ACTA UNIVERSITATIS LODZIENSIS

FOLIA OECONOMICA 68, 1987

Czesława Jackiewicz*, Elżbieta Żółtowska*

THE DISCONTINUOUS PRODUCTION FUNCTION WITH CONSTANT ELASTICITY OF SUBSTITUTION**

1. Introduction

One of the basic but controversial assumptions in the theory of production and the econometric models of production is the assumption of continuity of an analytic form of the relation between production and inputs. The lack of continuity of the production function (in given input levels and in given moments of time) will be manifested by adding the zero-one variable (or variables) to the technical relation between production and inputs. This lack causes a need for considering a way of determining the estimates of its parameters on the basis of the available statistical information. Our analysis is limited to the case when the dependence between production and employment as well as fixed assets is a function of the CES type.

2. Discontuity of the Production Function

The classical theory of the production function assumes that the input-output relation is described by a function belonging to the class C^2. If we take Y for production and $X_1, X_2, \ldots, X_k$ for the production factors, then the gradient of the function

(1) $Y = f(X_1, \ldots, X_k)$,

---

* Lecturer, Institute of Econometrics and Statistics of the University of Łódź.

** The article is based in the research conducted within the problem R. III.9.3.2.

is a vector with positive components, and the hessian of this function is a positive definite matrix. This means that at a fixed level of arbitrary (k-1) production factors, Y treated as a function of a non-fixed (variable) factor is an increasing curve whose rate of increase is constantly diminishing. It is also assumed that if any of X_i ($i = 1, 2, \ldots, k$) is zero, then Y is also zero.

The continuity assumption causes a lot of doubts and discussions among the economists. These doubts come mainly from the fact that the production factors are not infinitely divisible, and the level of production determines not only the level of production factors, but also the efficiency of their utilization.

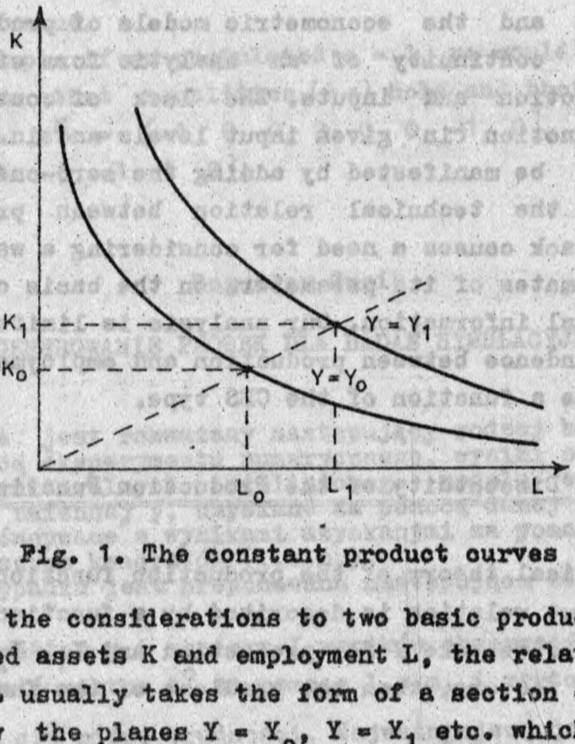

Fig. 1. The constant product curves

Limiting the considerations to two basic production-creating factors: fixed assets K and employment L, the relation between Y and K and L usually takes the form of a section of the surface $Y = f(K, L)$ by the planes $Y = Y_0$, $Y = Y_1$ etc. which are the constant product curves.

The points (K, L) lying on the same curve correspond to different production techniques, and the possage from the point (L_0, K_0) to (L_1, K_1) corresponds to the appearance of independent technological and organizational progress. This passage can be of almost continuous character, or it can be markedly discrete. In the latter case the function surface $Y = f(K, L)$ will cease to be a continuous surface, which is easy to observe on the $L = L_0$ or $K = K_0$ vertical planes sections of the production surface (Fig. 2). We can assume that the discrete change of production from Y_o to Y_1 occurs after passing certain threshold values K^* and L^*, and that the latter levels are the functions (not necessarily continuous functions, of K and L respectively. Let us assume that we make a vertical section of the surface $Y = f(K, L)$ with the plane $L = L_0$.

