ACTA UNIVERSITATIS LODZIENSIS
FOLIA OECONOMICA 192, 2005

Joanna Bruzda, Tomasz Koźliński***

NON-LINEARITY AND THE PURCHASING POWER PARITY HYPOTHESIS FOR EXCHANGE RATE JPY/USD

Abstract. In the paper we verify the purchasing power parity (PPP) hypothesis for the exchange rate JPY/USD for different time periods and price indexes. We build several forecasting models for this exchange rate including some non-linear specifications, for example the univariate BL-GARCH model and fundamental VEC models based on a long run equilibrium relationship. We have found that under some specific conditions the PPP hypothesis does hold. An adjustment process towards a long run equilibrium turned out to be of a non-linear nature. However, the impact of this adjustment on short-run dynamics of the exchange rate has a linear error correction form.

Keywords: purchasing power parity, forecasting exchange rates, non-linear adjustment, non-linear error correction models.

JEL Classification: C22, C53, E310, F310.

1. INTRODUCTION

Forecasting exchange rates is one of the most important activities of all financial institutions. Volume of all transactions on international and domestic money markets is considerably bigger than turnover on capital markets. International organizations, national banks, multinational companies and individual investors are often more connected or familiar with currencies than with stocks or derivative instruments. In case of international corporations forecasting exchange rates is a crucial matter in a budgeting and risk management process.

* Dr (Ph.D., Assistant Professor), Department of Econometrics and Statistics, Nicolaus Copernicus University in Toruń. The author acknowledges the support of the Polish Foundation for Science under the scholarship for young scientists in 2004.

** Mgr (M.A., Ph.D. student), Department of Finance Management, Nicolaus Copernicus University in Toruń.

Time series of exchange rates have some short- and long-term properties. Short-term data depend on financial, economic and political information. Investors can use technical analysis to try to predict its volatility in a day-to-day or week-to-week quotations. But there are also theories based on economics which indicate that exchange rates should be determined by some significant processes and information in a long run. In our paper we examine factors influencing long-term fluctuations of the exchange rate JPY/USD. Japanese and United States' economies have been in deep and long economic and political relations since the Second World War I. The exchange rate was floating almost all time except for some interventions by the National Bank of Japan to counteract yen appreciation against US dollar.[1]

2. PURCHASING POWER PARTY HYPOTHESIS

In economics there is a law of one price that states that price of one commodity or basket of products should be equal, after exchange rate adjustment, to strictly the same goods or basket of commodities in another country. This can be mathematically written as:

$$P_{Japan} = (JPY/USD) \cdot P_{US}, \tag{1}$$

where: P_{Japan} is a price of commodity or basket of goods in Japan, P_{US} is a price of commodity or basket of goods in the United States, JPY/USD means a nominal exchange rate, amount of Japanese yens paid for one US dollar.

This law is a foundation of the purchasing power parity (PPP) hypothesis. The hypothesis states that the exchange rate is driven in a long run by prices measured by different price indexes or differences between them in two countries. This implicates that there are, generally, two models of PPP. The first one is an absolute version of the PPP hypothesis and the next one is a relative approach (cf. Rosenberg 1996, p. 14):

$$JPY/USD = P_{Japan}/P_{US}, \tag{2}$$

$$\%\Delta JPY/USD = \%\Delta P_{Japan} - \%\Delta P_{US}. \tag{3}$$

There are many model representations of the two versions of PPP. For example model (4) is a modification of the absolute version by adding an

[1] In spite of huge amount of some intervention by the National Bank of Japan long-term appreciation could not be broken.

intercept and taking logarithms and another one (5) is based on differences – in this case an intercept does not have to be significant (Strzała 2002, pp. 106–107):

$$\ln(JPY/USD) = \alpha_0 \ln C + \alpha_1 \ln P_{Japan} - \alpha_2 \ln P_{US}, \tag{4}$$

$$\ln(\Delta JPY/USD) = \alpha_0 \ln C + \alpha_1 \ln(\Delta P_{Japan}) - \alpha_2 \ln(\Delta P_{US}). \tag{5}$$

Burda and Wyplosz (2000, pp. 262–264) describe the relative PPP (6) under assumption of the constant real exchange rate ($r(JPY/USD) = C \neq 1$) and transform this equation to (7):

$$(JPY/USD) = r(JPY/USD) \cdot (P_{Japan}/P_{US}), \tag{6}$$

$$\frac{\Delta r(JPY/USD)}{r(JPY/USD)} = \frac{\Delta(JPY/USD)}{JPY/USD} + \frac{\Delta P_{US}}{P_{US}} - \frac{\Delta P_{Japan}}{P_{Japan}}. \tag{7}$$

If price indexes are integrated of order 1, empirical testing of the hypotheses (4)–(5) is based on cointegration. Similar results should arise from testing stationarity of the real exchange rate series $r(JPY/USD)$. In test equations it does not appear a deterministic trend because it means that the real exchange rate is not constant.

$$r(JPY/USD) = (JPY/USD) \cdot (P_{US}/P_{Japan}), \tag{8}$$

$$\ln[r(JPY/USD)] = \alpha_0 \ln(JPY/USD) + \alpha_1 \ln P_{US} - \alpha_2 \ln P_{Japan}. \tag{9}$$

where: $r(JPY/USD)$ is the real exchange rate calculated from a nominal exchange rate after adjustment to inflation indexes in Japan and the United States.

