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SOME PROPERTIES OF THE ROBUST TREND TESTS

Abstract. Formal testing of whether a time series contains a trend is greatly complicated by the fact that in practice it is not known whether the trend is embedded in an *I*(0) or *I*(1), series, that is, within a weakly or strongly autocorrelated series. In this article we would like to present the properties of behavior of the robust (to the order of integration of the data) trend tests of Bunzel and Vogelsang (2005), Harvey et al. (2007) and Perron and Yabu (2009). These statistics are termed 'robust' in the sense that the asymptotic critical values for testing hypotheses on the trend coefficient.

Key words: the trend coefficient, the robust trend, the robust trend tests

INTRODUCTION

In time series analysis, autoregressive integrated moving average (ARIMA) models have found extensive use since the publication of Box and Jenkins (1976). Regression models are also frequently used in finance and econometrics research and applications. As "factor" models for empirical asset pricing research and for parsimonious covariance matrix estimation in portfolio risk models. Often ARIMA models and regression models are combined by using an ARIMA model to account for serially correlated residuals in a regression model, resulting in REGARIMA models. In reality, most time series data are rarely completely well behaved and often contain outliers and level shifts, which is especially true for economic and financial time series. The classical maximum likelihood estimators of both ordinary regression model parameters and ARIMA model parameters are not robust in that they can be highly influenced by the presence of even a small fraction of outliers and/or level shifts in a time series. It is therefore not surprising that classical maximum likelihood estimators of REGARIMA models also lack robustness toward outliers and/or level shifts.

Formal testing of whether a time series contains a trend is greatly complicated by the fact that in practice it is not known whether the trend is embedded in an *I*(0) or *I*(1), series, that is, within a weakly or strongly autocorrelated series. If one knew that the shocks were *I*(0) then one could test for the presence of a linear trend using levels data. Similarly, if it were known that the shocks were *I*(1) then one could perform tests on the first differences of the data (growth rates). However, tests based on growth rates display very poor power properties relative to those based on levels when the shocks are in fact *I*(0).

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In this article we would like to present the properties of sample behavior of the robust (to the order of integration of the data) trend tests of Bunzel and Vogelsang (2005), Harvey et al. (2007) and Perron and Yabu (2009). These statistics are termed 'robust' in the sense that the asymptotic critical values for testing hypotheses on the trend coefficient.

I. ROBUST CHANGE DETECTION

1.1.ARMA(p, q) Models

A very rich and practically useful class of stationary and ergodic processes is the autoregressive-moving average (ARMA) class of models made popular by Box and Jenkins (1976). ARMA(p, q) models take the form of a p^{th} order stochastic difference equation

$$y_t - \mu = \phi_1(y_{t-1} - \mu) + ... + \phi_p(y_{t-p} - \mu) + \varepsilon_t + \theta_1\varepsilon_{t-1} + ... + \theta_q\varepsilon_{t-q} \qquad (1)$$
$$\varepsilon_t \sim \mathrm{WN}(0, \sigma^2)$$

ARMA(p, q) models may be thought of as parsimonious approximations to the general Wold form of a stationary and ergodic time series.

The presentation of time series models is simplified using *lag operator* notation. The lag operator L is defined such that for any time series $\{y_t\}$, $Ly_t = y_{t-1}$. The lag operator has the following properties:

$L^2y_t = L \cdot Ly_t = y_{t-2}$,

$L^0 = 1$ and $L^{-1}y_t = y_{t+1}$.

The operator $\Delta = 1 - L$ creates the first difference of a time series: $\Delta y_t = (1 - L)y_t = y_t - y_{t-1}$. The ARMA(p, q) model may be compactly expressed using lag polynomials.

Define

$\phi(L) = 1 - \varphi_1 L - \cdot\cdot\cdot - \varphi_p L_p$ and $\theta(L) = 1+\theta_1 L + \cdot\cdot\cdot + \theta_q L_q$.