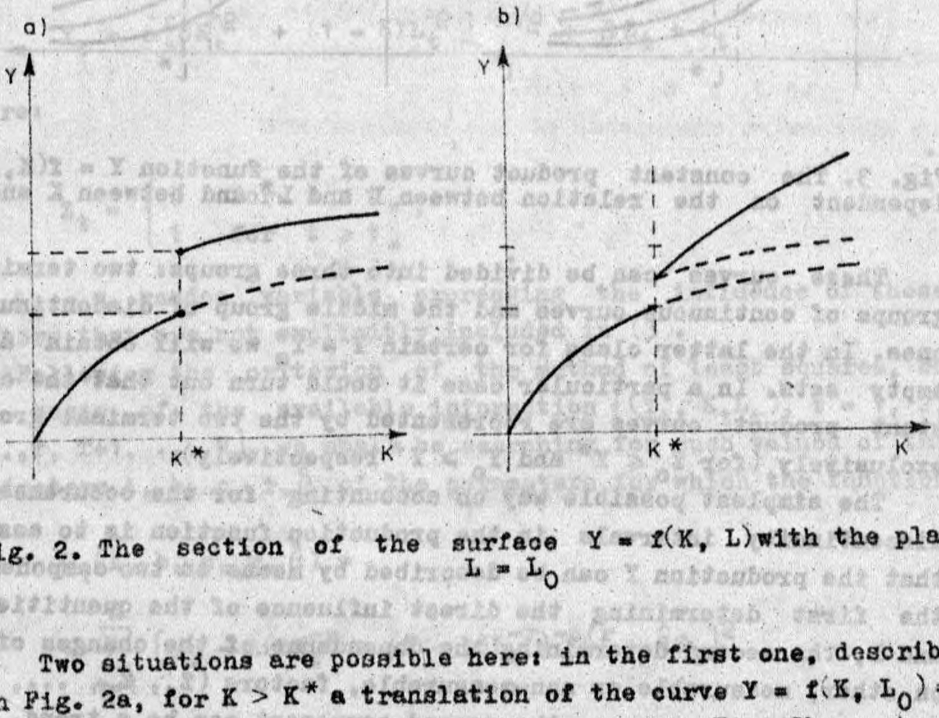

Fig. 2. The section of the surface $Y = f(K, L)$ with the plane $L = L_0$

Two situations are possible here: in the first one, described in Fig. 2a, for $K > K^*$ a translation of the curve $Y = f(K, L_0) = 0$ occurs, and in the second one, described in Fig. 2b, a change of the rate of increase of the curve takes place for $K > K^*$, apart from the translation. Now if we assume that we determine the sections of the surface $Y = f(K, L)$ with the horizontal planes $Y = Y_o$ in such a way that the succesive planes $Y = Y_1$, $Y = Y_2$, Y_3, ..., $Y = Y_p$ differ from one another by a constant distance $\Delta Y = Y_2 - Y_1 = Y_3 - Y_2, \ldots, = Y_p - Y_{p-1}$, we will obtain a system of constant product curves presented in Fig. 3.