3. VERIFICATION OF THE PURCHASING POWER PARITY HYPOTHESIS

It is interesting to test the above relations during different periods of time and by applying different price indexes' definitions. In case of the Japanese economy the real exchange rate calculated by the Bank of Japan has been published. It is the monthly average index of weighted average yen's real exchange rates versus 15 major world currencies officially used in 26 countries. Testing this time series for stationarity should indicate similar results as looking for unit roots in $r(JPY/USD)$ calculated according to the model (8), compare results in Tables 1 and 4.

Table 1. Results of unit root tests for real effective exchange rate for yen weighted by 15 major currencies

Specification	ADF		PP	KPSS
Include in test equation	AIC	HQ	Bartlett kernel	Bartlett kernel
Intercept	0.3903	0.3903	0.4092	1.569819
Trend and intercept	0.0314**	0.1137	0.2373	0.105905***
None	0.7936	0.7936	0.8538	–

Note: *** denotes rejection of the null hypothesis of a unit root at 1% significance level, ** and * at 5% and 10% accordingly. ADF – Augmented Dickey-Fuller, PP – Phillips-Perron, KPSS – Kwiatkowski, Phillips, Schmidt and Shin tests. AIC – Akaike information criterion, HQ – Hannan-Quinn criterion.

There are 6 main testing approaches to the verification of the PPP hypothesis (e.g. an overview in the paper by Strzała 2002). In our paper we concentrate on two of them, based on models (4) and (8). After Granger and Newbold (1974) results, cointegration theory has been mainly applied to the verification of the PPP hypothesis. If there is a cointegrating equation in the model (4) with an intercept, a relative PPP holds, if there is not an intercept in this equation, an absolute PPP is verified. In the second case after imposing $r(JPY/USD) = 1$, we get:

$$JPY/USD = \alpha_1 \cdot P_{Japan} - \alpha_2 P_{US}. \tag{10}$$

After choosing testing equations there is at least one more problem concerning inflation index definition. There are many price indexes, which have different base definitions, for example consumer, production, wholesale, import and export price indexes. For testing PPP hypothesis this index should be used, which describes price movements well and is the most often used in trade agreements between decision makers in Japan and the United States. Interesting results can be achieved with the help of import and export price indexes, that are weighted average prices of traded goods counted for the Japanese economy. In addition US Bureau of Labor and Statistics publishes one more import and export price index (IPIEPI), which measures price volatility of all goods by the location of origin from/to Japan.[2] In the last index definition of all commodities are taken into account, which logically can influence the exchange rate JPY/USD. This quotient can be used for testing cointegration with spot exchange rates and reverse

[2] IPIEPI denotes import/export price index for all commodities, which are exchanged between USA and Japan, location of origin is Japan (1993 January – 2003 December), taken from the US Department of Labor.

Table 2. Results of the Johansen cointegration test for SPOT, JWPI_SA, WPI_SA

Specifications			1975	1976	1977	1978	1979	1980	1981	1982	1983
Akaike	VAR	Lag intervals	1 4	1 4	1 4	1 4	1 4	1 4	1 2	1 2	1 2
Akaike	Johansen coint	Lag intervals	1 3	1 3	1 3	1 3	1 3	1 3	1 1	1 1	1 1
Data trend	Cointegration		Trace Max-Eig	Trace Max-Eig	Trace Max-Eig	Trace Max-Eig	Trace Max-Eig	Trace Max-Eig	Trace Max-Eig	Trace Max-Eig	Trace Max-Eig
1 None	No Intercept	No Trend	1 1	1 1	1 1	1 1	0 0	1 1	1 1	1 1	1 1
2 None	Intercept	No Trend	1** 1	1* 1	1* 1	1** 1	1** 2	1** 1	1* 1	1* 1	1* 1
3 Linear	Intercept	No Trend	0 0	0 0	0 0	0 1	1** 1	1** 1	0 0	0 0	0 0
4 Linear	Intercept	Trend	1 1	1 1	1 1	1 2	2 2	1 2	1 1	1 1	1 1
5 Quadratic	Intercept	Trend	1 1	1 1	1 1	2 2	2 2	3 3	1 1	1 1	1 1
			1984	1985	1986	1987	1988	1989	1990	1993	
Akaike	VAR	Lag intervals	1 2	1 2	1 2	1 2	1 2	1 1	1 2	1 1	
Akaike	Johansen coint	Lag intervals	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	
Data trend	Cointegration		Trace Max-Eig	Trace Max-Eig	Trace Max-Eig	Trace Max-Eig	Trace Max-Eig	Trace Max-Eig	Trace Max-Eig	Trace Max-Eig	
1 None	No Intercept	No Trend	1 1	2 2	2 2	1 1	1 1	1 1	1 1	2 2	
2 None	Intercept	No Trend	1* 1	1** 2	1** 1	1* 1	1* 1	1* 0	1** 1	2** * 0	
3 Linear	Intercept	No Trend	0 0	0 0	1* 1	0 0	0 0	0 0	0 0	0 0 SIC Rank (0) AIC (1)	
4 Linear	Intercept	Trend	1 1	1 1	1 2	1 1	1 0	1 1	0 0	0 0 SIC Rank (0)	
5 Quadratic	Intercept	Trend	1 1	1 1	2 2	1 1	1 0	3 0	0 0	0 0	

Note: VAR (1 3) means one to three lags interval for endogenous; Johansen cointegration (1 3) means one to three lags interval in first differences for endogenous; (1 1) means one cointegration equation according to trace test and one according to max-eigenvalue test, only for 2nd and 3rd options: ** trace test indicates 1 cointegrating equation at the 5% (1%) level, * trace test indicates 1 cointegrating equation at the 5% level; Max-Eig – max-eigenvalue test; for 1975–2002 and 1993–2002 if trace test did not indicate cointegration equation were written Schwarz and Akaike information criteria (SIC and AIC).

