ACTA UNIVERSITATIS LODZIENSIS
FOLIA OECONOMICA 192, 2005

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FORECASTING RETURNS USING THRESHOLD MODELS

Abstract. In this paper we present the problem of forecasting efficiency of the TAR models. Three methods of forecasting are considered to compare their accuracy: the Monte Carlo method, and the two versions the bootstrap technique. The basic models are two- or three- regimes stationary threshold autoregressive models with the endogenous or exogenus switching variable. The time series set consists of the weekly stock returns of the banking sector quoted at the Warsaw Stock Exchange.

Keywords: threshold models, foreasting, Monte Carlo, bootstrap.
JEL Classification: C15, C22.

1. INTRODUCTION

Forecasting financial prices as well as returns is not an easy task. Often application of even very complicated tools do not bring us to the conclusion that the forecasting accuracy is satisfactory. It can be especially seen when the prediction of the conditional mean is made (cf. Dunis ed. 2001). That is why the models of financial time series usually combine two parts: i.e. the conditional mean and the conditional variance. One of the simple univariate case is the ARIMA-GARCH representation. However, taking into account, that investors may react in one way in the case of high returns and in another when the returns are low, the threshold autoregressive

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models (TAR) are considered (cf. Proietti 1998). The TAR models describe the conditional mean due to regimes given by the threshold parameter. It can be seen that the conditional variance can be still described by the GARCH-type models (cf. Osińska and Witkowski 2003).

In the presented paper we put our attention to the problem of forecasting efficiency of the TAR models. Three methods of forecasting are considered to compare their accuracy: one of them is the Monte Carlo method, and the two others are based on the bootstrap technique. The basic models are two or three regimes stationary threshold autoregressive models with the endogenous or exogenous switching variable. The time series set consists of the weekly stock returns of the banking sector quoted at the Stock Exchange in Warsaw, observed within January 1995 – September 2003.

The paper consists of six sections. In Section 2 the model is considered. Section 3 presents the statistical inference using the self-exciting threshold autoregressive model. Section 4 contains the methodology used in forecasting. The empirical results are presented in Section 5. The final remarks are summed up in Section 6.

2. THE MODEL

Let $\mathbf{Y}_t$ denotes k-dimensional random vector. Let us consider the model

(1) $$\mathbf{Y}_t = \mathbf{B}^{\mathbf{J}_t}\mathbf{Y}_t + \mathbf{A}^{\mathbf{J}_t}\mathbf{Y}_{t-1} + \mathbf{H}^{\mathbf{J}_t}\boldsymbol{\varepsilon}_t + \mathbf{C}^{\mathbf{J}_t},$$

where $\mathbf{J}_t$ is a random variable taking values of finite set of natural numbers $\{1, 2, 3, ..., p\}$, $\mathbf{B}^{\mathbf{J}_t}$, $\mathbf{A}^{\mathbf{J}_t}$, $\mathbf{H}^{\mathbf{J}_t}$ are $k \times k$-dimensional matrices of the coefficients, $\boldsymbol{\varepsilon}_t$ is the k-dimensional white noise, $\mathbf{C}^{\mathbf{J}_t}$ is a constant vector. The model (1) is called a canonical form of the threshold model. It defines a wide class of the models depending on the choice of $\mathbf{J}_t$.

When $\mathbf{J}_t$ is the function of Y_t. we obtain a SETAR model (self-exciting threshold autoregressive model). The SETAR (p; k_1, k_2, ..., k_p) model is defined in the following way:

(2) $$Y_t = \alpha_0^j + \sum_{i=1}^{k_j} \alpha_i^j Y_{t-i} + h^j \varepsilon_t,$$

conditionally on $Y_{t-i} \in R_j$, $j = 1, ...p$.

The more convenient form of (2) is the following

$$(3)\qquad Y_t=\begin{cases}\alpha_0^1+\alpha_1^1Y_{t-1}+\dots+\alpha_{k_1}^1Y_{t-k_1}+h^1\varepsilon_t & \text{for}\quad Y_{t-d}\leqslant r_1\\ \alpha_0^2+\alpha_1^2Y_{t-1}+\dots+\alpha_{k_2}^2Y_{t-k_2}+h^2\varepsilon_t & \text{for}\quad r_1<Y_{t-d}\leqslant r_2\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots & \\ \alpha_0^p+\alpha_1^pY_{t-1}+\dots+\alpha_{k_p}^pY_{t-k_p}+h^p\varepsilon_t & \text{for}\quad Y_{t-d}\leqslant r_{p-1}\end{cases}$$

The threshold variable is in (3) lagged Y_t, but it can be also an exogenous variable, say lagged Z_t.