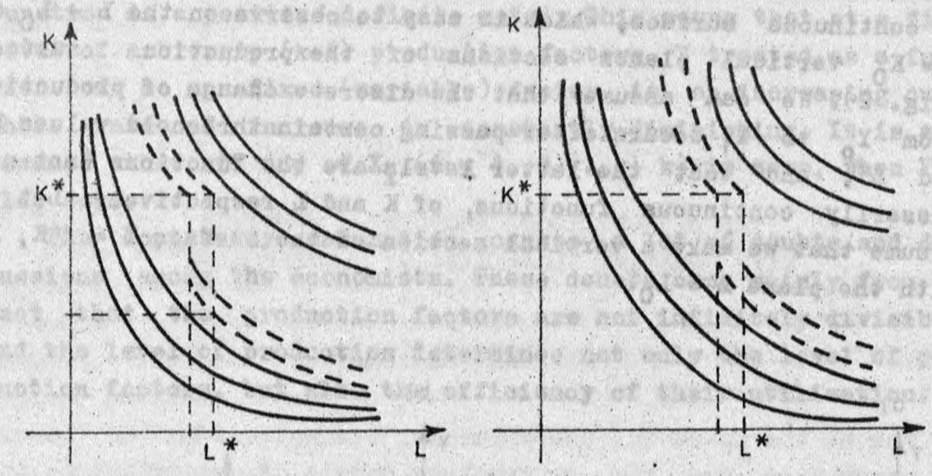

Fig. 3. The constant product curves of the function $Y = f(K, L)$ dependent on the relation between L and L^* and between K and K^*

These curves can be divided into three groups: two terminal groups of continuous curves and the middle group of discontinuous ones. In the latter class for certain $Y = Y_0$ we will obtain also empty sets. In a particular case it could turn out that the constant product curves are represented by the two terminal groups exclusively (for $Y_0 \leqslant Y^*$ and $Y_0 \geqslant Y^*$ respectively).

The simplest possible way on accounting for the occurence of discontinuity intervals in the production function is to assume that the production Y can be described by means on two components, the first determining the direct influence of the quantities K and L, the second determining the dependence of the changes of Y on other, measurable or non-measurable, factors $(Z_1, Z_2, \ldots, Z_s)$. In a particular case, the second component can be a trend. (In different, so far considered types of production function, the multplicative superposition of the trend on "pure" (dependent only on K and L) form of the function has been assumed). So we assume that

$$Y = f(K, L) + g(Z_1, \ldots, Z_s). \tag{2}$$

The present paper will consider a case where the variables $Z_1, \ldots, Z_s$ from (2) are artificial variables, more precisely zero-one variables, and the function G is a linear function.

3. The CES Production Function with a Zero-one Variable

We shall consider the estimation problem for the CES production function, assuming that in the observed time interval $\langle t_o, t_1 \rangle$ there is a translation of the function surface for $t \geq t_*$. Then we can assume that

$$(3) \quad Y_t = \alpha[\delta K_t^{-\varrho} + (1-\delta)L_t^{-\varrho}]^{-\nu/\varrho} + \beta Z_t + \varepsilon_t$$

where:

$$Z_t = \begin{cases} 0 & \text{for } t < t_*, \\ 1 & \text{for } t \geq t_* \end{cases}$$

ε_t - a random variable expressing the influence of these factors that are not explicitly included in (3).

Following the criterion of the method of least squares, on the basis of the available information $\{(Y_t, K_t L_t), t = 1, 2, 3 \ldots, T, T+1, \ldots, N\}$, we shall be searching for such values of the estimators A, b, c, r, $\hat{\beta}$ of the parameters for which the function

$$(4) \quad Q_1(A, b, c, r, \hat{\beta}) =$$
$$= \sum_{t=1}^{N} [Y_t - A(bK_t^{-r} + (1-b)L_t^{-r})^{-c/r} - \hat{\beta} Z_t]^2$$

will reach the minimum value. As far as the parameters are concerned, it is assumed that

$$A \in (0, +\infty),$$
$$b \in (0, 1),$$
$$c \in (0, +\infty),$$
$$r \in (-1, 0) \cup (0, +\infty),$$
$$(5) \quad \hat{\beta} \in (-\infty, +\infty).$$

Let us then assume that $T < t^*$ and that $T + 1 \geq t^*$, which means that for the first T observations the variable Z_t is equal to 0, and for the last N - T observations it is equal to 1.

Transforming (4) the function Q_1 can be written as

(6) $$\text{grad } Q_1(A, b, c, r, \hat{\beta}) = 0.$$

Thus the function $Q_1(A, b, c, r, \hat{\beta})$ is a non-linear function of five variables. We shall be searching for its stationary points where the hessian of the function Q_1 is a positive definite matrix.