Table 3. Results of the Johansen cointegration test

Specifications			SPOT CPIJAP SA CPIUSA SA		SPOT JEPI SA JIPI SA		SPOT IPIEPI SA
			1975–2002	1993–2002	1975–2002	1993–2002	1993–2002
Akaike	VAR	Lag intervals	1 2	1 1	1 2	1 2	1 4
Akaike	Johansen coint	Lag intervals	1 1	1 1	1 1	1 1	1 3
Data trend	Cointegration		Trace Max-Eig	Trace Max-Eig	Trace Max-Eig	Trace Max-Eig	Trace Max-Eig
1 None	No Intercept	No Trend	2 2	1 1	0 0 SC rank (0)	0 0 AIC SC rank(0)	0 0 SC rank (0)
2 None	Intercept	No Trend	2** 2	1** 1	0 0 SC rank (0)	0 0 AIC SC rank(0)	0 0 SC rank (0)
3 Linear	Intercept	No Trend	3** * * 1 SC rank(1)	0 0 SC rank(0) AIC rank(1)	0 0	0 0	0 0
4 Linear	Intercept	Trend	2 2	0 0 SC rank(0)	1* 0	0 0	1 1 AIC rank (1)
5 Quadratic	Intercept	Trend	1 1 AIC rank (1)	0 0	1* 1* AIC rank (1)	0 0	2 2

Note: As Table 2.

Table 4. Results of unit root tests for real exchange rates, for levels

	SPOT WPI SA JWPI SA		SPOT CPIUSASA CPIJAP SA		SPOT JIPI SA JEPI SA		SPOT EPIIPISA
Real exchange rate	1975-2002	1993-2002	1975-2002	1993-2002	1975-2002	1993-2002	1993-2002
ADF intercept	0.3863	0.7562	0.6703	0.6481	0.6703	0.6481	0.4134
ADF trend and intercept	0.7563	0.2805	0.7026	0.2285	0.7026	0.2285	0.2354

of it could be used for verifying stationarity in real exchange rate according to equation (8). This price index should be the best approximation of the one price theory, because it gets rid of some limitations, e.g. that all goods are not tradable, tradable goods are not from applicable country.

All series were extracted from the International Financial Statistics and the Bank of Japan databases cover the period January 1975 to May 2003. The last five observations have been left for forecasting accuracy evaluation, so the in-sample period ends in December 2002. The ADF test indicates that almost all series are $I(1)$. Lags for Johansen cointegration test and VEC models were determined using the multivariate Akaike information criterion. In the Johansen test we presume five different combination of assumptions. The occurrence of a cointegrating vector without an intercept proves the absolute PPP, whereas the presence of an intercept in a cointegrating equation implies the relative PPP (cf. results in Tables 2 and 3).

In about half of cases we succeeded in finding cointegrating vectors, however, there are periods, for which the PPP hypothesis does not hold. For longer time series cointegrating vectors occur more often. There is no unambiguous indication, which price index is the most useful in finding cointegrating relations but when WPI (wholesale price index) is applied cointegration appears more often. Real exchange rates seem to be non-stationary in levels, what rejects the PPP hypothesis (cf. Table 4).

The existence of a cointegrating vector enables to build vector error correction models. In three out of seven models an error correction term has a minus sign and there is no trend in a cointegrating equation, which makes it possible to interpret the models in a proper way. The best five-months-ahead forecasts were obtained with VAR models based on IPIEPI and WPI for the period 1993–2002, with the forecasting accuracy measured by Theil inequality coefficient. The best one-year-ahead forecasts were obtained with the VAR model for indexes JEPI – Japanese export price index – and JIPI – Japanese import price index – (1975–2002) and VEC model for WPI (1993–2002).[3] The results of forecasting accuracy

[3] Variables are described as and taken from: JWPI – wholesale prices index for Japan, period averages 1995 = 100, taken from the International Financial Statistics of the International Monetary Fund (15863...ZF...); WPI – producer prices index for the USA, period averages 1995 = 100, extracted from the IFS IMF (11163...ZF). VAR WPI means vector autoregression model with seasonally adjusted indexes JWPI_SA and WPI_SA. VEC WPI is explained similar and generally means vector error correction model with use of JWPI_SA and WPI_SA. JEPI – export price index for Japan – index on yen basis calculated on 222 items, 2000 year average = 100 – all commodities, taken from the Bank of Japan; JIPI – import price index for Japan – index on yen basis calculated on 227 items, 2000 year average = 100 - all commodities, extracted from the Bank of Japan; IPIEPI – import/export price index for the USA for all

comparison as well as estimation outputs are available upon request from the authors.

4. NON-LINEARITY IN EXCHANGE RATE JPY/USD – METHODOLOGY

There are many empirical studies, which document the failure of linear exchange rate models. Also theoretical extensions of linear exchange rate behavior to non-linear framework have grown in econometric literature (for example the concept of bubbles and self-fulfilling expectations, target zone models and other nonlinear adjustment models – e.g. for details Sarno and Taylor 2002). There is a growing recognition that the introduction of non-linearities in the modeling framework enables to explain the slowness of the exchange rates adjustment process toward its long-run equilibrium (e.g. Ma and Kanas 2000, Baum et al. 2001, Dufrenot 2002). Non-linear framework for modeling exchange rates allows taking into account such phenomena like multiple long-run equilibria, presence of target zones, abrupt changes in adjustment speed and different adjustments according to the sign and size of the deviation from parity. All these factors imply either a non-linear relationship between exchange rate and its fundamentals or a non-linear adjustment mechanism. Besides, it is interesting to examine the impact of the non-linear adjustment on short-run dynamics of exchange rates in a non-linear error correction form.

