

*Tadeusz Gerstenkorn**

A COMPOUND OF AN INFLATED PASCAL DISTRIBUTION WITH THE POISSON ONE

Abstract

In this paper there is presented a compound of an inflated Pascal distribution with the Poisson one.

In the introductory part of the paper is giving an overview of the last results in topic of compounding of distributions, considering also the Polish results. In succeeding Sections, probability function of the compound distribution Pascal-Poisson, factorial, crude and incomplete moments as well recurrence relations of this distribution are presented.

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Key words: compound distributions, compound inflated Pascal-Poisson distribution, inflated Pascal distribution, Poisson distribution, factorial, crude and incomplete moments, recurrence relations for the moments.

I. INTRODUCTION

The problem of the compounding of probability distributions goes back to the twenties of the 20th century. It is worth here to mention the papers by M. Greenwood and G.U. Yule (1920; they took the parameter of the Poisson distribution as a gamma variate) and E.S. Pearson (1925; he gave a device of making the parameter of a distribution of Bayes theorem). In forties the problem was dealt with by L. Lundberg (1940; the Pólya distribution as a compound one), F.E. Satterthwaite (see a remark on the comparison of Satterthwaite's idea of the generalized Poisson distribution with a compound distribution in W. Feller's paper, 1943; 390), W. Feller (1943; some compound distributions as "contagious" ones), G. Skellam (1948; the binomial-beta giving Pólya-Eggenberger distribution), M.E. Cansado

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(1948; compound Poisson distributions). Some interrelations among compound and generalized distributions gave J. Gurland (1957). Important theoretical problems are contained in papers by H. Teicher (1960, 1962), W. Molenaar (1965) and W. Molenaar and W.R. Zwet (1966). Also in sixties we find the results by T. Śródka (1964; Rayleigh with gamma, gamma with Rayleigh; 1966 – Laplace with normal $N(0, 3)$ left truncated at zero, with gamma, Rayleigh, Weibull, exponential), by S.K. Katti and J. Gurland (1967; Poisson-Pascal), D.S. Dubey (1968; compound Weibull). Then we have papers by D. S. Dubey (1970; compound gamma, beta and F), T. Gerstenkorn and Śródka (1972; generalized gamma with gamma), H. Jakuszenkow (1973; Poisson with normal $N(m, \sigma)$ left truncated at zero, with generalized two-parameter gamma, with Maxwell and Rayleigh), W. Dyczka (1973; binomial with beta (incomplete moments)) T. Gerstenkorn (1982; binomial with generalized beta), T. Gerstenkorn (1994; generalized gamma with exponential), T. Gerstenkorn (1996; Pólya with beta), G.E. Wilmot (1989; Poisson-Pascal and others). M.L. Huang and K.Y. Fung (1993; D compound Poisson distribution as a new extension of the Neyman Type A distribution).

A wide list of references to the problem considered one can find in review studies by N.L. Johnson and S. Kotz (1969; chapter 8: 183–215), G. P. Patil et al. (1968) and nowadays by G. Wimmer and G. Altmann (1999).

II. THE COMPOUNDING OF PROBABILITY DISTRIBUTIONS

Below, we give a definition and relations needed for the compounding of distributions.

Definition 1. The compound distribution

Let a random variable X_y have a distribution function $F(x|y)$ depending on a parameter y . Suppose that the parameter y is considered as a random variable Y with a distribution function $G(y)$. Then the distribution having the distribution function of X defined by

$$H(x) = \int_{-\infty}^{\infty} F(x|cy)dG(y) \quad (1)$$

will be called compound, with that c is an arbitrary constant or a constant bounded to some interval (Gurland, 1957; 265).

The occurrence of the constant c in (1) has a practical justification because the distribution of a random variable, describing a phenomenon,

often depends on the parameter which is a realization of another random variable multiplied, however, by a certain constant.

The variable which has distribution function (1) will be written down in symbol as " $X \wedge Y$ " and called a compound of the variable X with respect to the "compounding" Y . Relation (1) will be written symbolically as follows:

$$H(x) \equiv F(x|cy) \wedge_y G(y). \quad (2)$$

Consider the case when both variables are discrete, the first one with a probability function $P(X = X_i|kn)$ depending on a parameter n being a random variable N with a probability function $P(N = n)$.