Then ARMA(p, q) model may be expressed as

$$\phi(L)(y_t - \mu) = \theta(L)\varepsilon_t \qquad (2)$$

ARMA(p, q) models often arise from certain aggregation transformations of simple time series models. An important result due to Granger and Morris (1976) is that if y_{1t} is an ARMA(p_1, q_1) process and y_{2t} is an ARMA(p_2, q_2) process, which may be contemporaneously correlated with y_{1t}, then $y_{1t} + y_{2t}$ is an ARMA(p, q) process with $p = p_1 + p_2$ and $q = \max(p_1 + q_2, q_1 + p_2)$. For example, if y_{1t} is an AR(1) process and y_2 is a AR(1) process, then $y_1 + y_2$ is an ARMA(2,1) process.

High order ARMA(p, q) processes are difficult to identify and estimate in practice and are rarely used in the analysis of financial data. Low order ARMA(p, q) models with p and q less than three are generally sufficient for the analysis of financial data.

1.2.ARIMA(p, d, q) Models

The specification of the ARMA(p, q) model (1) assumes that y_t is stationary and ergodic. If y_t is a trending variable like an asset price or a macroeconomic aggregate like real GDP, then y_t must be transformed to stationary form by eliminating the trend. Box and Jenkins (1976) advocate removal of trends by differencing. Let $\Delta = 1-L$ denote the *difference operator*. If there is a linear trend in y_t then the first difference $\Delta y_t = y_t - y_{t-1}$ will not have a trend.

If there is a quadratic trend in y_t, then Δy_t will contain a linear trend but the second difference $\Delta^2 y_t = (1 - 2L + L^2) y_t = y_t - 2y_{t-1} + y_{t-2}$ will not have a trend.

The class of ARMA(p, q) models where the trends have been transformed by differencing *d* times is denoted ARIMA(p, d, q) .

1.3 REGARIMA Models

The REGARIMA model takes the following form:

$$y_t = x_t^{'}\beta + \varepsilon_t \text{, for } t = 1,\ldots,T \tag{3}$$

where x_t is a $k \times 1$ vector of predictor variables, and β is a $k \times 1$ vector of regression coefficients. The error term ε_t follows a seasonal ARIMA process:

$$\Phi(L)(1-L)^d (1-L^s)^D \varepsilon_t = (1-\theta^* L^s)\Theta(L) u_t \tag{4}$$

where:

L is the lag (or backshift) operator,

d the number of regular differences,

D the number of seasonal differences,

s the seasonality frequency,

$\Phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$ a stationary autoregressive operator of order p,

$\Theta(L) = 1 - \theta_1 L - \cdots - \theta_q L^q$ a moving average operator of order q

θ^* a seasonal moving average parameter.

Note that currently only one seasonal moving average term is allowed in the discussions in this chapter. The innovations u_t are assumed to be identically independent distributed random variables with distribution F. In practice, observed time series data are rarely well behaved as assumed in the REGARIMA model (3) and (4). An observed time series y_t^* is usually some kind of variant of y_t in equation (3). When the observed time series y_t^* might be influenced by some outliers, the classical maximum likelihood estimates are not robust. Furthermore, it will detect three kinds of outliers in the original data y_t^*:

Additive outliers (AO):

An additive outlier occurs at time t_0 if $y_{t_0}^* = y_{t_0} + c,$ where c is a constant. The effect of this type of outlier is restricted to the time period t_0.

Innovation outliers (IO):

An innovation outlier occurs at time t_0 if $u_{t_0} = v_{t_0} + c$, where v_{t_0} is generated by the distribution F. Usually it is assumed that F is the normal distribution N(0, σ^2). Note that the effect of an innovation outlier is not restricted to time t_0 because of the structure of an ARIMA model. It also has influence on the subsequent observations.

Level shifts (LS):

If one level shift occurs at time t_0, the observed series is $y_t^* = y_t + c$ for all $t \geq t_0$, with c being a constant. Note that if the series y_t^* has a level shift at t_0, the differenced series $y_t^* - y_{t-1}^*$ has an additive outlier at t_0.

In all those three cases c is the size of the outlier or level shift. Without any potential confusion, the general term "outlier" may refer to any of the three types of behavior.