In our paper we concentrate much more on forecasting properties of considered models than on their explanatory power. At the beginning, we consider several univariate non-linear specifications to model logarithms and logarithmic returns of the exchange rate JPY/USD. The models are non-linear in the conditional variance and/or in the conditional mean of the process. They are as follows:

- random walk with drift and GARCH errors

$$y_t = \mu + y_{t-1} + \varepsilon_t, \qquad \sigma^2(\varepsilon_t|\Omega_{t-1}) = h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{11}$$

- AR(1) for logarithmic returns with GARCH errors

$$\Delta y_t = \gamma\Delta y_{t-1} + \varepsilon_t, \qquad \sigma^2(\varepsilon_t|\Omega_{t-1})h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{12}$$

- GARCH-M:

$$\Delta y_t = \mu + \theta h_t + \varepsilon_t, \qquad \sigma^2(\varepsilon_t|\Omega_{t-1}) = h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{13}$$

---

commodities originated from Japan, acquired from US Department of Labor; SPOT – market exchange rate JPY/USD means yens per US dollar, end of period, taken from the IFS IMF (158..AE.ZF...).

- autoregressive subdiagonal bilinear model with GARCH errors

$$\Delta y_t = \mu + \gamma \Delta y_{t-1} + \theta \Delta y_{t-1} \varepsilon_{t-2} + \varepsilon_t,$$
$$\sigma^2(\varepsilon_t | \Omega_{t-1}) = h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}. \tag{14}$$

In the above models we assume t-Student conditional distributions for residual terms. The model (14) has been chosen from the whole family of bilinear models with GARCH disturbances as the most promising in the sense of fitting and forecasting accuracy. Such specification makes it possible to merge the general form of non-linearity in the mean value of a process with non-linearity in the conditional variance of the process into one model.

As a basis for construction of multivariate fundamental models for exchange rate determination we consider a model based on the monetarist interpretation of exchange rate movements (e.g. Sarno and Taylor 2002). The specification includes such explanatory variables like money supplies measured as M1, outputs approximated by industrial production, inflation rates and long-term interest rates. Apart from the above variables we include other three factors emerging from behavioral equilibrium exchange rate models (cf. for details Rosenberg 1996; Rubaszek 2003): total reserves minus gold, exports and imports. Additionally we consider share prices for industry.

All variables are taken in logarithms and the ratios of the domestic country to the foreign country value of the variables are calculated. On the basis of the transformed variables several VAR (VEC) models are estimated. In the specification of these models we use results of Granger causality tests and we perform a cointegration analysis for chosen collections of variables with the Johansen methodology.

Deviations from the exchange rate long-run relationship represent the so-called adjustment process. As we have pointed out earlier in the text it is often of a non-linear nature. The non-linear adjustment can be characterised in terms of a smooth transition autoregressive (STAR) model of Granger and Teräsvirta (1993). We assume that deviations from our long-run equilibria can be described by an exponential STAR (ESTAR) model of the form:

$$z_t = k + \sum_{j=1}^{p} \pi_j z_{t-j} + \left(k^* + \sum_{j=1}^{p} \pi_j^* z_{t-j} \right) F(\cdot) + \varepsilon_t, \tag{15}$$

where the transition function $F(\cdot)$ is U-shaped:

$$F(z_{t-d}) = 1 - \exp \lfloor -\gamma (z_{t-d} - c^*)^2 / \hat{\sigma}_z^2 \rfloor. \tag{16}$$

The ESTAR model can be viewed as a generalization of a double-threshold TAR model of Tong (1990). When $z_{t-d} = c^*$, we get $F(\cdot) = 0$ and (15) takes a form of standard a AR(p) representation. For extreme deviations from long-run equilibrium (15) becomes a different AR(p) model with intercept $k + k^*$ and autoregressive parameters $\pi_j + \pi_j^*$. In our analysis a convenient reparametrization of (15) is the following:

$$(17) \quad \Delta z_t = k + \lambda z_{t-1} + \sum_{j=1}^{p-1} \varphi_j \Delta z_{t-j} + \left(k^* + \lambda z_{t-1} + \sum_{j=1}^{p-1} \varphi_j^* \Delta z_{t-j}\right) F(\cdot) + \varepsilon_t.$$

The equation (18) corresponds with the usual testing equation for augmented Dickey-Fuller test:

$$(18) \quad \Delta z_t = k' + \lambda z_{t-1} + \sum_{j=1}^{p-1} \varphi'_j \Delta z_{t-j} + \varepsilon_t.$$

In the ESTAR model the coefficient λ governs the adjustment for small deviations from the equilibrium, when the coefficient λ^* corresponds to large deviations. To ensure the stability and mean-reverting properties of the adjustment process the quantity $\lambda + \lambda^*$ must be negative. This means that the stable z_t process may follow a unit root or even explosive behavior for small deviations ($\lambda \geqslant 0$), but it should be mean-reverting for large errors, so that the condition $\lambda + \lambda^* < 0$ is fulfilled.