For two regimes we have the following $I(y)$ function

$$(4)\qquad I(y)=\begin{cases}0 & \text{when}\quad y\leqslant 0\\ 1 & \text{when}\quad y>0\end{cases}$$

and the corresponding SETAR (2, k, k) model

(5)

$$Y_t=(\alpha_0+\alpha_1Y_{t-1}+\dots+\alpha_kY_{t-k}+(\beta_0+\beta_1Y_{t-1}+\dots+\beta_kY_{t-k})\cdot I(Y_{t-d})+\varepsilon_t$$

When all β_0, β_1, ..., β_k parameters are zeros then (5) becomes the linear autoregressive model.

Leting ε_t to be a martingale difference sequence, instead of the white noise, we can modify the classic SETAR model by allowing conditional heteroscedasticity. Let us consider the case when the conditional variance changes over time, but it does not changes within the regimes. As the result we have the second equation defining a GARCH-type model

$$\varepsilon_t|\Psi_{k-1}\sim N(0,\ h_t),$$

where:

$$(6)\qquad h_t=\alpha_0+\sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2+\sum_{i=1}^{p}\beta_ih_{t-i}$$

$p\geqslant 0$, $q>0$ and $\alpha_0>0$, $\alpha_i\geqslant 0$ for $i=1,\ 2,\ \dots,\ q$, $\beta_i\geqslant 0$ for $i=1,\ 2,\ \dots,$ p (cf. Bollerslev (1986).

3. STATISTICAL INFERENCE WITHIN THE TAR FRAMEWORK

3.1. Testing for the TAR Model vs. the Linear one in the Presence of ARCH

Testing for threshold non-linearity vs. the linear alternative (e.g. $H_0: \alpha = \beta$ in (5)) one has to remember that the threshold parameter r is unknown and unidentified, as a růle. Thus the asymptotic distribution of LM statistics is non-standard. Usually the LR type tests are used. The testing procedure while the residuals constitute the white noise process is described in Tong (1990), Osińska and Witkowski (1997).

Hansen (1996, 1997) indicates, that the presence of ARCH affects the testing for non-linearity in the TAR models. In the case of changing conditional variance the following procedure is recommended. An appropriate test is the Wald statistics, which is consistent in the case of heteroscedasticity. It is constructed for particular values of the threshold parameter r. The test has the following form:

$$W_n(r) = (R\theta(r))'[R(M_n(r)^{-1}V_n(r)M_n(r)^{-1})R']^{-1}, \tag{7}$$

where:

$\theta = [\alpha, \beta]$;
$R = [I - I]$
$M_n(r) = \Sigma y_t(r) y_t(r)'$;
$V_n(r) = \Sigma y_t(r) y_t(r)' \varepsilon_t^2$;
$y_t(r)$ – is a set of lagged values of Y_t in each regime.

An appropriate statistics for H_0 is

$$W_n = \sup_{r \in R} \mathrm{W}_n(r). \tag{8}$$

The critical values are generated using the bootstrap technique in the following way: let u_t^* be a sequence of random numbers such as $u_t^* \sim n.i.d.$, $t = 1, 2, ..., n$ and let $x_t^* = \varepsilon_t u_t^*$. Using empirical observations y_t, regress x_t^* conditional on y_t and $y_t(r)$. Taking the first regression we obtain the residual variance σ_t^{*2}, and the second regression gives $\sigma_t^{*2}(r)$. Assuming that W_n statistics converges to F distribution, which is the limit distribution when the threshold parameter r is known, we may compute $F_n^*(r) = n(\sigma_n^{*2} - \sigma_n^{*2}(r))/\sigma_n^{*2}$ and $F_n^* = \sup_{r \in R} F_n^*(r)$. Hansen (1996) showed, that the distribution of F_t^* converges to W_n distribution, then repeating the bootstrap procedure, and computing F_n^* we obtain the asymptotic distribution of W_n. The asymptotic p-values are given by adding the ratio of bootstrap samples for which the F_n^* exceeds the computed value of W_n.