Then (1) is expressed by the formula

$$h(x_i) = P(X = x_i) = \sum_{n=0}^{\infty} P(X = x_i|N = kn) \cdot P(N = n). \quad (3)$$

III. THE INFLATED PASCAL \wedge POISSON

Definition 2. The distribution of the form

$$P_s(X = i|Y) = \begin{cases} 1 - s + s \cdot P(X = i_0|Y) & \text{for } i = i_0, \\ s \cdot P(X = i|Y) & \text{for } i = 0, 1, 2, \dots, i_0 - 1, i_0 + 1, \dots, n, \end{cases} \quad (4)$$

where $0 \leq s \leq 1$ and the variable Y , being in the condition, has a distribution $G(y)$, we call an inflated conditional distribution. It is a discrete distribution with an inflation of the distribution $P(X = i|Y)$ at a point $i = i_0$.

Distribution (4) were introduced, named as inflated, by S.N. Singh (1963) and M.P. Singh (1965–66, 1966) and later discussed by many authors. Some information about these results one can find in T. Gerstenkorn (1977) and G. Wimmer, G. Altmann (1999).

Lemma 1. The compounding of an inflated distribution with another distribution gives a distribution with the same inflation.

The proof, as rather easy one, is here omitted.

Theorem 1. We assume that X is a variable with inflated Pascal distribution $Pa_s(nk, p)$ depending on a parameter n , i.e.

$$Pa_s(nk, p) = Pa_s(X = i | N = n) = \begin{cases} 1 - s + s \frac{(nk)^{[i_0, -1]}}{i_0!} p^{i_0} q^{nk} & \text{for } i = i_0 \\ s \frac{(nk)^{[i, -1]}}{i!} p^i q^{nk} & \text{for } i = 0, 1, 2, \dots, i_0 - 1, i_0 + 1, \dots \end{cases}$$

where $p > 0$, $k > 0$, $p + q = 1$ and

$$x^{[l, -1]} = x(x+1)(x+2)\dots(x+l-1)$$

is the so-called ascending factorial polynome.

Let N be a variable with the Poisson distribution $P_o(n, \lambda)$

$$P_o(N = n, \lambda) = \frac{\lambda^n}{n!} e^{-\lambda}, \quad n = 0, 1, 2, \dots, \quad \lambda > 0.$$

By using the formula (3) we get

$$h_s(i) = \begin{cases} 1 - s + s \frac{p^{i_0} \mu_{[i_0]}}{i_0!} \exp(\theta - \lambda) & \text{for } i = i_0, \\ s \frac{p^i \mu_{[i]}}{i!} \exp(\theta - \lambda) & \text{for } i = 0, 1, \dots, i_0 - 1, i_0 + 1, \dots \end{cases} \quad (5)$$

where $p > 0$, $\lambda > 0$, $\theta = q^k$ and

$$\mu_{[i]} = \int_{-\infty}^{\infty} x^{[i, -1]} dF(x) \quad (6)$$

is the so-called ascending (or reverse) factorial moment determined by the equality:

$$\mu_{[i]} = E[(kX)^{[i, -1]}] = \sum_{n=0}^{\infty} (kn)^{[i, -1]} \frac{\theta^n}{n!} \exp(-\theta), \quad (7)$$

Proof. At first we determine $h_s(i_0) = P(X = i_0)$:

$$\begin{aligned} h_s(i_0) &= P_s(X = i_0) = \\ &= \sum_{n=0}^{\infty} \left(1 - s + s \frac{(nk)^{[i_0, -1]}}{i_0!} p^{i_0} q^{nk} \right) \frac{\lambda^n}{n!} \exp(-\lambda) = \\ &= 1 - s + s \frac{p^{i_0} \exp(-\lambda)}{i_0!} \sum_{n=0}^{\infty} (nk)^{[i_0, -1]} \frac{(q^k \lambda)^n}{n!}. \end{aligned}$$

Analogously we calculate $h_s(i)$.

