($3.997929 \leqslant$ *shift* $\leqslant 4.002071$) should be detected), the CUSUM chart is followed (Fig. 2).

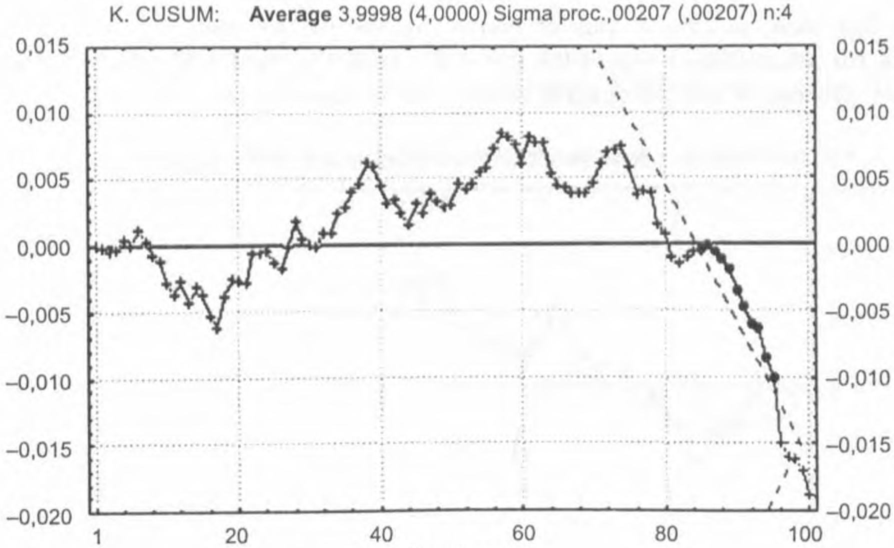

Figure 2. The CUSUM chart for $\mu_0 \pm 1 \times \sigma = 0.002071$

The CUSUM control chart indicates the process is out of control in 96 sample.
For *detectedshift* $= \mu_0 \pm 0.9 \times \sigma = 4 \pm 0.001864$ we have the following results (Fig. 3).

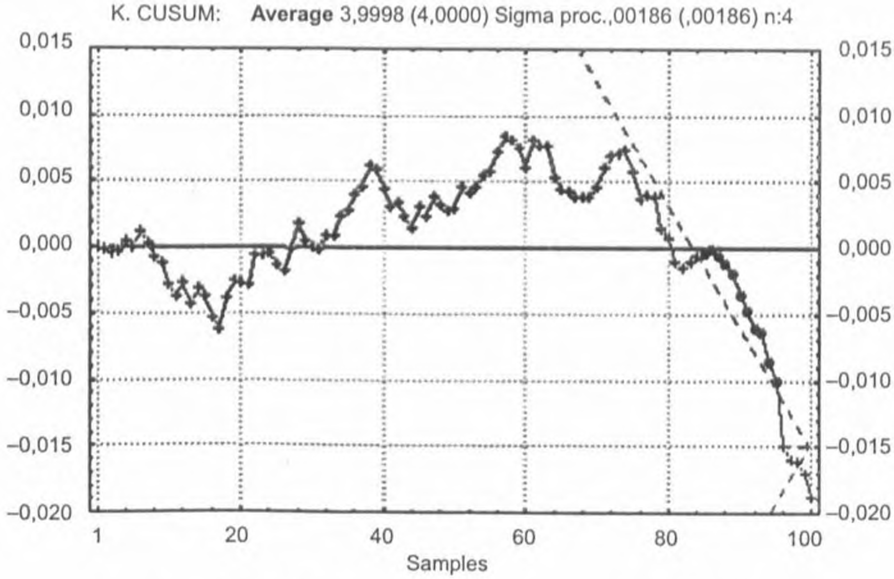

Figure 3. The CUSUM chart for $\mu_0 \pm 0.9 \times \sigma = 4 \pm 0.001864$

In this case, process is out of control in 96 sample, too.
But for $detectedshift = \mu_0 \pm 0.4 \times \sigma = 4 \pm 0.000828$ the CUSUM chart shows that process is out of control earlier, in 94 sample. (see Fig. 4).