3.2. The Parameter Estimation of the TAR Model

The parameters of the TAR models are estimated using the OLS method, conditional on whether the parameters d, r and k are known or not. The parameters are usually not known and have to be estimated (cf. Witkowski 1999).

Let us consider the following modification of (3) model:

$$\text{(9)} \qquad \mathrm{Y}_t = \begin{cases} a_0^1 + a_1^1 Y_{t-1} + \dots + a_{k_1}^1 Y_{t-k_1} + h_1 \varepsilon_t & \text{for} \quad Y_{t-d} < r \\ a_0^2 + a_1^2 Y_{t-1} + \dots + a_{k_2}^2 Y_{t-k_2} + h_2 \varepsilon_t & \text{for} \quad Y_{t-d} \geqslant r. \end{cases}$$

The estimation proceeds in two steps (cf. Tong 1983, 1990):
1. The estimation of parameters standing with lagged variables with fixed d, r, k_1, k_2.
Let

$$\text{(10)} \qquad \bar{\mathbf{a}}_i = [\mathbf{a}_0^1, \mathbf{a}_1^i, \dots, \mathbf{a}_{k_i}^i] i = 1, 2,$$

$$\text{(11)} \qquad k = \max(k_1, k_2, d).$$

The data $[\mathbf{y}_{k+1}, \dots, \mathbf{y}_N]$ may be divided into two groups $\overline{y}_1$, $\overline{y}_2$ satisfying:

$$\text{(12)} \qquad \begin{aligned} \mathbf{y}_j \in \overline{\mathbf{y}}_1 &\Leftrightarrow y_{j-d} < r, \\ y_j \in \overline{y}_2 &\Leftrightarrow y_{j-d} < r. \end{aligned}$$

Let

$$\text{(13)} \qquad \begin{aligned} &\overline{\mathbf{y}}_1 = [\mathbf{y}_{j_1}^1 \cdot \mathbf{y}_{j_2}^1, \dots, \mathbf{y}_{j_{n_1}}^1], \quad \overline{\mathbf{y}}_2 = [\mathbf{y}_{j_1}^2 \cdot \mathbf{y}_{j_2}^2, \dots, \mathbf{y}_{j_{n_2}}^2], \\ &n_1 + n_2 = N - k, \end{aligned}$$

and

$$\text{(14)} \qquad \mathbf{A}_i = \begin{bmatrix} 1 & \mathrm{y}_{j_1-1}^i & \mathrm{y}_{j_1-2}^i & \cdots & \mathrm{y}_{j_1-k_i}^i \\ 1 & \mathrm{y}_{j_2-1}^i & \mathrm{y}_{j_2-2}^i & \cdots & \mathrm{y}_{j_2-k_i}^i \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \mathrm{y}_{j_{n_i}-1}^i & \mathrm{y}_{j_{n_i}-2}^i & \cdots & \mathrm{y}_{j_{n_1}-k_i}^i \end{bmatrix} \quad i = 1, 2.$$

The estimate of $\bar{a}_1$ may be expressed in the following way:

$$\hat{\mathbf{a}}_i = (\mathbf{A}_i^T \mathbf{A}_i)^{-1} \mathbf{A}_i^T \bar{\mathbf{y}}_i, \quad i = 1, 2. \tag{15}$$

2. The estimation of all parameters. Let d, r be fixed at d_0, r_0 (model 3). Let L denote maximum order for each linear autoregressive model within the regimes. Denote:

$$AIC(d_0, r_0) = AIC(\hat{k}_1) + AIC(\hat{k}_2). \tag{16}$$

where:

$$\begin{aligned} AIC(\hat{k}_1) &= \min_{0 \leqslant k_1 \leqslant L} [n_1 \ln\{\|\varepsilon_1\|^2/n_1\} + 2(k_1 + 1)], \\ AIC(\hat{k}_2) &= \min_{0 \leqslant k_2 \leqslant L} [n_2 \ln\{\|\varepsilon_2\|^2/n_2\} + 2(k_2 + 1)], \end{aligned} \tag{17}$$

$$\bar{\boldsymbol{\varepsilon}}_i = \bar{\mathbf{y}}_i - \mathbf{A}_i \bar{\mathbf{a}}_i \quad i = 1, 2. \tag{18}$$

Hence, minimising (16) we obtain $\hat{k}_1$ and $\hat{k}_2$ with fixed d, r. Under (16), $AIC(d_0, r_0)$ is determined.

