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APT MODEL FOR ELECTRICITY PRICES ON THE DAY AHEAD MARKET OF THE POLISH POWER EXCHANGE

Abstract

In this paper we presented the model of the dependence of the electricity price on macroeconomic factors such as changes in the dollar price, the Deutsche mark price, the rate of inflation, the rate of unemployment, price changes in the mining industry, the production of the manufacturing sector, the output of the mining industry and weather conditions. The aim of this article was the empirical verification of the price model on the Day Ahead Market (DAM) of the Polish Power Exchange in 2001 based on the principal components method.

The results were compared with the results for the APT model, selected by means of the graph analysis method and the optimum choice method proposed by Z. Hellwig. The aim of this work was to choose the best model for the description of price trends on the DAM.

Key words: The Day Ahead Market, Arbitrage Pricing Theory, factors analysis, the principal components, eigenvalue, eigenvector, the graph analysis method, and the optimum choice method proposed by Z. Hellwig

I. INTRODUCTION

The Day Ahead Market (DAM) was the first market, which was established on the Polish Power Exchange. This whole-day market consists of twenty-four separate, independent markets, where participants can freely buy and sell electricity. The breakthrough in the development of the Polish Power Exchange was made on 1st July 2000, when the first transactions were completed on the Day Ahead Market.

The energy market is not a neutral isolated structure. In this paper we attempt to answer the question, how the electricity price depends on the

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macroeconomic factors. To examine these dependencies we use the APT model. While selecting the variables for the model of price changes we use three methods: the factors analysis, the optimum choice method proposed by Z. Hellwig and the graph analysis method.

II. THE APT MODEL

The Arbitrage Pricing Theory is based on the 'one price' rule. The differences in prices of the same goods can be affected both by systematic risk factors and accidental risk factors. We take into account the following items:

R_t – rate of return from a price,

$E(R_t)$ – expected rate of return from a price,

f_{it} – rate of return of i^{th} systematic risk factor,

$\beta_i - i = 1, ..., k$ sensitivity coefficient of the rate of return against the i^{th} systematic risk factor,

ξ_t – accidental risk factor, with the mean $E(\xi_t) = 0$ and the covariance $D(\xi_t) = \sigma^2$, $\sigma \in R_+$.

In this paper we have applied the APT model:

$$R_t = E(R_t) + \beta_1 f_{1t} + \beta_2 f_{2t} + ... + \beta_k f_{kt} + \xi_t, \quad t = 1, ..., n. \tag{1}$$

The equation presented in example (1) can be expressed with the matrix equation. We notice:

$\mathbf{R} - (n \times 1)$ – vector of rates of return of prices,

$\mathbf{E(R)} - (n \times 1)$ – vector of expected value rates of return of prices,

$\mathbf{F} - (n \times k)$ – matrix of rates of return of systematic factors of risk, where $\mathbf{E(F)} = 0$, $\mathbf{E(F\,F^T)} = \mathbf{I}$,

$\boldsymbol{\beta} - (k \times 1)$ – vector of coefficients sensitivity of rates of return of systematic factors of risk,

$\boldsymbol{\xi} - (n \times 1)$ – vector of error terms with n – normal distribution $E(\xi) = \mathbf{0}$, $D(\xi) = \sigma^2 \mathbf{I}$, $\sigma \in R_+$.

With the supplementary assumptions:

$$E(\xi\, \mathbf{F}^T) = \mathbf{0}, \tag{2}$$

$$E[R - E(R)] = \mathbf{0}, \tag{3}$$

we can write the model APT:

$$R = E(R) + F\beta + \xi. \tag{4}$$

The price arbitrage will not be possible when the number of values grows. So we can write this model with the equation:

$$R_t = \beta_0 + \beta_1 f_{1t} + \beta_2 f_{2t} + \dots + \beta_k f_{kt} + \xi_t, \quad t = 1, \dots, n. \tag{5}$$

III. FACTOR ANALYSIS

In the model we use the variables which we can observe and measure. These variables often depend on other factors which we cannot immediately describe. These factors are called latent variables or factors. To separate these factors we use the factor analysis. In this paper we separate the latent variables, which we have applied, making use of the principal components method. We notice:

$\mathbf{F} = (f_1, f_2, \dots, f_k]^T$ – vector of observational variables,

$\mathbf{X} = [X_1, X_2, \dots, X_k]^T$ – vector of principal components,

$\mathbf{A} = [a_1, a_2, \dots, a_k]$ – orthogonal and normality matrix.