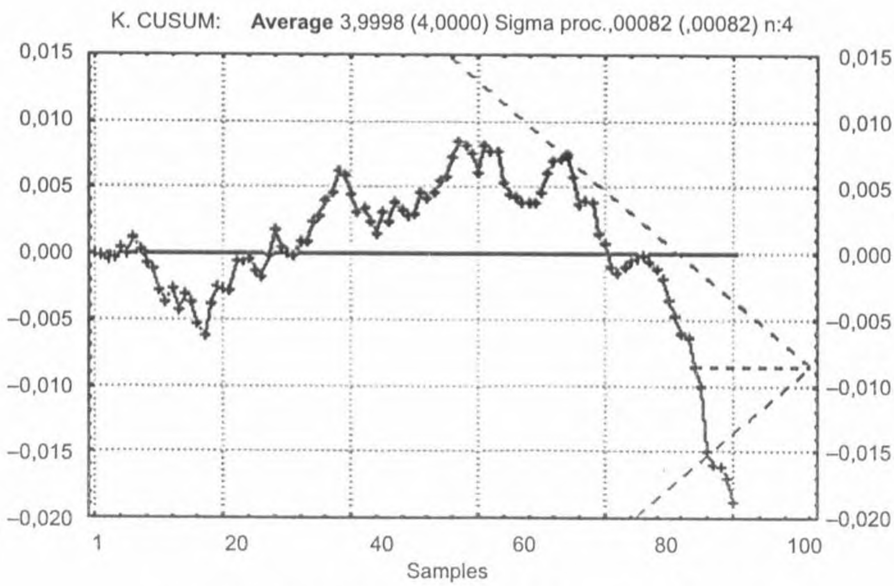

Figure 4. The CUSUM chart for $\mu_0 \pm 0.4 \times \sigma = 4 \pm 0.000828$

In Table 2 we demonstrate the performance of four CUSUM schemes, with different choices of α, β, *detectedshift*.

Table 2. Example of CUSUM control schemes

No. of case	1	2	3	4
α	0.003	0.003	0.003	0.05
β	0.003	0.003	0.003	0.05
detectedshift	4 ± 0.0006213	4 ± 0.0004142	4 ± 0.0002071	4 ± 0.0002071
Sample out-of control	95	97	process is in-control	80

As you can notice, for different parameters we become so different results. The choice of these parameters is very important to have reliable results.

According to results, then we get, we may believe that process is out of control in 94, 95, 96 and 97 sample. We should stop this process, find the reason of the shift and delete it. Then we could start new analysis of this process.

We may consider that there was false alarm in 80 sample; there was only random shift of the process (the probabilities: α (type I error) and β (type II error) are high).

Let's make an EWMA analysis for the data from Table 1.

For $\lambda = 0.2$ and $L = 2.86$ we have the following EWMA chart (Fig. 5).

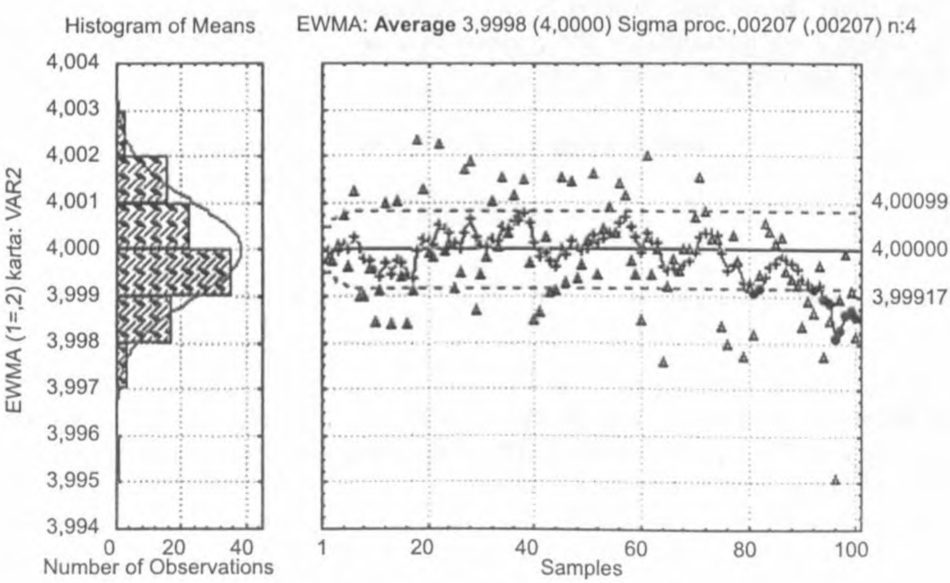

Figure 5. The EWMA chart for $\lambda = 0.2$ and $L = 2.86$

According to this chart, the process is out-of-control in 94 sample. For $\lambda = 0.2$ and $L = 2.4$ the situation has changed, as follows (Fig. 6).