Finally, we estimate delay parameter d and threshold parameter r:

$$AIC(\hat{d}, \hat{r}) = \min_{d \in \{1, 2, \ldots, T\}} \left\{ \min_{r \in \{\tau_1, \tau_2, \ldots, \tau_m\}} AIC(d, r) \right\}, \tag{19}$$

where T means maximum value of d and $\{\tau_1, \tau_2, \ldots, \tau_m\}$ is a set of potential candidates for estimation of r.

4. FORECASTING PROCEDURES USING THRESHOLD MODELS

Forecasting based on the non-linear models is mostly often based on the Monte Carlo method (cf. Brown and Mariano 1984), Clements and Smith 1997. The MC method gives an asymptotically unbiased predictor, while the standard deterministic predictor is usually biased. Taking a great number of replications the MC predictor is usually more efficient – taking the mean squared error – then the deterministic one. There are, however, some disadvantages. The strong requirement of the MC method is a prior assumption of the innovations distribution. While the distribution is improperly specified, the predictor becomes asymptotically biased. The alternative method is based on the bootstrap technique, which uses the estimated residuals of the model instead of the generated innovations.

Three methods of forecasting the threshold models are discussed below: the mean squared error method, the Monte Carlo and the bootstrap.

4.1. The Mean Squared Forecast Error Method

The mean squared forecast error method allows to compute forecasts using any type of the TAR model. For the model (5) the practical way of taking the forecast is to compute a weighted average of the forecasts given separately from the first and second regimes. The weights are usually the probabilities that the forecasted series is in the first or in the second regime within the forecast horizon. Thus we have:

(20)

$$\hat{Y}_{n+k} = p_{k-1}\hat{Y}_{1,n+k} + (1 + p_{k-1}) \cdot \hat{Y}_{2,n+k} + (a_{2,1} - a_{1,1})\hat{\sigma}_{n+k-1}\varphi\left(\frac{r - \hat{Y}_{n+k-1}}{\sigma_{n+k-1}}\right)$$

$$k = 2,\ 3,\ \ldots,$$

where:

$$\hat{Y}_{1,n+k} = a_{1,0} + a_{1,1}\hat{Y}_{n+k-1}\hat{Y}_{2,n+k} = a_{2,0} + a_{2,1}\hat{Y}_{n+k-1},$$

$$p_{k-1} = \Phi\left(\frac{r - \hat{Y}_{n+k-1}}{\hat{\sigma}_{n+k-1}}\right)$$

Φ, φ – denote correspondingly the standard normal distribution and density $N(0, 1)$. The formula (20) is the recursive one. The first step of the procedure is as follows:

$$\hat{Y}_{n+1} = a_0 + a_1 Y_n + (b_0 + b_1 Y_n) \cdot I_n(r).$$

The formula (20) requires the standard error of prediction $\hat{\sigma}_{n+k-1}$ to be estimated. It can be computed in the following way:

$$\begin{aligned}\hat{\sigma}^2_{n+k} = {} & \left\{(a_{1,0} + a_{1,1}\hat{Y}_{n+k-1})^2 + a^2_{1,1}\hat{\sigma}^2_{n+k-1}\right\}p_{k-1} + \\ & + \left\{(a_{2,0} + a_{2,1}\hat{Y}_{n+k-1})^2 a^2_{2,1}\hat{\sigma}^2_{n+k-1}\right\} + \\ & + \left\{\begin{array}{l} a_{2,1}^2(r - \hat{Y}_{n+k-1}) + 2a_{2,1}(a_{2,0} + a_{2,1}\hat{Y}_{n+k-1}) - \\ (a^2_{1,1}(r - \hat{Y}_{n+k-1}) + 2a_{1,1}(a_{1,0} + a_{1,1}\hat{Y}_{n+k-1})) \end{array}\right\} \cdot \\ & \cdot \hat{\sigma}_{n+k-1}p_{k-1} + \sigma^2_{\varepsilon} - \hat{Y}^2_{n+k}.\end{aligned}$$

The above formula is proper only in the case when the residual variances in each regimes are mutually equal to σ^2_{ε}.