If in (5) we take $s = 1$ we obtain the compound Pascal-Poisson distribution without inflation

$$h(i) = \frac{p^i \mu_{[i]}}{i!} \exp(\theta - \lambda), \quad i = 0, 1, 2, \dots$$

R. Shumway and J. Gurland (1960) have also obtained this result but using a method of generalization of distributions.

The moment (6) or (7) is to be determined in dependence on crude or factorial moment.

In the paper by R. Shumway and J. Gurland (1960; 92) we find the following formula

$$\mu_{[r]} = \sum_{i=1}^r S_i^r k^i \sum_{j=1}^i S_j^i \alpha_{[j]}$$

where S_j^i are Stirling numbers of the second kind, k is a parameter and $\alpha_{[j]}$ is a factorial moment

$$\alpha_{[j]} = \int_{-\infty}^{\infty} x^{[j]} dF(x)$$

where $x^{[j]} = x(x-1)(x-2)\dots(x-i+1)$ and S_i^r are Stirling numbers of the third kind.

In general the factorial moment $\alpha_{[j]}$ are easier to be calculated than the crude ones. But one can pass from the factorial moments to the crude ones.

This problem has been exactly discussed in the paper by T. Gerstenkorn (1983) where one can also find a table of Stirling numbers of the third kind.

IV. THE COMPOUND PASCAL-POISSON DISTRIBUTION AND ITS MOMENTS

The compound Pascal-Poisson without inflation can be written symbolically

$$Pap(k, p, \lambda) \sim Pa(nk, p) \wedge_n P_o(\lambda)$$

This distribution has been presented in an equivalent form by S.K. Katti and J. Gurland (1961) as a generalization of the Poisson distribution with the Pascal one.

Corollary 1. The moment of the r^{th} order of the compound distribution Pascal-Poisson is the mean value of an appropriate moment of Pascal distribution if the parameter n appearing in the formulae of moments of the Pascal distribution is considering as a random variable of the Poisson distribution, i. e.

$$b_{[r]}(k, p, \lambda) = a_{[r]}(nk, p) \wedge_n P_o(\lambda)$$

$$b_r(k, p, \lambda) = a_r(nk, p) \wedge_n P_o(\lambda).$$

Taking in account these formulae, we have

Theorem 2. The factorial moment $b_{[r]}(k, p, \lambda)$ of the compound distribution Pascal-Poisson is expressed by

$$b_{[r]}(k, p, \lambda) = \left(\frac{p}{q}\right)^r \mu_{[r]}^{P_o} \quad (8)$$

where $\mu_{[r]}^{P_o}$ is the reverse factorial moment of the Poisson distribution, i.e.

$$\mu_{[r]}^{P_o} = \sum_{n=0}^{\infty} (nk)^{[r, -1]} \frac{\lambda}{n!} \exp(-\lambda). \quad (9)$$

Proof. $b_{[r]}(k, p, \lambda) = \left(\frac{p}{q}\right)^r (nk)^{[r, -1]} \wedge_n P_o(\lambda) = \left(\frac{p}{q}\right)^r ((nk)^{[r, -1]} \wedge_n P_o(\lambda)) = \left(\frac{p}{q}\right)^r \mu_{[r]}^{P_o}$.

Already in G. Bohlmann (1913; 398) we find the relation between the crude and factorial moments

$$b_r = \sum_{j=0}^r S_j^r b_{[j]} \quad (10)$$

where S_j^r are Stirling numbers of the second kind. The table of these numbers can be found, for instance, in A. Kaufmann (1968) or in J. Łukaszewicz and M. Warmus (1956).

Corollary 2. Directly from (8) and (10) we have

$$b_r(k, p, \lambda) = \sum_{j=0}^r S_j^r \left(\frac{p}{q}\right)^j \mu_{[j]}^{P_o} \quad (11)$$

where S_j^r are Stirling numbers of the second kind.