Controlling Outlier Detection

The outlier detection procedure is similar to those proposed by Chang, Tiao and Chen (1988) and Tsay (1988) for ARIMA models. The main difference with those procedures is that we use innovation residuals based on the filtered τ – estimates of β and λ, instead of the classical maximum likelihood estimates.

To detect the presence of an outlier at a given time t_0, the outlier detection procedure we compute:

$$\tau = \max_{t_0} \max \{T_{t_0,AO}, T_{t_0,LS}, T_{t_0,IO}\} \tag{5}$$

where $T_{t_0,AO}, T_{t_0,LS}, T_{t_0,IO}$ are the statistics corresponding to AO, LS and IO at time t_0 respectively.

The test statistic is defined as follows:

$$T = \frac{|\hat{\omega}|}{\hat{V}(\hat{\omega})^{1/2}} \tag{6}$$

where $\hat{\omega}$ is an estimate of ω, the size of the outlier, based on the residuals of the filtered τ –estimates and $\hat{V}(\hat{\omega})$ an estimate of its variance. If $\tau > \xi$, where ξ is a conveniently chosen critical value, one declares that there is an outlier. The time t_0 where the outlier occurs and the type of the outlier are those where the double maximum is attained.

The critical value ξ is similar to the constant used by Chang, Tiao and Chen (1988). They recommend using ξ = 3 for high sensitivity in outlier detection, ξ = 3,5 for medium sensitivity and ξ = 4 for low sensitivity, when the length of the series is less than 200. The critical value ξ is specified by the optional argument, the default value is set as follows:

$$\xi = \begin{cases} 3 & \text{if } \mathrm{T} \le 200, \\ 3{,}5 & \text{if } 200 < \mathrm{T} \le 500, \\ 4 & \text{if } \mathrm{T} > 500. \end{cases}$$

More details of this procedure can be found in Bianco, Garcia Ben, Martinez and Yohai (1996, 2001).

1.4. ARARMA Models

This is a non-parametric linear model posed by Grambsch and Stahel (1990) modeling and forecasting time series. These time series were characterized by downward sloping trends and step jumps. The median based estimate of trend is designed to be uninfluenced by outliers. In ARIMA notation for a time series, X_t, the model is

$$(1 - B)X_t = \theta_0 + a_t \tag{7}$$

an ARIMA (0,1,0) with a constant, i.e. a random walk with a deterministic trend, and the $\{a_j\}$ are independent, identically distributed stable random variables. The robustness of the estimate of the deterministic trend parameter, for θ_0 , is due to its being based on the median of the median based estimate of trend is designed outliers that are particularly common in the rather than the arithmetic mean. This protects the estimate of the trend parameter from being contaminated by the outliers that are particularly common in the telecommunications data. The details of forecasting and trend estimation are given here. At time *T*, the *k* step ahead

$$\hat{X}_{T+K} = X_T + k\hat{\theta}_{0|T}$$

where $\hat{\theta}_{0|T}$ is the estimate the trend At the time *T*. The robust estimate of the trend is

$$\hat{\theta}_{0|T} = M_T + \frac{m_T}{T}\sum_{t=2}^{T}\psi\left(\frac{Z_T - M_T}{m_T}\right),$$

where $Z_t = (1 - B)X_t$, M_T is the median of $(Z_2,\ldots, Z_T)$ and m_T is the median of $(|Z_2 - M_T|,\ldots, |Z_T - M_T|)$.

The response function ψ is of a "three part re-descending" type:

$$\psi(x) = sign(x)\max[\min\left(\left|\frac{2x}{3}\right|,1,2-\left|\frac{x}{3}\right|\right),0]$$

The design of the response function gives the method its robustness by preventing very large deviation from affecting the trend adjustment.

ARARMA stands for auto regressive moving average. This methodology proposed by Parzen (1982). For a times series X_t,, the first transformation is from long memory to the short memory:

$$Z_t = X_t - \phi_\tau X_{t-\tau} \tag{8}$$

where ϕ_τ and τ are chosen to minimize

$$Err(\tau) = \frac{\sum_{t=\tau+1}^{T}(X_t - \phi_\tau X_{t-\tau})}{\sum_{t=\tau+1}^{T} X_t^2}.$$

To achieve the transformation of the data to stationarity, Parzen preferred a long memory AR filter to the "harsher" differencing used ARIMA.