Then we can express the **X** vector of transformation

$$\mathbf{X} = \mathbf{A}^T\mathbf{F}. \tag{6}$$

The X_j principal components have the following properties:

$$D^2(X_1) > D^2(X_2) > \dots > D^2(X_k), \tag{7}$$

$$\sum_{j=1}^{k} D^2(f_j) = \sum_{j=1}^{k} D^2(X_j). \tag{8}$$

We determine X_j making use of eigenvalues and eigenvectors of the covariance matrix of f_j observational variables. If we define the λ_j-eigenvalue of the j^{th} eigenvector of the covariance matrix, then

$$D^2(X_j) = \lambda_j, \quad j = 1, \dots, k. \tag{9}$$

Equation (8) means that all the observational variables are described by all principal components and by eigenvalue (9). If w_j denotes the contribution of X_j to the explanation of observational variables, we can write:

$$w_j = \frac{\lambda_j}{\sum_{i=1}^{k} \lambda_i}, \quad j = 1, \dots, k. \tag{10}$$

In the model we use only these principal components which have the biggest share in explaining the variance of observational variables. In the paper we use two criteria to determine the number of principal components. The first one, developed by Keiser, takes into account only these principal components, eigenvalues of which are close to one or higher than one. The second criterion is proposed by Cattel. It eliminates these principal components, eigenvalues of which decrease very slightly (Ostasiewicz, (ed.) 1999).

IV. EMPIRICAL ANALYSIS

In order to verify the APT model for the DAM electricity price in 2001 we have used monthly changes in macroeconomic factors (Tab. 1).

Table 1. Description of symbols and definition of variables

Symbol of variable	Name of variable	Description of variable
RC_t	Change in electricity price	Monthly average DAM price of electricity (C_t) in PLN $RC_t = \frac{C_t - C_{t-1}}{C_{t-1}}$
RW_t	Change in electricity volume	Monthly average DAM electricity volume (W_t) in MWh $RW_t = \frac{W_t - W_{t-1}}{W_{t-1}}$
f_{1t}	Change in the rate of inflation	Rate of inflation (I%) $f_{1t} = I/100$
f_{2t}	Change in the rate of unemployment	Rate of unemployment (w%) $f_{2t} = w/100$
f_{3t}	Price changes in the mining industry	Coefficient of prices of sold production in the mining industry (GK_t) $f_{3t} = GK_t/100 - 1$
f_{4t}	Price changes in the manufacturing industry	Coefficient of prices of sold production in the manufacturing industry (PP_t) $f_{4t} = PP_t/100 - 1$
f_{5t}	Price changes in the industry producing and delivering energy, gas and water	Coefficient of prices of sold production in the industry producing and delivering energy, gas and water (EGW_t) $f_{5t} = EGW_t/100 - 1$

Table 1. (contd.)

Symbol of variable	Name of variable	Description of variable
f_{6t}	Change of dollar price	NBP D_t dollar price in PLN $f_{6t}=\frac{D_t-D_{t-1}}{D_{t-1}}$
f_{7t}	Change of Deutsche mark price	NBP DM_t Deutsche mark price in PLN $f_{7t}=\frac{DM_t-DM_{t-1}}{DM_{t-1}}$
f_{8t}	Change in the output of the mining industry	Output of the mining industry (WK_t) in thousands of tonnes $f_{8t}=\frac{WK_t-WK_{t-1}}{WK_{t-1}}$
f_{9t}	Temperature change	Average temperature noted in Warsaw (T_t) in °C $f_{9t}=\frac{T_t-T_{t-1}}{T_{t-1}}$
f_{10t}	Change in cloudiness	Average cloudiness noted in Warsaw (Z_t) in octanes $f_{10t}=\frac{Z_t-Z_{t-1}}{Z_{t-1}}$
f_{11t}	Change in sunlight exposition	Average sunlight exposition noted in Warsaw (U_t) in h $f_{11t}=\frac{U_t-U_{t-1}}{U_{t-1}}$

For these variables we use the factor analysis. In Table 2 we present the eigenvalue of the covariance matrix for these variables.