The search for the stationary points $(A_o, b_o, c_o, r_o, \hat{\beta}_o)$ corresponds to the solution of the equation:

$$\text{grad } Q_1(A, b, c, r, \hat{\beta}) = 0.$$

The successive components of the gradient are:

(7) $$\frac{\partial Q1}{\partial A} = -2 \sum_{t=1}^{N} (Y_t - AG_t)G_t + 2\hat{\beta} \sum_{t=T+1}^{N} G_t$$

(8) $$\frac{\partial Q_1}{\partial b} = \frac{2AC}{r} \sum_{t=1}^{N} (Y_t - AG_t)G_t \frac{K_t^{-r} - L_t^{-r}}{g_t} 2\hat{\beta} \frac{AC}{r} \sum_{t=T+1}^{N} G_t \frac{K_t^{-r}-L_t^{-r}}{g_t}$$

(9) $$\frac{\partial Q_1}{\partial C} = \frac{2A}{r} \sum_{t=1}^{N} (Y_t - AG_t)G_t \ln g_t - \frac{2A\hat{\beta}}{r} \sum_{t=T+1}^{N} G_t \ln g_t,$$

$$\frac{\partial Q_1}{\partial r} = -\frac{2AC}{r^2} \sum_{t=1}^{N} (Y_t - AG_t)G_t \ln g_t + \frac{2AC}{r} \sum_{t=1}^{N} (Y_t-AG_t)G_t \frac{g_t^*}{g_t} +$$

(10) $$+ \frac{2AC\hat{\beta}}{r^2} \sum_{t=T+1}^{N} G_t \ln g_t - \frac{2AC\hat{\beta}}{r} \sum_{t=T+1}^{N} G_t \frac{g_t^*}{g_t},$$

(11) $$\frac{\partial Q_1}{\partial \hat{\beta}} = -2 \sum_{t=T+1}^{N} Y_t + 2A \sum_{t=T+1}^{N} G_t + 2\hat{\beta}(N - T),$$

where

$$g_t = bK_t^{-r} + (1 - b)L_t^{-r}$$

$$G_t = (g_t)^{-c/r}$$

$$(12) \quad g_t^* = -(b \ln K_t K_t^{-r} + (1 - b)\ln L_t L_t^{-r}).$$

Assuming that $\frac{\partial Q_1}{\partial A} = 0$ and $\frac{\partial Q_1}{\partial \beta} = 0$, and by virtue of (7) and (11), we shall obtain

$$(13) \quad A = \frac{\sum_{t=1}^{N} Y_t G_t - \frac{\sum_{t=T+1}^{N} Y_t \sum_{t=T+1}^{N} G_t}{N - T}}{\sum_{t=1}^{N} G_t^2 - \frac{\left(\sum_{t=T+1}^{N} G_t\right)^2}{N - T}}$$

$$(14) \quad \hat{\beta} = \frac{\sum_{t=T+1}^{N} Y_t}{N - T} - A \frac{\sum_{t=T+1}^{N} G_t}{N - T}$$

Whereas, if we assume that $\frac{\partial Q_1}{\partial b} = 0$, $\frac{\partial Q_1}{\partial c} = 0$, $\frac{\partial Q_1}{\partial r} = 0$ and due to (8), (9) and (10), we obtain a system of equations with the unknown b, c, r, of the form:

$$(15) \quad \begin{cases} \sum_{t=1}^{N} (Y_t - AG_t)G_t \frac{K_t^{-r} - L_t^{-r}}{g_t} - \hat{\beta} \sum_{t=T+1}^{N} G_t \frac{K_t^{-r} - L_t^{-r}}{g_t} = 0 \\ \sum_{t=1}^{N} (Y_t - AG_t)G_t \ln g_t - \hat{\beta} \sum_{t=T+1}^{N} G_t \ln g_t = 0 \\ \sum_{t=1}^{N} (Y_t - AG_t)G_t \frac{g_t^*}{g_t} - \hat{\beta} \sum_{t=T+1}^{N} G_t \frac{g_t^*}{g_t} = 0, \end{cases}$$

where A and $\hat{\beta}$ are determined by (13) and (14) and are also functions of the variables b, c, r.