4.2. The Monte Carlo Method

The Monte Carlo method is a simple simulation based method of forecasting used to a broad class of the non-linear models. The forecast for one period ahead is identical to the one described in Section 4.1, i.e.

$$\hat{Y}_{n+1} = a_0 + a_1 Y_n + (b_0 + b_1 Y_n) \cdot I_n(r). \tag{21}$$

For longer forecast horizon a following sequence of the forecasts is computed $\hat{Y}^j_{n+2}, \hat{Y}^j_{n+3}, \ldots, \hat{Y}^j_{n+k}$, such as

$$\hat{Y}^j_{n+2} = a_0 + a_1 \hat{Y}_{n+1} + (b_0 + b_1 \hat{Y}_{n+1}) \cdot I_{n+1}(r) + \hat{\xi}^h_{2,j}, \tag{22}$$

$$\hat{Y}^j_{n+3} = a_0 + a_1 \hat{Y}^j_{n+2} + (b_0 + b_1 \hat{Y}^j_{n+2}) \cdot I_{n+2}(r) + \hat{\xi}^h_{3,j}, \tag{23}$$

and

$$\hat{Y}^j_{n+k} = a_0 + a_1 \hat{Y}^j_{n+k-1} + (b_0 + b_1 \hat{Y}^j_{n+k-1}) \cdot I_{n+k-1}(r) + \hat{\xi}^h_{k,j}, \tag{24}$$

$$j = 1, 2, 3, \ldots, N,$$

where $\hat{\xi}^h_{2,j}, \hat{\xi}^h_{3,j}, \hat{\xi}^h_{k,j}$ constitute a set of independent random variables, normally distributed, independent of ε. The superscript h means, that the variance of the random variable depends on the regime of the process, i.e. $\hat{\xi}^h_{i,j} \sim N(0, \sigma^2_h)$. Repeating the procedure given by the relations (22)–(24) for $j = 1, 2, 3, \ldots, N$ we are able to compute the final result as

$$\hat{Y}_{n+k} = \frac{1}{N} \sum_{j=1}^{N} \hat{Y}^j_{n+k}. \tag{25}$$

4.3. The Bootstrap Method

The idea of the method is very similar to the Monte Carlo method, the difference is that the set $\hat{\xi}^h_{2,j}, \hat{\xi}^h_{3,j}, \hat{\xi}^h_{k,j}$ is the result of the independent sampling from the estimated error vectors $\hat{\varepsilon}_1, \hat{\varepsilon}_2$.

5. FORECASTING RATES OF RETURN USING THRESHOLD MODELS – SOME EMPIRICAL RESULTS

The parameter estimates were obtained using EViews 4.0 software. The following assumptions were made:

- there is one or two threshold parameters (i.e. two or three regimes);
- the minimum and maximum value of parameter d is equal to one and three respectively;
- the maximum order for each linear autoregressive model is equal to 6.

The examples of the estimated models (for BPH and Kredytbank) are presented below:

$$BPH_t = \begin{cases} -0{,}000453 \leqslant + h_1\varepsilon \\ WIG_{t-1} \leqslant -0{,}009375 \\ 0{,}00629 - 0{,}1007 \cdot BPH_{t-1} - 0{,}04223 \cdot BPH_{t-2} - 0{,}492 \cdot BPH_{t-3} + 0.065 \cdot BPH_{t-4} + 0{,}193 \cdot BPH_{T-5} + h_2\varepsilon \\ -0{,}000453 < WIG_{t-1} \leqslant 0{,}069951 \\ 0{,}03308 + h_3\varepsilon_t \\ WIG_{t-1} > 0{,}069951 \end{cases}$$

$$KRT_t = \begin{cases} -0{,}002306 - 0{,}13607 \cdot KRT_{t-1} + h_1\varepsilon \\ KRT_{t-2} \leqslant 0{,}013351 \\ 0{,}00949 - 0{,}16998 \cdot KRT_{t-1} - 0{,}297835 \cdot KRT_{t-2} + 0{,}29448 \cdot KRT_{t-3} + 0{,}338059 \cdot KRT_{t-4} + h_2\varepsilon \\ -0{,}013351 < KRT_{t-1} \leqslant 0{,}030687 \\ -0{,}010465 + h_3\varepsilon_t \\ KRT_{t-2} > 0{,}0306871 \end{cases}$$

In the first model the Warsaw Stock Exchange index lagged by 1 was the threshold variable and in the second case we can see the SETAR model with the threshold variable lagged by 2.