Table 2. Eigenvalue of factors

Number of factor X_j	Eigenvalue	% total variance	Cumulative	Cumulative variance in %
1	3.31	27.57	3.31	27.57
2	2.66	22.13	5.96	49.69
3	1.96	16.29	7.92	65.98
4	1.34	11.19	9.26	77.18
5	0.95	7.92	10.21	85.10
6	0.70	5.85	10.91	90.95
7	0.48	4.01	11.39	94.96
8	0.43	3.59	11.83	98.55
9	0.10	0.83	11.93	99.38
10	0.06	0.51	11.99	99.89
11	0.01	0.11	12.00	100.00

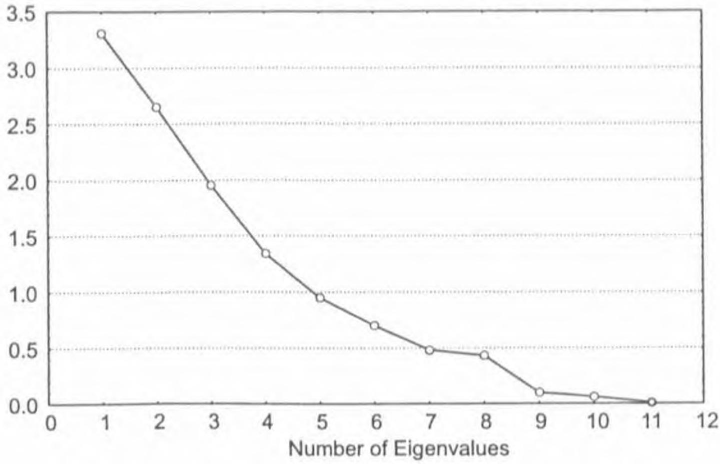

Figure 1. Plot of eigenvalues

Based on Cattel's criterion we use the first eight principal components. We incorporate these macroeconomic variables which have the greatest loadings in relation to latent variables into the APT model.

Table 3. Factor loadings (varimax normalized)

Variable	Factor X_1	Factor X_2	Factor X_3	Factor X_4	Factor X_5	Factor X_6	Factor X_7	Factor X_8
RW_t	0.06	0.01	**–0.95**	–0.01	0.01	–0.15	0.11	–0.23
f_{1t}	0.17	0.14	–0.11	0.04	0.17	**–0.94**	–0.09	–0.09
f_{2t}	0.12	**–0.85**	0.06	–0.18	0.04	0.08	0.33	0.31
f_{3t}	**–0.75**	–0.12	–0.20	–0.25	–0.35	0.31	0.16	–0.02
f_{4t}	**–0.86**	0.13	0.34	0.04	0.11	0.01	–0.21	0.19
f_{5t}	0.02	0.17	0.00	**0.97**	0.06	0.08	0.12	–0.09
f_{6t}	–0.27	0.00	0.27	0.43	0.03	0.69	–0.26	0.01
f_{7t}	**–0.76**	–0.06	–0.09	0.18	0.10	0.56	–0.04	–0.20
f_{8t}	0.04	0.16	–0.27	0.09	0.18	–0.04	0.14	**–0.92**
f_{9t}	–0.07	0.22	0.12	–0.11	0.08	0.01	**–0.95**	0.13
f_{10t}	0.03	–0.12	0.02	–0.04	**–0.97**	0.09	0.06	0.17
f_{11t}	0.15	**0.78**	0.06	0.12	0.54	–0.14	–0.04	0.08

Bold – Alpha reliabilities at $\alpha = 0{,}05$.

We have built the APT model on the ground of Keiser's criterion. We have selected the following variables as the first four principal components: change in electricity volume, change in the rate of inflation, change in the rate of unemployment, price changes in the mining industry, price changes in the manufacturing industry, price changes in the industry producing and delivering energy, gas and water, change in the Deutsche mark price, change in the output of the mining industry. This model is insignificant with alpha reliabilities of $\alpha = 0.05$ based on the Fisher-Snedecor test.