The system (15) is a non-linear system of equations and its solution can be determined by approximate - iterative methods[1]. The determined quantities b_o, c_o, r_o, $A_o = A(b_o, c_o, r_o)$ and $\hat{\beta}_o = \beta(b_o, c_o, r_o)$ define only the stationary point of the function $Q(A, b, c, r, \hat{\beta})$. The sufficient condition of the existence of the minimum of the function Q_1 in the stationary point needs checking whether the hessian of Q_1 in this point is positively determined. The determining of the matrix of the second partial derivatives of the function Q_1 will be presented in another paper.

4. The CES Production Function with Quarterly Fluctuation Included

In the practice of estimation of econometric models we often face a situation where the sample size is small. In many cases the only possibility of increasing the sample size is using the data for shorter periods, e.g. quarterly. It quadruples the number of observations, but is usually causes the need to allow for the occurence of seasonal fluctuations in the explained variable. These fluctuations can be constant or varying. We shall at first consider the CES production function with the assumption that the variable Y is subject to constant quarterly changes. Then

$$Y_t = \alpha\left[\delta K_t^{-\varrho} + (1-\delta)L_t^{-\varrho}\right]^{-\nu/\varrho} + \alpha_1(Z_1 - Z_4) + \alpha_2(Z_2 - Z_4) + \alpha_3(Z_3 - Z_4) + \varepsilon_t \tag{16}$$

where

$$Z_i = \begin{cases} 1 & \text{for the i-th quarter (i.e. three month period),} \\ 0 & \text{for all other quarters} \end{cases}$$

---

[1] Similar to those used in the research whose results are presented in the work [3] and [4].

$\alpha_1, \alpha_2, \alpha_3$ - determine the seasonal effects for the first three quarters, whereas the effect for the fourth quarter is $\alpha_4 = -(\alpha_1 + \alpha_2 + \alpha_3)$.

Let us assume that we have a statistical sample $\{(Y_t, K_t, L_t)\ t = 1, \ldots, N\}$; consisting of N quarterly observations for $n = N/4$ years. Determining the estimators A, b, c, r, a_1, a_2, a_3 of the parameters $\alpha, \delta, \nu, \varrho, \alpha_1, \alpha_2, \alpha_3$ of the model (16) according to the idea of the method of least squares we are looking for the minimum of the function

$$Q_2(A, b, c, r, a_1, a_2, a_3) = \sum_{t=1}^{N} (Y_t - f(K_t L_t) -$$

$$- \alpha_1 U_1 - \alpha_2 U_2 - \alpha_3 U_3)^2, \tag{17}$$

where: $f(K_t, L_t) = A\left[bK_t^{-r} + (1 - b)L_t^{-r}\right]^{-c/r}$,

$$U_i = Z_i - Z_4 \quad \text{for } i=1, 2, 3$$

under the condition that $A \in (0, +\infty)$, $b \in (0, 1)$, $c \in (0,\infty)$, $r \in (-1, 0) \cup (0, +\infty)$, $\alpha_i \in (-\infty, +\infty)$ for i=1, 2, 3.

Noticing that

$$U_i = \begin{cases} 1 & \text{when } t \equiv i \pmod 4 \\ 0 & \text{when } t \not\equiv i \pmod 4 \text{ for } i=1, 2, 3 \\ -1 & \text{when } t \equiv 0 \pmod 4 \end{cases} \tag{18}$$

and transforming (17) we obtain

$$Q_2(A, b, c, r, a_1, a_2, a_3) = \sum_{t=1}^{N} (Y_t - f(K_t, L_t))^2 +$$

$$-2a_1 \sum_{t=1}^{N} (Y_t - f(K_t, L_t))U_1 - 2a_2 \sum_{t=1}^{N} (Y_t - f(K_t, L_t)U_2 +$$

$$+ 2a_3 \sum_{t=1}^{N} (Y_t - f(K_t, L_t))U_3 + a_1^2 \sum_{t=1}^{N} U_1^2 + a_2^2 \sum_{t=1}^{N} U_2^2 +$$

$$+ a_3^2 \sum_{t=1}^{N} U_3^2 + 2a_1a_2 \sum_{t=1}^{N} U_1U_2 + 2a_1a_3 \sum_{t=1}^{N} U_1U_3 +$$