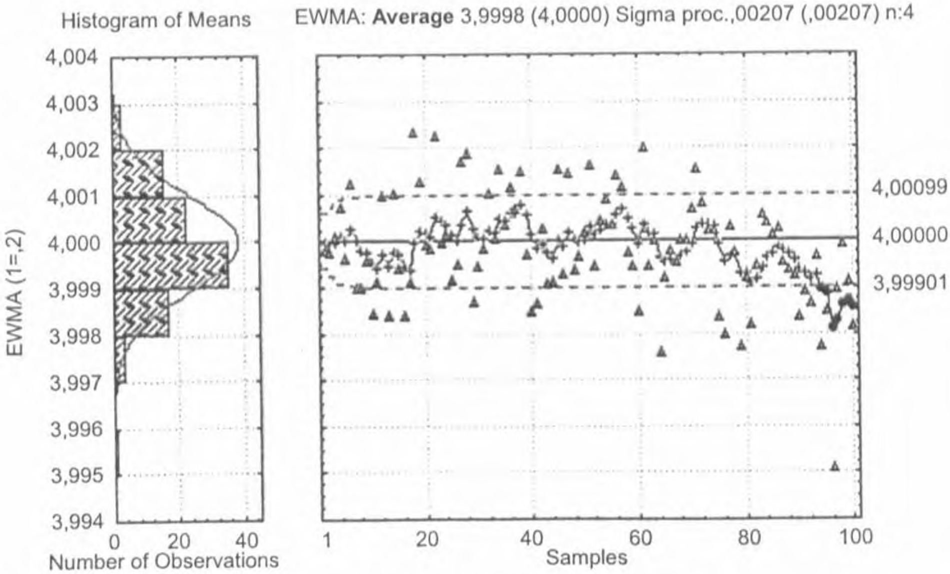

Figure 6. The EWMA chart for $\lambda = 0.2$ and $L = 2.4$

This chart shows that process is out-of-control in 80 sample.
In Table 3 we demonstrate the performance of four EWMA schemes, with different choices of λ and L for $\mu_0 = 4$.

Table 3. Example of EWMA control schemes

No. of case	1	2	3	4
λ	0.2	0.25	0.15	0.02
L	2.5	2.8	3	3
Sample out-of control	80; 94	94	94	96

As you can see, the appropriate selection of λ and L is critical for effective application of this charting technique. These control charts show, that we may believe that process is out-of-control in sample 94. There was unimportant (for whole process) shift in 80 sample.

V. CONCLUSIONS

Finally, we have the similar results using CUSUM and EWMA charts analysis. CUSUM charts consider all observed samples with the same wage. Sometimes (when the quantity of samples is too much) CUSUM chart may detect shift with delay (the shift was in 94 sample – CUSUM chart detected this just in 96, 97 sample).

CUSUM are less effective for large shifts than EWMA. But EWMA is more complicated and less tolerant for bad parameters.

The analysis of ARL for our CUSUM and EWMA charts seems to be essential to compare these methods. Average Run Length (ARL) is the average time until a shift of a specified size is detected (shift specified in terms of standard deviation of the charted characteristic to eliminate scale effects). ARL (0) is average time until false alarm occurs (no shift is occurred). ARL (1) is average time until a true shift is detected. The good chart analysis has a small ARL(0) and ARL(1).

ARL for EWMA is very sensitive to the selection of weighting factors. Therefore, it is very important to choose correct value of α to get desired ARL. Unfortunately, the calculations of ARL are very complicated and can't be done without special program.

Each charting technique has certain advantages and disadvantages. To detect small shifts in the process, both of charts (CUSUM and EWMA) are effective. Using these charts we should remember that the choice of parameters is very important to make correct decision.

Using simultaneously Shewhart's charts (good for large shifts) and CUSUM (or EWMA) charts seems to be reasonable for improving process monitoring.

REFERENCES

Bower, K.M., Using Exponentially Weighted Moving Average (EWMA) charts, http://www.minitab.com/company/virtualpressroom/Articles/UsingEWMAcharts.pdf

Hryniewicz O. (1996), *Nowoczesne metody statystycznego sterowania jakością*, Instytut Badań Systemowych PAN, Warszawa.

Iwasiewicz A., Paszek Z. (2000), *Statystyka z elementami statystycznych metod sterowania jakością*, Wyd. Akademii Ekonomicznej w Krakowie, Kraków.

Lucas J. (1976), The design and use V-mask control schemes, *Journal of Quality Technology*, **8**, 1–12

Montgomery D. (1991), *Introduction to Statistical Quality Control*, John Wiley & Sons, New York.

Thompson J.R., Koronacki J. (1994), *Statystyczne sterowanie procesem. Metoda Deminga etapowej optymalizacji jakości*, Akademicka Oficyna Wydawnicza PLJ, Warszawa.

Statsoft, Inc. (1997), *Statistica for Windows*, vol. 4: Industrial Statistics Computer program manual, Tulsa.

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ZASTOSOWANIE KART KONTROLNYCH DO WYKRYWANIA NIEWIELKICH ZAKŁÓCEŃ KONTROLOWANEGO PROCESU

Streszczenie

Niezwykle ważny dla efektywności zastosowań statystycznego sterowania procesem jest dobór odpowiednich kart kontrolnych. Użycie kart kontrolnych Shewharta w celu wykrycia niewielkich zakłóceń procesu jest nieefektywne. W niniejszym artykule przedstawiono zastosowanie karty sum skumulowanych (CUSUM) oraz karty wykładniczo ważonych ruchomych średnich (EWMA) do wczesnego wykrywania niewielkich zakłóceń procesu produkcyjnego.