The model built based on the first principal component, is insignificant. If we increase the number of principal components, we obtain insignificant models, too. In the next step we analyse only these factors, which have the absolute value of loadings higher than 0.8. This way the model comprises: change in electricity volume, change in the rate of unemployment, price changes in the manufacturing industry, price changes in the industry producing and delivering energy, gas and water, change in cloudiness, change in the rate of inflation, change in the output of the mining industry, and temperature change. We use the ordinary least squares method (OLS) to estimate the parameters of the model:

$$RC_t = \alpha_0 + \alpha_1 f_{1t} + \alpha_2 f_{2t} + \alpha_4 f_{4t} + \alpha_5 f_{5t} + \alpha_8 f_{8t} + \alpha_9 f_{9t} + \alpha_{10} f_{10t} + \alpha_{12} RW_t + \xi_t, \tag{11}$$

With alpha reliabilities at $\alpha = 0,05$ of T-statistics and F-statistics we receive the model:

$$\hat{RC}_t = \underset{(0.067)}{0.272} - \underset{(0.408)}{1.788_{2t}} - \underset{(0.306)}{1.850 f_{4t}} + \underset{(0.120)}{0.855 f_{5t}} - \underset{(0.018)}{0.260 f_{8t}} - \underset{(0.006)}{0.020 f_{10t}} + \underset{(0.002)}{0.032 RW_t} \tag{12}$$

with change in the rate of unemployment, price changes in the manufacturing industry, price changes in the industry producing and delivering energy, gas and water, change in the output of the mining industry, change in cloudiness, and change in electricity volume.

In the econometric model we use the method which is proposed by Hellwig (Ganczarek, 2002). We choose the variable combination with three variables: change in electricity volume, price changes in the industry producing and delivering energy, gas and water and change in the Deutsche mark price. The integral capacity of this combination is the greatest of all possible combinations of the variables from Table 1.

$$H = 0,5545.$$

We use the ordinary least squares estimate parameters of the model for this combinations:

$$RC_t = \alpha_0 + \alpha_5 f_{5t} + \alpha_7 f_{7t} + \alpha_{12} RW_t + \xi_t, \tag{13}$$

and we receive:

$$\hat{RC}_t = -0.016 + 0.762 f_{5t} - 0.236 f_{7t} + 0.023 RW_t. \quad (14)$$
$$(0.008)\quad (0.564)\quad (0.194)\quad (0.007)$$

In the third model we select the variables based on the graph analysis method and receive the following variables: change in the rate of unemployment, price changes in the mining industry, the manufacturing industry, and the industry producing and delivering energy, gas and water, as well as change in the output in the mining industry (Ganczarek 2002). The APT model for electricity prices can described with the equation:

$$RC_t = \beta_0 + \beta_2 f_{2t} + \beta_3 f_{3t} + \beta_4 f_{4t} + \beta_5 f_{5t} + \beta_8 f_{8t} + \beta_{12} RW_t + \xi_t. \quad (15)$$

We use the ordinary least squares estimate parameters for this model and with alpha reliabilities at $\alpha = 0{,}05$ of T-statistics and F-statistics we receive:

$$\hat{RC}_t = 0.219 - 1.455 f_{2t} + 0.806 f_{5t} - 0.234 f_{8t} + 0.034 RW_t. \quad (16)$$
$$(0.072)\quad (0.444)\quad (0.522)\quad (0.022)\quad (0.003)$$

V. COMPARISON OF THE EFFECTIVENESS OF THE MODELS

We compare the three models with the changes in electricity price listed on the Day Ahead Market in 2001 and the results of the analysis are presented below.