To test for linearity of the adjustment mechanism we apply Teräsvirta (1994) linearity test against ESTAR alternative. If the delay parameter d is known, the test consists of estimating an artificial regression of the form

$$(19) \quad z_t = \beta_{00} + \sum_{j=1}^{p} (\beta_{0j} z_{t-j} + \beta_{1j} z_{t-j} z_{t-d} + \beta_{2j} z_{t-j} z_{t-d}^2) + \varepsilon_t,$$

and testing the hypothesis

$$(20) \quad \mathrm{H}_0 : \beta_{1j} = \beta_{2j} = 0 \quad (j = 1, \ldots, p).$$

The order of autoregression p is chosen on the basis of serial correlation Ljung-Box test on residuals. To determine the delay parameter Michael, et al. (1997) suggest repeating the linearity test for different values of d and choosing the delay parameter corresponding to the smallest p-value of the test statistic. When linearity is rejected, we proceed with estimation of ESTAR models. Since in our application the process z_t is zero-mean, we can assume that $k = k^* = c$.

As we have noticed before, besides modeling the non-linear adjustment process solely (this approach dominates in the existing econometric literature), it is interesting to examine an impact of the possibly non-linear misalignments on the short-run dynamics of exchange rate process. It can be done in a non-linear error correction (NEC) framework. The relation between cointegration and error correction is well recognized in a linear context. The extension of it to the non-linear case seems to be one of the most important challenges for econometricians. Escribano and Mira (2002) start to fill this gap by partially extending the Granger representation Theorem to the non-linear framework. They start considerations with new concepts of $I(0)$ processes and linear cointegration, which are based on a definition of so-called near epoch dependent (NED) process. The definition of a NED process is a relaxation of the concept of a mixing process. Heuristically, a process is mixing, if it is short-range dependent, i.e. when the dependence between past and future events becomes negligible with the time span converging to infinity. Escribano and Mira found that, if variables are $I(1)$ with a non-linear error correcting mechanism, then they are linearly cointegrated under certain conditions on the non-linear adjustment. In particular, they give sufficient conditions for the NEC model to be well specified and balanced.

A non-linear error correction (NEC) model can be written as follows:

$$\Delta y_1 = \sum_{j=1}^{q} \delta_j \Delta X_{t-j} + \sum_{j=1}^{p} \upsilon_j \Delta y_{t-j} + \lambda_1 z_{t-1} + \lambda_2 f(z_{t-1}) + \varepsilon_t, \tag{21}$$

$$z_t = y_t - \beta X_t, \tag{22}$$

where X_t is a vector of explanatory variables and δ_j are vectors of parameters respectively, the transition function $f(\cdot)$ satisfies some regularity conditions and z_t is NED. In what follows we consider two formulations of the transition function (cf. for comparison Chaouachi et al. 2003):

- an exponential smoothing transition function:

$$f(z_{t-1}) = 1 - \exp \lfloor -\gamma(z_{t-1}^2 - c) \rfloor, \tag{23}$$

- a cubic polynomial function

$$f(z_{t-1}) = \alpha_1 z_{t-1} + \alpha_2 z_{t-1}^2 + \alpha_3 z_{t-1}^3. \tag{24}$$

The first transition function allows for discrimination between corrections with regard to small and large deviations from the equilibrium path. Additionally, the correction changes in a smooth way. NEC models based on cubic polynomials allow examining general form of asymmetric dynamics between overvaluation and undervaluation periods. This type of non-linearity is connected with varying strength of attraction to a linear attractor. For example, the attractor may be stronger on one side than on the other.

5. NON-LINEARITY IN EXCHANGE RATE JPY/USD – EMPIRICAL RESULTS

As previously all series are extracted from the IMF's IFS and the Bank of Japan databases and cover the period January 1975 to December 2002.[4] Explanatory variables have been seasonally adjusted with the Census X-12 method. In the first step we conducted pairwise Granger causality tests with the number of lags 4. Because of a non-stationarity of the variables, results of these tests are only approximately valid. On the basis of these results several VAR model were specified. For further analysis we chose IMP, TRES and WPI as explanatory variables. Results corresponding to other specifications are available upon request from the authors. Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests indicate that all variables, which we chose after causality tests, are $I(1)$ processes. The KPSS test (Kwiatkowski et al. 1992) gives other results indicating that TRES is an $I(2)$ process. In Tables 5–8 below estimation outputs for the univariate models (11)–(14) are presented.

Table 5. Estimation output for the random walk model with drift and t-distributed GARCH errors

Specification	Coefficient	Std. error	z-Statistic	Prob.
MU	–0.001326	0.001607	–0.824827	0.4095
OMEGA	2.25E–05	1.18E–05	1.908074	0.0564
ALPHA	0.036637	0.021870	1.675222	0.0939
BETA	0.950848	0.020699	45.93696	0.0000
TDF	4.946433	1.605981	3.080006	0.0021
Log likelihood	672.0810	Akaike info criterion		–3.982573
Hannan–Quinn crit.	–3.959878	Schwarz criterion		–3.925646
$Q(2) = 3.2830$ $(p = 0.194)$	$Q(4) = 5.7905$ $(p = 0.215)$			
$Q(8) = 11.6170$ $(p = 0.169)$				

[4] We use the following notation: natural logarithms of monthly exchange rate JPY/USD (end of the month) are denoted by SPOT; the remaining variables are logarithms of the ratios of the domestic value to the foreign value total reserves minus gold (TRES), money supply measure M1 (M1), federal founds rate (INTREST), share prices for industry (SHARES), wholesale price index (WPI), hourly earnings (WAGES), industrial production (IP), exports (EXP), imports (IMP).