$$(19) \quad + 2a_2a_3 \sum_{t=1}^{N} U_2U_3.$$

Since $\sum_{t=1}^{N} U_iU_j = \begin{cases} n/2 & \text{for } i = j (i=1, 2, 3) \\ n/4 & \text{for } i \neq j (i, j = 1, 2, 3) \end{cases}$

hence

$$Q_2(A, b, c, r, a_1, a_2, a_3) = \sum_{t=1}^{N} (Y_t - f(K_t, L_t))^2 +$$

$$- 2a_1 \sum_{t=1}^{N} (Y_t - f(K_t, L_t))U_1 - 2a_2 \sum_{t=1}^{N} (Y_t - f(K_t, L_t))U_2 +$$

$$+ 2a_3 \sum_{t=1}^{N} Y_t - f(K_t, L_t))U_3 + \frac{N}{2} \; a_1^2 + a_a^2 +$$

$$(20) \quad + a_3^3 + a_1a_2 + a_1a_3 + a_2a_3).$$

In order to determine the estimators of the parameters of the function (16) we are searching for the stationary points of the function $Q_2(A, b, c, r, a_1, a_2, a_3)$, i.e. the set $(A, b, c, r, a_1, a_2, a_3)$.

The components of the vector grad Q_2 are the following:

$$(21) \quad \frac{\partial Q_2}{\partial A} = - 2 \sum_{t=1}^{N} (Y_t - AG_t)G_t + 2 \sum_{t=1}^{N} G_t(a_1u_1 + a_2u_2 + a_3u_3)$$

$$\frac{\partial Q_2}{\partial b} = \frac{2AC}{r} \sum_{t=1}^{N} (Y_t - AG_t)G_t \frac{K_t^{-r} - L_t^{-r}}{g_t} + \sum$$

$$(22) \qquad -\frac{2AC}{r}\sum_{t=1}^{N} G_t \frac{K_t^{-r} L_t^{-r}}{g_t}(a_1u_1 + a_2u_2 + a_3u_3)$$

$$\frac{\partial Q_2}{\partial c} = \frac{2A}{r}\sum_{t=1}^{N}(Y_t - AG_t)G_t \ln g_t -$$

$$(23) \qquad -\frac{2A}{r}\sum_{t=1}^{N}(G_t \ln g_t \cdot (a_1u_1 + a_2u_2 + a_3u_3))$$

$$\frac{\partial Q_2}{\partial r} = -\frac{2AC}{r^2}\sum_{t=1}^{N}(Y_t - AG_t)G_t \ln g_t +$$

$$+\frac{2AC}{r}\sum_{t=1}^{N}(Y_t - AG_t)G_t\frac{g_t^*}{g_t} +$$

$$+\frac{2AC}{r^2}\sum_{t=1}^{N}G_t \ln g_t\,(a_1u_1 + a_2u_2 + a_3u_3) -$$

$$(24) \qquad -\frac{2\,AC}{r}\sum_{t=1}^{N}G_t\frac{g_t^*}{g_t}(a_1u_1 + a_2u_2 + a_3u_3)$$

$$(25) \qquad \frac{\partial Q_2}{\partial a_1} = -2\sum_{t=1}^{N}(Y_t - AG_t)u_1 + Na_1 + \frac{N}{2}a_2 + \frac{N}{2}a_3$$

$$(26) \qquad \frac{\partial Q_2}{\partial a_2} = -2\sum_{t=1}^{N}(Y_t - AG_t)u_2 + \frac{N}{2}a_1 + Na_2 + \frac{N}{2}a_3$$

$$(27) \qquad \frac{\partial Q_2}{\partial a_3} = -2\sum_{t=1}^{N}(Y_t - AG_t)u_3 + \frac{N}{2}a_1 + \frac{N}{2}a_2 + Na_3,$$

where

$$g_t = b\,K_t^{-r} + (1 - b)L_t^{-r},$$

$$G_t = g_t^{-c/r},$$

$$g_t^* = -(b \ln K_t K_t^{-r} + (1 - b) \ln L_t L_t^{-r}).$$

Solving the vectorial equation grad $Q_2 = 0$ we will obtain an equivalent system of normal equations for the function Q_2