The forecasting process was concentrated on two methods: the Monte Carlo and two versions of the bootstrap method. In the Monte Carlo method the innovations of the model were generated from the standard normal distribution $N(0, 1)$.

The bootstrap sampling was applied in two versions: BS1 – when the innovations came from the whole sample of the estimated residuals and BS2 – when the innovations were taken from separated regimes. The forecast horizon was 10 periods ahead. For each period 400 replications were made and the forecast was taken at the mean level and at the median level, respectively. The distributions of the forecast values in each replication, for 1, 2, etc. periods ahead were usually skewed.

The forecasting accuracy was measured using mean squared error (MSE) and the mean absolute percentage error (MAPE) and the measures of the direction accuracy such as (cf. Brzeszczyński and Kelm 2002)

$$Q1 = \frac{N(Y_t\hat{Y}_t > 0)}{N(Y_t\hat{Y}_t \neq 0)}. \tag{26}$$

where:

Y_t, $\hat{Y}_t$ – the observed and the theoretical value of Y_t, respectively;

$N(Y_t\hat{Y}_t > 0)$ – number of observations where the direction of the forecast and empirical values was the same;

$N(Y_t\hat{Y}_t \neq 0)$ – number of non-zero products of the observed and theoretical values.

In the Tables 1 and 2 the squared roots of the MSE and the MAPE results are reported, respectively.

Table 1. The computed squared roots of the MSE forecast errors using threshold models (10 periods ahead)

Model	Squared roots of MSE					
	BS1		BS2		MC	
	mean	median	mean	median	mean	median
BIG	0.05402	0.05291	0.05435	0.05420	0.05586	0.05683
BOS	0.02002	0.01920	0.01998	0.01837	0.02083	0.02139
BSK	0.01681	0.01703	0.01778	0.01694	0.01732	0.01612
HANDLOWY	0.03915	0.03993	0.03790	0.04005	0.03891	0.03844
KREDYT	0.10754	0.10837	0.10705	0.10708	0.10740	0.10699
KREDYT*	0.10770	0.10783	0.10670	0.10607	0.10636	0.10519
WIG	0.04340	0.04341	0.04309	0.04426	0.04389	0.04292
BPH	0.04327	0.04189	0.04244	0.04203	0.04295	0.04322
BPH*	0.04421	0.04314	0.04392	0.04065	0.04268	0.04316
BRE	0.06081	0.06097	0.06128	0.06104	0.06137	0.06172
BZWBK	0.06628	0.06533	0.06431	0.06400	0.06654	0.06792
PEKAO	0.04090	0.04049	0.04097	0.03998	0.04192	0.04322

* Denotes two-regime version of the model, the remained are three regime models.

The first seven rows in Tables 1 and 2 concern the SETAR models and the 5 last concern the TAR models in which lagged rate of return of WIG index is the threshold variable. Taking into account that we had to predict the threshold variable first, it is understandable that the results based on the TAR models are worse. Additionally the forecasts for the WIG index were the worst of all forecasts based on the SETAR models.

Table 2. The computed MAPE for the forecasts using threshold models (10 periods ahead)

Model	MAPE					
	BS1		BS2		MC	
	mean	median	mean	median	mean	median
BIG	-0.15928	0.123184	-0.31656	0.016525	-0.30889	-0.00525
BOS	-0.77150	-0.11910	-0.74446	-0.07428	-0.85644	-0.17040
BSK	-0.95451	-0.13407	-0.90165	-0.12981	-0.76976	-0.22786
HANDLOWY	-0.68099	-0.05093	-0.44648	0.07791	-0.74387	-0.10452
KREDYT	-0.28967	-0.23101	-0.35089	-0.13910	-0.37211	-0.22673
KREDYT*	-0.42369	-0.38126	-0.24637	-0.18847	-0.42120	-0.27121
WIG	1.185956	0.574644	2.39827	2.333291	2.252129	0.964737
BPH	0.92199	1.00117	0.75706	-1.01767	1.35056	1.08186
BPH*	-1.12421	-1.41804	-0.58183	0.26272	-1.62506	-1.41652
BRE	-0.23200	-0.17912	-0.18270	-0.13152	-0.30956	-0.20334
BZWBK	-0.73616	-0.66071	-0.71549	-0.66670	-0.67518	-0.64100
PEKAO	-0.52122	-0.44558	-0.54569	-0.39966	-0.51608	-0.48785

* Denotes two-regime version of the model, the remained are three regime models.