Table 6. Estimation output for the AR(1) model for logarithmic returns with t-distributed GARCH errors

Specification	Coefficient	Std. error	z-Statistic	Prob.
GAMMA	0.106509	0.053719	1.982684	0.0474
OMEGA	3.28E–05	1.66E–05	1.978339	0.0479
ALPHA	0.037195	0.023327	1.594481	0.1108
BETA	0.940415	0.025469	36.92378	0.0000
TDF	5.544558	1.952147	2.840236	0.0045
Log likelihood	654.8985	Akaike info criterion		–3.987108
Hannan–Quinn crit.	–3.963930	Schwarz criterion		–3.929026
$Q(2) = 0.5917$ $(p = 0.744)$	$Q(4) = 2.9972$ $(p = 0.558)$			
$Q(8) = 8.0296$ $(p = 0.431)$				

Table 7. Estimation output for the GARCH–in–mean model

Specification	Coefficient	Std. error	z–Statistic	Prob.
MU	–0.677642	0.144882	–4.677199	0.0000
THETA	14.21781	3.214230	4.423394	0.0000
OMEGA	0.000148	6.41E–05	2.303252	0.0213
ALPHA	0.008084	0.003336	2.423005	0.0154
BETA	0.930834	0.027708	33.59490	0.0000
TDF	2.683520	0.533691	5.028229	0.0000
Log likelihood	644.1574	Akaike info criterion		–3.855936
Hannan–Quinn crit.	–3.828448	Schwarz criterion		–3.787016
$Q(2) = 2.340$ $(p = 0.309)$	$Q(4) = 4.4549$ $(p = 0.348)$			
$Q(8) = 10.900$ $(p = 0.207)$				

Table 8. Estimation output for the subdiagonal bilinear model

Specification	Coefficient	Std. error	z–Statistic	Prob.
MU	–0.001708	0.001655	–1.032021	0.3021
GAMMA	0.114806	0.055459	2.070113	0.0384
THETA	1.988684	1.420782	1.399711	0.1625
OMEGA	2.59E–05	1.28E–05	2.027156	0.0426
ALPHA	0.039438	0.022882	1.723516	0.0848
BETA	0.943596	0.023795	39.65593	0.0000
TDF	5.821522	2.037318	2.857443	0.0043
Log likelihood	660.6572	Akaike info criterion		–3.985714
Hannan–Quinn crit.	–3.953418	Schwarz criterion		–3.904766
$Q(2) = 1.2520$ $(p = 0.535)$	$Q(4) = 4.2085$ $(p = 0.379)$			
$Q(8) = 8.9321$ $(p = 0.348)$				

The next step in our analysis was estimation of fundamental models for exchange rate determination. In Table 9 we present results of the Johansen cointegration test for SPOT, TRES, WPI and IMP. The lag length of the unrestricted VAR model in levels was determined by using the multivariate Akaike information criterion allowing for a maximum lag length of 10. The multivariate Schwarz criterion was also employed but generally it underestimated the VAR lag length, resulting in serially correlated residuals.

Table 9. Results of Johansen cointegration test

Unrestricted Cointegration Rank Test				
Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5% Critical Value	1% Critical Value
None *	0.078949	52.76163	47.21	54.46
At most 1	0.049315	24.96461	29.68	35.65
At most 2	0.022689	7.871120	15.41	20.04
At most 3	0.000337	0.113845	3.76	6.65
Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5% Critical Value	1% Critical Value
None *	0.078949	27.79701	27.07	32.24
At most 1	0.049315	17.09349	20.97	25.52
At most 2	0.022689	7.757275	14.07	18.63
At most 3	0.000337	0.113845	3.76	6.65
*(**) denotes rejection of the hypothesis at the 5%(1%) level				

Residuals from the cointegrating equation were tested for linearity with Teräsvirta linearity test against ESTAR alternative. Results of this test indicate the value of the delay parameter $d = 1$ (cf. Table 10). For this value of d an ESTAR model for residuals was estimated (Table 11).

Table 10. Results of linearity tests

D	p	F-statistic	p-value
1	5	2.4949	0.0599
2	7	2.0828	0.1499
3	5	0.8592	0.4244

The output in Table 11 indicates that the analyzed adjustment process is of a non-linear nature and can be described by an ESTAR model. The next question was whether the short-run dynamic of logarithms of the exchange rate JPY/USD can be described by a non-linear error correction model. Tables 12 and 13 include results of the estimation of NEC models with exponential smooth transition and cubic transition functions.

Table 11. Results of the estimation of an ESTAR model for residuals

$D(Z) = C(1) \cdot Z(-1) + C(2) \cdot D(Z(-1)) + C(3) \cdot D(Z(-2)) + C(4) \cdot D(Z(-3)) + C(5) \cdot D(Z(-4)) + (C(6) \cdot Z(-1) + C(7) \cdot D(Z(-1)) + C(8) \cdot D(Z(-2)) + C(9) \cdot D(Z(-3)) + C(10) \cdot D(Z(-4))) \cdot (1-EXP(-C(11) \cdot Z(-1) \wedge 2))$

	Coefficient	Std. error	t-Statistic	Prob.
$C(1)$	−0.448822	0.566375	−0.792448	0.4287
$C(2)$	−0.703820	0.198410	−3.547310	0.0004
$C(3)$	−0.771434	0.241977	−3.188050	0.0016
$C(4)$	0.011856	0.166023	0.071413	0.9431
$C(5)$	−0.254250	0.170146	−1.494309	0.1361
$C(6)$	0.319664	0.570963	0.559869	0.5760
$C(7)$	0.593508	0.212359	2.794834	0.0055
$C(8)$	0.811723	0.249751	3.250134	0.0013
$C(9)$	0.011488	0.190713	0.060239	0.9520
$C(10)$	0.346184	0.188299	1.838478	0.0669
$C(11)$	1824.641	939.3330	1.942486	0.0530
R-squared	0.150912	Akaike info criterion		−3.490890
S.E. of regression	0.041556	Schwarz criterion		−3.364536
Log likelihood	588.7424	Durbin-Watson stat.		2.034494