$$(28) \quad \begin{cases} A \sum_{t=1}^{N} G_t^2 - \sum_{t=1}^{N} Y_t G_t + a_1 \sum_{t=1}^{N} G_t u_1 + a_2 \sum_{t=1}^{N} G_t u_2 + \\ \qquad + a_3 \sum_{t=1}^{N} G_t u_3 = 0, \\ \sum_{t=1}^{N} G_t \frac{K_t^{-r} - L_t^{-r}}{g_t} (Y_t - AG_t - a_1 u_1 - a_2 u_2 - a_3 u_3) = 0, \\ \sum_{t=1}^{N} G_t \ln g_t (Y_t - AG_t - a_1 u_1 - a_2 u_2 - a_3 u_3) = 0, \\ \sum_{t=1}^{N} G_t \frac{g_t}{g_t} (Y_t - AG_t - a_1 u_1 - a_2 u_2 - a_3 u_3) = 0, \\ A \sum_{t=1}^{N} G_t u_1 + \frac{N}{2} a_1 + \frac{N}{4} a_2 + \frac{N}{4} a_3 - \sum_{t=1}^{N} Y_t u_1 = 0, \\ A \sum_{t=1}^{N} G_t u_2 + \frac{N}{4} a_1 + \frac{N}{2} a_2 + \frac{N}{4} a_3 - \sum_{t=1}^{N} Y_t u_2 = 0, \\ A \sum_{t=1}^{N} G_t u_3 + \frac{N}{4} a_1 + \frac{N}{4} a_2 + \frac{N}{2} a_3 - \sum_{t=1}^{N} Y_t u_3 = 0, \end{cases}$$

Taking into account the first, fifth, sixth, and seventh equation of this system we obtain the following system of equations, equivalent to (28)

$$
(29)\quad \begin{cases}
A = \dfrac{N \sum_{t=1}^{N} G_t Y_t - wp + 4u}{N \sum_{t=1}^{N} G_t^2 - p^2 + 4q} \\[2ex]
a_1 = \frac{1}{N}\left(4 \sum_{t=1}^{N} Y_t u_1 - W\right) - A\frac{1}{N}\left(4 \sum_{t=1}^{N} G_t u_1 - u\right) \\[2ex]
a_2 = \frac{1}{N}\left(4 \sum_{t=1}^{N} Y_t u_2 - W\right) - A\frac{1}{N}\left(4 \sum_{t=1}^{N} G_t u_2 - u\right) \\[2ex]
a_3 = \frac{1}{N}\left(4 \sum_{t=1}^{N} Y_t u_3 - W\right) - A\frac{1}{N}\left(4 \sum_{t=1}^{N} G_t u_3 - u\right) \\[2ex]
\sum_{t=1}^{N} G_t \dfrac{K_t^{-r} - L_t^{-r}}{g_t} (Y_t - AG_t - a_1u_1 - a_2u_2 - a_3u_3) = 0 \\[2ex]
\sum_{t=1}^{N} G_t \ln g_t (Y_t - AG_t - a_1u_1 - a_2u_2 - a_3u_3) = 0 \\[2ex]
\sum_{t=1}^{N} G_t \dfrac{g_t^*}{g_t} (Y_t - AG_t - a_1u_1 - a_2u_2 - a_3u_3) = 0
\end{cases}
$$

where

$$p = \sum_{t=1}^{N} G_t(u_1 + u_2 + u_3),$$

$$q = \left(\sum_{t=1}^{N} G_t u_1\right)^2 + \left(\sum_{t=1}^{N} G_t u_2\right)^2 + \left(\sum_{t=1}^{N} G_t u_3\right)^2.$$

$$W = \sum_{t=1}^{N} Y_t(u_1 + u_2 + u_3),$$

$$u = \sum_{t=1}^{N} Y_t u_1 \sum_{t=1}^{N} G_t u_1 + \sum_{t=1}^{N} Y_t u_2 \sum_{t=1}^{N} G_t u_2 +$$