Recurrence relations for crude moments

Theorem 3. Crude relations of the compound Pascal-Poisson $Pap(k, p, \lambda)$ distribution are expressed by

$$1) \quad b_{r+1}(k, p, \lambda) = \sum_{i=0}^r \binom{r}{i} \sum_{j=0}^i S_j^i b_{[j+1]}(k, p, \lambda), \quad (12)$$

$$2) \quad b_{r+1}(k, p, \lambda) = \sum_{j=0}^r S_j^r b_{[j+1]}(k, p, \lambda) + \frac{p}{q} \frac{d}{d\left(\frac{p}{q}\right)} b_r(k, p, \lambda). \quad (13)$$

Proof

Ad 1)

$$\begin{aligned} b_{r+1}(k, p, \lambda) &= a_{r+1}(nk, p) \wedge_n P_o(\lambda) = \\ &= nk \frac{p}{q} \sum_{i=0}^r \binom{r}{i} a_i(nk+1, p) \wedge_n P_o(\lambda) = \\ &= \frac{p}{q} \sum_{i=0}^r \binom{r}{i} (nk a_i(nk+1, p) \wedge_n P_o(\lambda)). \end{aligned} \quad (14)$$

Now we will calculate the expression in parentheses using (10), some properties of factorial polynomials, and the following proposition:

Proposition 1. Factorial moment of the Pascal $P_a(nk, p)$ distribution is given by

$$\alpha_{[r]} = \left(\frac{p}{q}\right)^r (nk)^{[r, -1]}$$

(see: Dyczka, 1973: 223 (63), (64)).

$$\begin{aligned} nk a_i(nk+1, p) \wedge_n P_o(\lambda) &= \\ &= nk \sum_{j=0}^i S_j^i \left(\frac{p}{q}\right)^j (nk+1)^{[j, -1]} \wedge_n P_o(\lambda) = \\ &= \sum_{j=0}^i S_j^i \left(\frac{p}{q}\right)^j (nk(nk+1))^{[j, -1]} \wedge_n P_o(\lambda) = \\ &= \sum_{j=0}^i S_j^i \left(\frac{p}{q}\right)^j (nk)^{[j+1, -1]} \wedge_n P_o(\lambda) = \sum_{j=0}^i S_j^i \left(\frac{p}{q}\right)^j \mu^{[j+1]}. \end{aligned}$$

Taking account this result and (11) in (13) we have (12).

Ad 2) W. Dyczka (1973: 225 (70)) has shown that

$$a_{r+1}(nk, p) = \frac{p}{q} (nka_r(nk+1, p) + \frac{d}{d\left(\frac{p}{q}\right)} a_r(nk, p)).$$

Using this formula in (14) and regarding the first component, we have

$$\begin{aligned} nk \frac{p}{q} a_r(nk+1, p) \wedge_n P_o(\lambda) &= nk \frac{p}{q} \sum_{j=0}^r S_j^r \left(\frac{p}{q}\right)^j (nk+1)^{[j, -1]} \wedge_n P_o(\lambda) = \\ &= \sum_{j=0}^r S_j^r \left(\frac{p}{q}\right)^{j+1} (nk)^{[j+1, -1]} \wedge_n P_o(\lambda) = \\ &= \sum_{j=0}^r S_j^r \left(\frac{p}{q}\right)^{j+1} \mu_{[j+1]} = \sum_{j=0}^r S_j^r b_{[j+1]}(k, p, \lambda). \end{aligned}$$

Then regarding the second component we have

$$\begin{aligned} \frac{p}{q} \cdot \frac{d}{d\left(\frac{p}{q}\right)} a_r(nk, p) \wedge_n P_o(\lambda) &= \\ &= \frac{p}{q} \cdot \frac{d}{d\left(\frac{p}{q}\right)} \sum_{j=0}^r S_j^r \left(\frac{p}{q}\right)^j (nk)^{[j, -1]} \wedge_n P_o(\lambda) = \\ &= \frac{p}{q} \cdot \frac{d}{d\left(\frac{p}{q}\right)} \sum_{j=0}^r S_j^r \left(\frac{p}{q}\right)^j \mu_{[j]} = \frac{p}{q} \cdot \frac{d}{d\left(\frac{p}{q}\right)} b_r(k, p, \lambda). \end{aligned}$$

The result (13) is then evident.

Incomplete moments

An incomplete (truncated, generalized) crude moment $m_r(t)$ of order r of a distribution function $F(x)$ is defined by

$$m_r(t) = \int_t^{\infty} x^r dF(x).$$

For incomplete factorial moment we have analogously

$$m_{[r]}(t) = \int_t^\infty x^{[r]} dF(x).$$

In the discrete case we assume that t in the sum is an integer. It is quite evident that a complete moment is a special case of the incomplete one.