Table 4. Comparison of the values of changes in electricity price on the Day Ahead Market in 2001 with the values of the presented models

Month	Change in price on DAM	Factor analysis (12)	Hellwig's method (14)	Graph analysis (16)	The residuals of model (12)	The residuals of model (14)	The residuals of model (16)
1	0.0355	0.0380	0.0444	0.0379	–0.0025	–0.0089	–0.0024
2	0.0088	0.0095	–0.0119	0.0055	–0.0007	0.0207	0.0033
3	0.0069	0.0056	0.0212	0.0079	0.0012	–0.0143	–0.0010
4	0.0248	0.0268	0.0031	0.0281	–0.0019	0.0217	–0.0033
5	–0.0135	–0.0168	–0.0074	–0.0150	0.0033	–0.0061	0.0015
6	–0.0142	–0.0141	–0.0191	–0.0155	–0.0001	0.0050	0.0014
7	–0.0030	–0.0026	–0.0045	–0.0054	–0.0004	0.0014	0.0024
8	–0.0194	–0.0202	–0.0274	–0.0257	0.0009	0.0080	0.0063
9	–0.0532	–0.0523	–0.0349	–0.0442	–0.0009	–0.0183	–0.0090
10	–0.0433	–0.0428	–0.0055	–0.0431	–0.0004	–0.0378	–0.0002
11	0.0402	0.0362	0.0195	0.0405	0.0040	0.0207	–0.0002
12	–0.0050	–0.0025	–0.0128	–0.0062	–0.0025	0.0078	0.0012

Table 5. Errors of the models

Symbol of variable	The APT model (12) Factor analysis	The APT model (24) Model of Hellwig	The APT model (16) Model of analysis of graph
R_W	0.997	0.777	0.991
R^2	0.994	0.605	0.982
S_u	0.004	0.021	0.006
Durbin-Watson	1.386	1.470	1.450

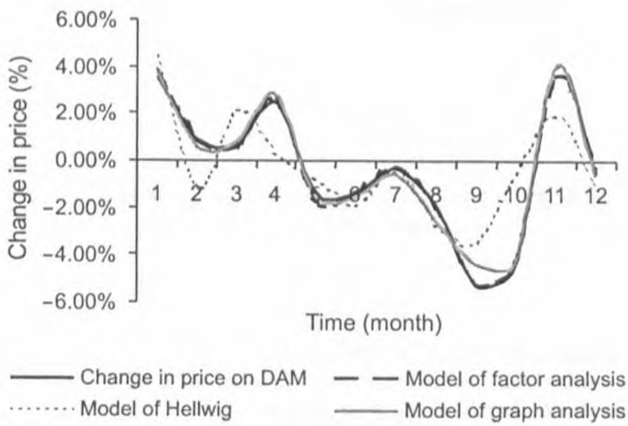

Figure 2. Values of empirical and theoretical changes in electricity price on DAM

Figure 2 shows that the APT model (12), which is based on the factor analysis, describes changes in the electricity price on DAM in 2001 most accurately. The standard deviation (S_u) of this model is smaller than in other models. The determination coefficient $R^2 = 0.994$ means that this model describes the price change on DAM with the accuracy of 99.4%. But we should not compare the determination coefficients (R^2) and multi-correlation coefficient (R_W), because the models have a different number of variables. Durbin-Watson test has the lowest value for the model (12), so this model has the highest autocorrelation of residuals.

In this paper we have presented three methods of selecting variables for econometric models and we developed three different models. In my opinion, the model which we developed with the principal components method is the best. But the selection depends on individual preferences and can be different.

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MODEL APT DLA CENY ENERGII ELEKTRYCZNEJ NA RDN GIEŁDY ENERGII SA

Streszczenie

W pracy przedstawiliśmy model zależności zmiany ceny energii elektrycznej od czynników makroekonomicznych, takich jak zmiany: kursu dolara, kursu marki, inflacji, bezrobocia, cen produkcji w górnictwie, kopalnictwie oraz przetwórstwie przemysłowym, wydobyciu węgla kamiennego oraz czynników pogodowych. Przedmiotem badań jest empiryczna weryfikacja modelu ceny na RDN Giełdy Energii SA w 2001 r. z wykorzystaniem metody głównych składowych.

Otrzymane wyniki skonfrontowaliśmy z wynikami uzyskanymi dla modelu APT, w którym do doboru składowych modelu zastosowaliśmy metodę analizy grafów i metodę optymalnego wyboru predyktant zaproponowanych przez Z. Hellwiga. Celem tej pracy jest wyłonienie modelu efektywniej opisującego kształtowanie się cen na RDN.