Taking the nominal values of the predicted returns we observe that they are rarely consistent with the realisations. However, some values of MAPE related to the median may be found quite satisfactory. In general, the median was a better basis of comparison then the mean, which results from the asymmetry of the forecasts distribution. There are not significant differences between the forecasting methods applied, however the bootstrap 2 (sampling within regimes) is recommended. The direction accuracy of the forecasts is presented in Table 3.

Table 3. The results of measuring the direction of forecast consistency using threshold models

Model	Method	Percentage when the direction was consistent		
		1 period ahead	5 periods ahead	10 periods ahead
BIG	BS1 – mean	+	80	70
	BS1 – median	+	80	70
	BS2 – mean	+	60	70
	BS2 – median	+	20	60
	MC – mean	+	40	50
	MC – median	+	40	40
Handlowy	BS1 – mean	+	80	60
	BS1 – median	–	60	40
	BS2 – mean	+	80	80
	BS2 – median	–	40	30
	MC – mean	–	60	60
	MC – median	+	80	60
Kredyt	BS1 – mean	+	60	50
	BS1 – median	+	80	50
	BS2 – mean	+	60	50
	BS2 – median	+	80	50
	MC – mean	+	80	60
	MC – median	+	80	70
Kredyt 2	BS1 – mean	+	80	40
	BS1 – median	+	80	40
	BS2 – mean	–	80	60
	BS2 – median	+	100	70
	MC – mean	+	80	70
	MC – median	+	80	80
BPH	BS1 – mean	+	80	60
	BS1 – median	+	60	40
	BS2 – mean	+	60	60
	BS2 – median	+	40	50
	MC – mean	+	60	60
	MC – median	+	60	60

Table 3. (cont.)

Model	Method	Percentage when the direction was consistent		
		1 period ahead	5 periods ahead	10 periods ahead
BRE	BS1 - mean	-	80	60
	BS1 - median	-	60	40
	BS2 - mean	-	60	60
	BS2 - median	-	40	50
	MC - mean	-	60	60
	MC - median	-	60	60

The consistency of the forecasts direction was satisfactory in general. It was independent of the chosen method of forecasting. In many cases the forecast direction was the same as the realisation in 80%, and occasionally in 100%. The forecasting using threshold stationary models is recommended for shorter horizons (up to 5 periods ahead).

6. FINAL REMARKS

The aim of the paper was to analyze the efficiency of forecasting using stationary threshold models. Two methods of forecasting in three variants were applied; each of them seems to be useful in prediction economic time series. Predicting weekly returns of some stocks quoted at the Stock Exchange in Warsaw at the level of the conditional mean is very difficult. However, we have found great usefulness of the threshold autoregressive models in *ex-ante* predicting the directions of the changes. In many cases the direction of the forecasts was consistent with the empirical data in 80%, especially for short (up to 5 weeks) forecast horizon. Taking weekly returns we have found that the ARCH effect was not too strong, so we decided to skip it in our investigation. We expect that adding forecasts of the conditional variances, may improve the results.

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WYKORZYSTANIE MODELI PROGOWYCH DO PROGNOZOWANIA STÓP ZWROTU

(Streszczenie)

Celem artykułu jest porównanie metod prognozowania nieliniowych modeli progowych. Wykorzystane zostały dwie metody prognozowania: metoda bootstrap w dwóch wariantach oraz metoda Monte Carlo. Przedmiotem analizy są tygodniowe stopy zwrotu spółek sektora bankowego, notowanych na GPW w Warszawie. W konkluzji stwierdza się, że przewidywanie dokładnych wartości stóp zwrotu jest bardzo trudne, natomiast modele progowe dają bardzo dobre wyniki w zakresie przewidywania kierunków zmian w przyszłości.