W. Dyczka (1973: 212, (22)) has shown that incomplete crude moments are expressed by the incomplete factorial ones in the following way:

$$m_r(t) = \sum_{k=0}^r S_r^k m_{[k]}(t).$$

where S_r^k are Stirling numbers of the second kind.

Using this notion for our case, we have

Theorem 4. Incomplete factorial moments of the compound Pascal-Poisson $Pap(k, p, \lambda)$ distribution are expressed by

$$b_{[r]}(k, p, \lambda, t) = b_{[r]}(k, p, \lambda) - p^r \exp(\theta - \lambda) \sum_{l=0}^{t-r-1} \frac{p^l}{l!} \mu_{[r+l]}$$

where $\theta = q^k \lambda$ and $\mu_{[r+l]}$ is the reverse factorial moment given by (9).

Proof. W. Dyczka (1973: 227, (73)) has shown that

$$\begin{aligned} a_{[r]}(nk, p, t) &= a_{[r]}(nk, p) a_0(nk + r, p, t - r) = \\ &= \left(\frac{p}{q}\right)^r (nk)^{[r, -1]} \sum_{l=t-r}^\infty \frac{(nk+r)^{[l, -1]}}{l!} p^l q^{nk+r} = \\ &= \left(\frac{p}{q}\right)^r (nk)^{[r, -1]} - \sum_{l=0}^{t-r-1} \frac{p^{r+l}}{l!} (q^{nk}(nk)^{[r+l, -1]}). \end{aligned}$$

Compounding this expression with $P_o(\lambda)$, we have

$$\left(\frac{p}{q}\right)^r (nk)^{[r, -1]} \wedge_n P_o(\lambda) - \sum_{l=0}^{t-r-1} \frac{p^{r+l}}{l!} (q^{nk}(nk)^{[r+l, -1]}) \wedge_n P_o(\lambda) = A.$$

Let us calculate the expression in parantheses:

$$\begin{aligned} q^{nk}(nk)^{[r+l, -1]} \wedge_n P_o(\lambda) &= \sum_{n=0}^\infty (nk)^{[r+l, -1]} \frac{(q^k \lambda)^n}{n!} \exp(-\lambda) = \\ &= \exp(\theta - \lambda) \sum_{n=0}^\infty (nk)^{[r+l, -1]} \frac{\theta^n}{n!} \exp(-\theta) = \exp(\theta - \lambda) \mu_{[r+l]}. \end{aligned}$$

Taking in account these calculations, we have

$$\begin{aligned}
 A &= \left(\frac{p}{q}\right)^r \mu_{[r]} - \sum_{l=0}^{t-r-1} \frac{p^{r+l}}{l!} \exp(0-\lambda) \mu_{[r+l]} = \\
 &= b_r(k, p, \lambda) - \exp(0-\lambda) p^r \sum_{l=0}^{t-r-1} \frac{p^l}{l!} \mu_{[r+l]}.
 \end{aligned}$$

As a simple conclusion, we have

Corollary 3. Incomplete crude moments of the compound Pascal-Poisson $Pap(k, p, \lambda)$ distribution are expressed by

$$b_r(k, p, \lambda, t) = \sum_{j=0}^r (S_j^r b_{[j]}(k, p, \lambda) - p^j \exp(0-\lambda) \sum_{l=0}^{t-r-1} \frac{p^l}{l!} \mu_{[j+l]}).$$

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*Tadeusz Gerstenkorn***ZŁOŻENIE INFLACYJNEGO ROZKŁADU PASCALA
Z ROZKŁADEM POISSONA**

Streszczenie

W pracy prezentowane jest złożenie inflacyjnego rozkładu Pascala z rozkładem Poissona.

W części wstępnej pracy podany jest przegląd wyników badawczych dotyczących tematu złożenia rozkładów ze szczególnym uwzględnieniem polskich autorów.

W dalszych rozdziałach podano funkcję prawdopodobieństwa rozkładu złożonego Pascal-Poisson oraz jego momenty silniowe, zwykłe, niekompletne oraz związki rekurencyjne.