For seasonal series, the data were deseasonalised by routines provided by Hibon, the forecasts prepared and then reseasonalised. In order to distinguish between series that exhibit seasonality and those whose observations are merely monthly or quarterly the following procedure was adopted. The last six available observations were forecast out of sample under the assumptions that series was seasonal and that the series was non-seasonal. The assumption that provided the better mean absolute percentage error was used to provide the final forecast.

II. ROBUST TEST FOR TREND

We present a simple test procedure (Harvey et al. 2007) for a linear trend which does not require knowledge of the form of serial correlation in the data, is robust to strong serial correlation, and has a standard normal limiting null distribution under either $I(0)$ or $I(1)$ shocks[1]. In contrast to other available robust linear trend tests, our proposed test achieves the Gaussian asymptotic local power envelope in both the $I(0)$ and $I(1)$ cases. For near- $I(1)$ errors our proposed procedure is conservative and a modification for this situation is suggested. An estimator of the trend parameter, together with an associated confidence interval, which is asymptotically efficient, again regardless of whether the shocks are $I(0)$ or $I(1)$, is also provided.

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[1] $\{y_t\}$ is an *integrated process* of order 1 denoted $y_t \sim I(1)$, if it has the form $y_t = y_{t-1} + u_t$ where u_t I s a stationary time series. The first difference is stationary $\Delta y = y_t - y_{t-1} = u_t$. Because of this property $I(1)$ is called *difference stationary* process. Starting at y_0 the y_t can be representing as an integrated sum of stationary innovations $y_t = y_0 + \sum_{j=1}^{t} u_j$. The integrated sum $\sum_{j=1}^{t} u_j$ is called *stochastic trend*. In contrast to deterministic trend are not perfectly predictable. Since the stationary process u_t does not need to be differenced, it is called an integrated process of order zero and is denoted $u_t \sim I(0)$.

2.1. Motivation for the test procedure

To fix ideas, we start with a highly simplified model and testing problem. Consider, therefore, the Gaussian AR(1) model

$$y_t = \alpha + \beta t + u_t\,,\; t = 1,\ldots T$$

$$u_t = \rho u_{t-1} + \varepsilon_t\,,\; t = 1,\ldots T\,,\; u_1 = \varepsilon_1 \tag{9}$$

where ε_t is assumed to be NIID(0; σ^2).

We suppose that the $I(0)$ scenario for u_t is represented by $\rho = 0$ and the $I(1)$ scenario by $\rho = 1$, with no other possibilities assumed to exist for the present.

Our interest centers on testing

$$H_0\colon \beta = \beta_0$$

against either a two-sided alternative,

$$H_1\colon \beta \neq \beta_0,$$

or either of the two one-sided alternatives

$$H_1\colon \beta > \beta_0 \text{ or } H_1\colon \beta < \beta_0,$$

but without assuming knowledge of whether u_t in (9) is $I(0)$ or $I(1)$. The case of leading empirical relevance is the no trend null hypothesis, given by $\beta_0 = 0$, although other values of β_0 may also be of practical interest, for example testing whether the growth rate in a particular country coincides with some hypothetical or desired growth rate. As is customary in this kind of testing problem, we partition H_1 into two scaled components $H_{1;0} : \beta = \beta_0 + \kappa T^{-3/2}$ when u_t is $I(0)$ and $H_{1;1} : \beta = \beta_0 + \kappa T^{-1/2}$ when u_t is $I(1)$, where κ is a finite constant, which provide the appropriate Pitman drifts on β under $I(0)$ and $I(1)$ errors, respectively. Notice that both $H_{1;1}$ and $H_{1;0}$ reduce to H_0 when $\kappa = 0$.