Table 12. Estimation of the NEC model with an exponential smooth transition function

$D(\text{SPOT}) = C(1) + C(2) \cdot D(\text{SPOT}(-1)) + C(3) \cdot D(\text{IMP}(-1)) + C(4) \cdot D(\text{TRES}(-1)) + C(5) \cdot D(\text{WPI}(-1)) + C(6) \cdot R1(-1) + C(7) \cdot (1 + \text{EXP}(C(8) \cdot ((R1(-1)) \wedge 2)))$

	Coefficient	Std. error	t-Statistic	Prob.
$C(1)$	−0.000845	0.010547	−0.080164	0.9362
$C(2)$	0.103890	0.055262	1.879945	0.0610
$C(3)$	−0.026565	0.031115	−0.853764	0.3939
$C(4)$	−0.001947	0.032795	−0.059381	0.9527
$C(5)$	−0.119628	0.317199	−0.377139	0.7063
$C(6)$	−0.056916	0.016359	−3.479173	0.0006
$C(7)$	−0.001754	0.009983	−0.175705	0.8606
$C(8)$	−4134.328	53296.97	−0.077572	0.9382
R-squared	0.044027	Mean dependent var		−0.002609
Adjusted R-squared	0.023500	S.D. dependent var		0.033715
S.E. of regression	0.033316	Akaike info criterion		−3.941872
Sum squared resid	0.361855	Schwarz criterion		−3.850587
Log likelihood	666.2926	Durbin-Watson stat.		2.005954

Table 13. Estimation of the NEC model with a cubic transition function

$D(\mathrm{SPOT}) = C(1) + C(2) \cdot D(\mathrm{SPOT}(-1)) + C(3) \cdot D(\mathrm{IMP}(-1)) + C(4) \cdot D(\mathrm{TRES}(-1)) + C(5) \cdot D(\mathrm{WPI}(-1)) + C(6) \cdot R1(-1) + C(7) \cdot R1(-1)2 + C(8) \cdot R1(-1) \wedge 3$

	Coefficient	Std. error	t-Statistic	Prob.
$C(1)$	-0.003480	0.002512	-1.385276	0.1669
$C(2)$	0.108562	0.055199	1.966726	0.0501
$C(3)$	-0.026474	0.031043	-0.852828	0.3944
$C(4)$	-0.003746	0.032799	-0.114203	0.9091
$C(5)$	-0.124487	0.313562	-0.397009	0.6916
$C(6)$	-0.040895	0.028837	-1.418129	0.1571
$C(7)$	0.075367	0.121329	0.621182	0.5349
$C(8)$	-0.582498	0.762447	-0.763984	0.4454
R-squared	0.046451	Mean dependent var		-0.002609
Adjusted R-squared	0.025976	S.D. dependent var		0.033715
S.E. of regression	0.033274	Akaike info criterion		-3.944411
Sum squared resid	0.360937	Schwarz criterion		-3.853126
Log likelihood	666.7166	Durbin-Watson stat.		2.008753

Because the non-linear error correction is not significant, we conclude that our error correction model has a linear form. Table 14 includes results of forecasting accuracy comparison between the univariate models and VEC and VAR models for the exchange rate JPY/USD (cf. also Figure 1).

Table 14. Forecasting accuracy comparison

Specification	RW	AR(1)	M-GARCH	BL	VEC FUND.	VAR IPIEPI
Root mean square error	0.008576	0.009118	0.022635	0.007913	0.015518	0.00827
Mean absolute error	0.007463	0.007367	0.019323	0.006973	0.013767	0.00770
Mean absolute precentage error	0.001563	0.001543	0.004041	0.001459	0.002883	0.00161
Theil coefficient	0.000401	0.000426	0.001061	0.00037	0.000725	0.00041
$I1$	0.208838	0.375955	0.728753	0.002787	0.787029	–
$I2$	0.480989	0.757265	0.000134	0.424275	0.143971	–
$I3$	0.406371	0.018233	0.271140	0.657793	0.097795	–