$$+ \sum_{t=1}^{N} Y_t u_3 \sum_{t=1}^{N} G_t u_3.$$

In order to determine the stationary points of the function $Q_2(A, b, c, r, a_1, a_2, a_3)$ we have to find out the coordinates b^o, c^o, r^o of the stationary point. Then the rest of the coordinates, i.e. A^o, a_1^o, a_2^o, a_3^o are immediately determined by the first four equations of (29). The estimator a_4 of the seasonal effect for the fourth quarter can be calculated from the condition

$$a_4 = -(a_1 + a_2 + a_3)$$

which gives

$$a_4 = \frac{1}{N}(Au - w). \qquad (30)$$

The problem to be solved is determining the b^o, c^o, r^o that will satisfy the last three conditions of the system (29) and checking the sufficient condition of the existence of the minimum of the function Q_2 in the determined stationary point.

While determining the estimates of the parameters δ, ν, ϱ of the model (16) we have to use iterative methods of solving the system of non-linear equations in which there are realizations of the random variable Y. Hence, the obtained estimates of the parameters δ, ν, ϱ as well as of the parameters α, α_1, α_2, α_3, α_4 (the functions of the estimates b, c, r, and the realization of the variable Y) will depend both on the numerical properties of the computing process and on the distribution of the variable Y.

From the point of view of application of the model in question to the description of the production process it is important to know at least the simplest properties of the estimators of its parameters. The basis for getting this information will be the results of suitable experiments of Monte Carlo type. Further, investigations on this problem will concern the numerical and statistical properties of the estimators of the parameters of this model.

Bibliography

[1] B a c k m a n F. S. (1960): The Solution of Linear Equations by the Conjugate Gradient Method, [in:] A. R a l s t o n, H. W i l f (ed.),Mathematical Methods for Digital Computers, New York.

[2] B i r k h o f f G., M a c L a n e S. (1963):Przegląd algebry współczesnej, Warszawa.

[3] F l e t c h e r R. (1965): Function Minimization without Evaluating Derivatives: a Review, "The Computer Journal", 8(1), p. 33-41.

[4] J a c k i e w i c z Cz., Ż ó ł t o w s k a E.,K l e p a c z H. (1978): Estymacja funkcji produkcji typu CES przy wykorzystaniu informacji a priori uzyskanych z różnych metod estymacji. Metoda podwójnej iteracji, research work within the contract R. III.9.5.4.

[5] J a c k i e w i c z Cz., Ż ó ł t o w s k a E. (1982): Wyznaczanie estymatorów parametrów nieciągłych funkcji produkcji o stałej elastyczności substytucji. Cz. I, research work within the contract R. III.9.3.2.

[6] J u s z c z a k G., Ż ó ł t o w s k a E. (1982): Analiza wpływu jednostek pomiaru zmiennych na oszacowania parametrów funkcji CES, "Wiadomości Statystyczne", 3, p. 27-29.

[7] K l e p a c z H. (1982): Wpływ błędów w zmiennej objaśniającej na wyniki oszacowań parametrów funkcji CES. Cz. II, research work within the contract R. III.9.3.2.

[8] Z a n g w i l l W. I. (1974): Programowanie nieliniowe, Warszawa.

Czesława Jackiewicz, Elżbieta Żółtowska

NIECIĄGŁA FUNKCJĄ PRODUKCJI O STAŁEJ ELASTYCZNOŚCI SUBSTYTUCJI

Artykuł nawiązuje do kontrowersyjnego, w teorii funkcji produkcji i ekonometrycznych modeli produkcji, założenia o ciągłości analitycznej postaci relacji między produkcją a nakładami. Pokazano, że w pewnych sytuacjach utrata ciągłości relacji nakłady - produkcja może być opisana poprzez uwzględnienie zmiennych sztucznych, przy specyfikacji analitycznej postaci funkcji produkcji. Przyjmując zmodyfikowaną funkcję produkcji typu CES wyprowadzono wzory na estymatory parametrów występujących przy zmiennych sztucznych oraz podano równania (nieliniowe), których iteracyjne rozwiązanie pozwala wyznaczyć oceny pozostałych parametrów strukturalnych.