If u_t is *known* to be $I(0)$ then $u_t = \varepsilon_t$, t = 1, . . . ,T, and a test which rejects for large values (large positive or large negative values for a two-tailed test, large negative (positive) values for a lower- (upper-) tailed test) of the (centred) *t-ratio*, associated with the OLS estimator of β in the estimated model (9), is an optimal (uniformly most powerful in the case of one sided alternatives and uniformly most powerful unbiased in the case of the two-sided alternative) test of H_0 against $H_{1;0}$, and is consistent against fixed alternatives.

Letting $\hat{\alpha}$ and $\hat{\beta}$ denote the OLS estimators from (9), this *t-ratio* is therefore given by

$$z_0 = \frac{\hat{\beta} - \beta_0}{s_0} \tag{10}$$

$$s_0 = \sqrt{\frac{\hat{\sigma}_u^2}{\sum_{t=1}^{T}(t-\bar{t})^2}} \text{ where } \hat{\sigma}_u^2 = \frac{\sum_{t=1}^{T}\hat{u}_t^2}{(T-2)} \text{ and } \hat{u}_t = y_t - \hat{\alpha} + \hat{\beta}t .$$

Standard results we show H_0: $z_0 \rightarrow N(0,1)$ while under $H_{1;0}$: $z_0 \rightarrow \kappa/(\sigma\sqrt{12})$ + (N(0,1)

Correspondingly, if u_t is *known* to be $I(1)$ then the optimal test of H_0 against $H_{1;1}$ is based on the *t*-ratio associated with the (centered) OLS estimator of β in model (9) estimated in first differences

$$\Delta y_t = \beta + v_t \quad t = 2; \ldots ;T, \tag{11}$$

where $v_t =: \Delta u_t = \varepsilon_t$,.

The *t*-ratio is therefore given by

$$z_1 = \frac{\tilde{\beta} - \beta_0}{s_1}, \tag{12}$$

$$s_1 = \sqrt{\frac{\tilde{\sigma}_v^2}{T-1}}$$

where $\tilde{\beta}$ is the OLS estimator of β in (11): $\tilde{\beta} = \frac{\sum_{t=1}^{T}\Delta y_t}{(T-1)} = \frac{y_T - y_1}{(T-1)}$ and $\tilde{\sigma}_v^2 = \frac{\sum_{t=1}^{T}\tilde{v}_t^2}{(T-2)} = \frac{\sum_{t=1}^{T}(\Delta y_t - \tilde{\beta})^2}{(T-2)}$.

Once more, standard results show that under H_0: $z_1 \to N(0,1)$, while under $H_{1;1}$: $z_1 \to \kappa/\sigma + N(0,1)$. Again the test is consistent against fixed alternatives.

Now consider the behavior of the statistic z_0 of (10) when u_t is in fact $I(1)$. It is entirely straightforward to establish that under both H_0 and $H_{1;1}$, z_0 is of $O_e(T^{1/2})$. That is, it diverges regardless of whether H_0 or $H_{1;1}$ is true.

As for the behavior of z_1 of (12) when u_t is $I(0)$, it is easy to show that under H_0 and $H_{1;0}$, z_1 is of $O_p(T^{-1/2})$ and, hence, converges in probability to zero, again regardless of whether H_0 or $H_{1;0}$ holds. The pertinent features of these findings are that z_0 does not control size under H_0 when u_t is $I(1)$ (its asymptotic size is unity), and z_1 does not control size when u_t is $I(0)$ (its asymptotic size is zero).

2.2. The model and robust trend tests

Here we pursue an approach based on a data-dependent weighted average of z_0 of (10) and z_1 of (12) where the weights used are based on a consistent estimator of $d \in \{0; 1\}$, the (unknown) order of integration of u_t. The estimator of d which we propose is constructed from unit root and stationarity test statistics. In generic notation, let U denote some unit root statistic used for testing the $I(1)$ null that $\rho = 1$ against the $I(0)$ alternative, which corresponds to $\rho = 0$ in the present simplified context.

Similarly, let S denote some stationarity statistic for testing the $I(0)$ null that $\rho = 0$ against the $I(1)$ alternative $\rho = 1$.