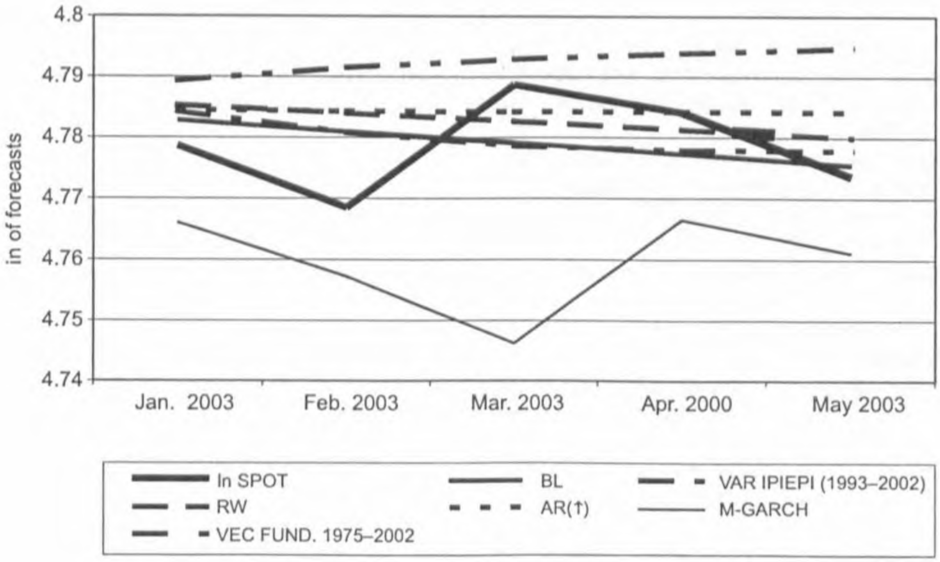

Fig. 1. Forecasting accuracy comparison

6. CONCLUSIONS

We have found that in some periods the exchange rate JPY/USD is formed by changes in price levels in Japan and the United States and the purchasing power parity hypothesis holds under some specific conditions for the exchange rate. For longer time series the cointegrating equations occur more often. There is a problem in selection of a proper price index, which should be used for modeling, but WPI seems to be the best solution for it. The best forecasting model for the exchange rate JPY/USD is a non-linear univariate BL-GARCH model. It turns out that the adjustment process towards a long-run equilibrium for this exchange rate is of a non-linear nature and can be effectively described by an ESTAR model. However, the impact of this adjustment process on short-run dynamics of the exchange rate has a linear error correction form. Usually VAR models, especially based on import/export price index for goods from/to Japan, give better approximation of the spot exchange rate than VEC models.

REFERENCES

Baum, C. F., Barkoulas, J. T. and Caglayan, M. (2001), "Nonlinear Adjustment to Purchasing Power Parity in the Post-Bretton Woods Era", *Journal of International Money and Finance*, **20**.

Burda, M. and Wyplosz, Ch. (2000), *Macroeconomics* (in Polish), Warszawa: PWE.

Chaouachi, S., Dufrenot, G. and Mignon, V. (2003), "Modelling the Misalignments of the Dollar-Sterling Real Exchange Rate: A Nonlinear Cointegration Perspective", University of Paris XII: *Working Paper*.

Dufrenot, G., Mathieu, L., Mignon, V. and Peguin-Feissolle, A. (2002), "Persistent Misalignments of the European Exchange Rates: Some Evidence from Nonlinear Cointegration", University of Paris XII: *Working Paper*.

Dufrenot, G. and Mignon, V. (2002), *Recent Developments in Nonlinear Cointegration with Applications to Macroeconomics and Finance*, Boston: Kluwer Academic Publishers.

Escribano, A. and Mira, S. (2002), "Nonlinear Error Correction Models", *Journal of Time Series Analysis*, **23**.

Granger, C. W. J. and Newbold, P. (1974), "Spurious Regressions in Econometrics", *Journal of Econometrics*, **2**.

Granger, C. W. J. and Teräsvirta, T. (1993), *Modelling Nonlinear Economic Relationships*, Oxford: Oxford University Press.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P. and Shin Y. (1992), "Testing the Null Hypothesis of Stationarity Against the Alternative of A Unit Root", *Journal of Econometrics*, **54**.

Ma, Y. and Kanas, A. (2000), "Testing for a Nonlinear Relationship Among Fundamentals and Exchange Rates in the ERM", *Journal of International Money and Finance*, **19**.

Michael, P., Nobay, A. R. and Peel, D. A. (1997), "Transaction Costs and Nonlinear Adjustment in Real Exchange Rates: An Empirical Investigation", *Journal of Political Economy*, **105**.

Rosenberg, M. (1996), *Currency Forecasting: A Guide to Fundamental and Technical Models of Exchange Rate Determination*, Chicago: Irwin Professional Pub.

Rubaszek, M. (2003), "Balance of Payments Equilibrium Model. Application to PLN Exchange Rate", in Polish, *Bank i Kredyt*, **5**.

Sarno, L., and Taylor, M. R. (2002), *The Economics of Exchange Rates*, Cambridge: Cambridge University Press.

Strzała, K. (2002), "Verification of Macroeconomic Hypothesis – Evolution of Approaches on an Example of PPP" (in Polish). In: Kufel, T., Piłatowska, M. (eds.), *Analysis of Economic Time Series at the Beginning of XXI Century*, Toruń: Nicholas Copernicus University.

Teräsvirta, T. (1994), "Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models", *Journal of the American Statistical Association*, **89**.

Tong, H. (1990), *Non-Linear Time Series: A Dynamical System Approach*, Oxford: Clarendon Press.

Joanna Bruzda, Tomasz Koźliński

NIELINIOWOŚĆ I HIPOTEZA PARYTETU SIŁY NABYWCZEJ PIENIĄDZA DLA KURSU WALUTOWEGO JPY/USD

(Streszczenie)

W artykule przedstawiamy wyniki weryfikacji hipotezy parytetu siły nabywczej pieniądza dla kursu walutowego JPY/USD dla różnych okresów i różnych indeksów cenowych. Prezentujemy kilka modeli prognostycznych dla tego kursu, włączając pewne specyfikacje nieliniowe, jak np. jednowymiarowy model BL-GARCH oraz fundamentalne modele VEC, oparte na zależności w długookresowym położeniu równowagi. Wyniki empiryczne wskazują, że przy pewnych specyficznych warunkach hipoteza PPP zachodzi. Proces dostosowania do długookresowego położenia równowagi okazuje się mieć postać nieliniową. Jednakże wpływ tego dostosowania na krótkookresową dynamikę kursu walutowego JPY/USD ma formę liniowej korekty błędem.