Consider the case where we have a sample of T observations generated according to the data-generating process (DGP):

$$y_t = \mu + \beta_T t + u_t \text{, t = 1,…,T} \tag{13}$$

$$u_t = \rho_T u_{t-1} + \varepsilon_t \text{, t = 1,…,T} \tag{14}$$

The statistics we consider to test the null hypothesis $\beta_T = 0$ against $\beta_T \neq 0$ in (13) are the z_λ statistics of Harvey et al.(2007), the $t^{RQF}{}_b$ statistic of Perron and Yabu (2009), and the Dan-J statistic of Bunzel and Vogelsang (2005).

The **z_λ statistic** of Harvey et al. (2007) employs a switching-based strategy that attains the local limiting Gaussian power envelope for this testing problem (under the assumption of an asymptotically negligible initial condition) irrespective of whether u_t is an exact $I(1)$ process or is $I(0)$, the latter occurring where

$\rho_\tau = \rho$ with $|\rho| < 1$. The test statistic is also asymptotically standard normal under the null in both cases. It is calculated as:

$$z_\lambda := (1-\lambda^*)z_0 + \lambda^* z_1 \tag{15}$$

$$z_0 = \frac{\hat{\beta}_T}{\sqrt{\dfrac{\hat{\omega}_u^2}{\sum_{t=1}^{T}(t-\bar{t})^2}}} \text{ and } z_1 = \frac{\tilde{\beta}_T}{\sqrt{\dfrac{\tilde{\omega}_v^2}{(T-1)}}} \tag{16}$$

In equation (4), $\hat{\beta}_T$ denotes the ordinary least square (OLS) estimator of β_T from equation (13) and $\hat{\omega}_u^2$ is a long-run variance estimator formed using $\hat{u}_t = y_t - \hat{\mu} + \hat{\beta}_T t$, $\hat{\mu}$ the corresponding OLS estimator of μ from equation (13), whereas $\tilde{\beta}$ is the OLS estimator of β from equation (13) estimated in first differences, that is, from $\Delta y_t = \beta_T + v_t$, $t = 2,\ldots,T$ and $\tilde{\omega}_v^2$ is a long-run variance estimator based on $\tilde{v}_t = \Delta y_t - \tilde{\beta}_T$.

The weight function λ^* is defined as:

$$\lambda^* = \exp\left(-0.00025\left(\frac{ERS}{KPSS}\right)^2\right)$$

where ERS is the with-trend local generalized least squares (GLS) unit root test statistic of Elliott et al. (1996) and KPSS is the with trend stationarity test statistic of Kwiatkowski et al. (1992).

The **the t^{RQF}_b statistic** of Perron and Yabu (2009) takes the form of an autocorrelation-corrected t-ratio on the OLS estimate of β_T obtained from the quasi-GLS regression

$$y_t - \tilde{\rho}_{MS} y_{t-1} = (1-\tilde{\rho}_{MS})\mu + \beta_T[t - \tilde{\rho}_{MS}(t-1)] + (u_t - \tilde{\rho}_{MS} u_{t-1}) \tag{17}$$

for $t = 2,\ldots,T$, along with $y_1 = \mu + \beta_T + u_1$. Here $\tilde{\rho}_{MS}$ is defined according to rule

$$\tilde{\rho}_{MS} = \begin{cases} 1 & if \left|\tilde{\rho}_{MS} - 1\right| < T^{-1/2} \\ \tilde{\rho}_{TWS} & otherwise \end{cases}$$

where $\tilde{\rho}_{TWS}$ is an autocorrelation robust-weighted symmetric least squares estimate of q (based on the OLS residuals $\hat{u}_t$) with one of two truncations applied as described by Roy and Fuller (2001) and Roy et al. (2004). The $t^{RQF}{}_b$ statistic is asymptotically standard normal under the null hypothesis when u_t is either exact $I(1)$ or is $I(0)$, and, has the same local asymptotic power as the z_λ statistic of Harvey et al. (2007) in the local-to-unity autoregressive root environment.

The ***Dan-J statistic*** of Bunzel and Vogelsang (2005) is essentially a modified version of the *t-PSW* test statistic of Vogelsang (1998) that employs a long-run variance estimator based on the 'fixed-b' asymptotic of Kiefer and Vogelsang (2005). Specifically, the statistic is

$$Dan - J = z_0^{'} \exp(-c_\xi J) \tag{18}$$

where z'_0 is z_0 as defined in (16) but with the implicit long run variance estimator, $\hat{\omega}_u^2$ constructed using the Daniell kernel with a data-dependent bandwidth. Specifically, the bandwidth is given by max($\hat{b}_{opt} T$; 2), where $\hat{b}_{opt} = b_{opt}(\hat{c})$. Here, $\hat{c} = T(1-\hat{\rho})$ with $\hat{\rho}$ obtained by OLS estimation of (2), and $b_{opt}(.)$ is a step function given in Bunzel and Vogelsang (2005). In the expressions for *Dan-J*, the z'_0 statistic is scaled by a function of the *J* unit root test statistic of Park (1990) and Park and Choi (1988). The constant c_ξ is chosen so that, at a given significance level, ξ, a particular test has the same critical value under both I(0) and I(1) errors. The value of constant c_ξ depends on $\hat{b}_{opt}$; Bunzel and Vogelsang (2005) provide a response surface for determining c_ξ for a given significance level, and $\hat{b}_{opt}$. The critical values for the test also depend on $\hat{b}_{opt}$, and again a response surface is provided by the authors for a variety of significance levels. The critical values for the test also depend on $\hat{b}_{opt}$, and again a response surface is provided by the authors for a variety of significance levels. Because *c* is not consistently estimated using $\hat{c}$, Bunzel and Vogelsang (2005) only provide a limiting distribution for *Dan-J* when it is assumed that c is known in the calculation of $\hat{b}_{opt}$. That is, when $\hat{b}_{opt} = b_{opt}(\hat{c})$ is replaced by $b_{opt}(c)$. Although this strictly means that their asymptotic results are based on the limiting behavior of an infeasible test, for the purposes of making comparisons tractable, in what follows the limit distribution for *Dan-J* is that using b_{opt}(c).

CONCLUSIONS

The ability to detect the presence of a deterministic linear trend in an economic time series is an important issue in applied econometrics for a number of reasons. The effectiveness of both policy modeling and forecasting is, for example, reliant on correct identification of the trend function. Correctly specifying the trend function is also of crucial importance in the context of unit root and stationarity testing. It is, for example, well known that an un-modeled linear trend effects non-similar and inconsistent unit root tests, while unnecessarily including a trend vastly reduces power to reject the unit root null under $I(0)$ (weakly dependent) errors.

We presented procedure that falls into the class of robust tests for the trend function. The statistic is based on taking a simple data-dependent weighted average of two trend test statistics, both conventional *t-ratios*, one that is appropriate when the data are generated by an $I(0)$ process and a second that is appropriate when the data are $I(1)$. Determined from an auxiliary statistic which consistently estimates the true order of integration of the data, the weights are designed to switch weight between the two trend statistics, depending on whether the data are generated by an $I(0)$ or $I(1)$ process. We show that the new weighted statistic has a standard normal limiting null distribution in both the $I(0)$ and $I(1)$ cases.

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WYBRANE WŁASNOŚCI TESTÓW W ODPORNIEJ ANALIZIE TRENDU

Formalne testowanie zagadnienia trendu w szeregu czasowym jest uzależnione od faktu znajomości postaci szeregu, w szczególności stopnia zintegrowania ($I(0)$ lub $I(1)$) szeregu czasowego, czyli od słabej lub silnej autokorelacji. W artykule przedstawimy odporne testy (na rząd integracji danych w szeregu czasowym) zaproponowane w pracach Bunzel i Vogelsang (2005), Harvey i inni (2007) oraz Perron i Yabu (2009). Testy te są odporne w sensie asympotycznych własności wartości krytycznych w testowaniu wartości współczynnika kierunkowego funkcji trendu.