A C T A   U N I V E R S I T A T I S   L O D Z I E N S I S
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VARIABILITY OF PARAMETERS IN LINEAR ECONOMETRIC MODEL AS AN EFFECT OF FILTERING OF ENDOGENOUS AND EXOGENOUS PROCESSES

One of the important problems in econometric model building for time series is taking a decision on using of the transformed or nontransformed data. It's one of the model specification problems. Zieliński[1] describes it from two points of view. The first is a question of effects of filtering of endogenous and exogenous processes, the second is a problem of choice of processes and filters for the complete description of variąbility of endogenous process. These problems are connected with the thesis that endogenous process Y_t depends on exogenous process X_t by means of a relationship between components of processes Y_t and X_t in the same section of frequencies.

Let processes Y_t and X_t be sums of components on a set of sections of frequencies P_i $(i = 1, 2, ..., k)$, it means

$$Y_t = \sum_{i=1}^{k} Y_{it} \tag{1}$$

and

$$X_t = \sum_{i=1}^{k} X_{it} \tag{2}$$

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[1] See: L. T a l a g a, Z. Z i e l i ń s k i, *Analiza spektralna w modelowaniu ekonometrycznym (Spectral Analysis in Econometric Modelling)*, PWN Warszawa 1986, p. 224.

where $(0, \pi) = \bigcup_{i=1}^{k} P_i$, and $P_i \cap P_j = \emptyset$ for all $i \neq j$. Analysing a relationship between Y_t and X_t, in the form

$$Y_t = \alpha \times X_t + \eta_t. \tag{3}$$

and a relationships between processes Y_{it} and X_{it} in the form

$$Y_{it} = \alpha_i \times X_{it} \qquad (i = 1, 2, \ldots, k), \tag{4}$$

the following question appears: what is the relationship between the parameter α and parameters α_i. Variance analysis of this processes leads to the formula:

$$\alpha^2 = \sum_{i=1}^{k} \alpha_i^2 \times \frac{\sigma_{Xi}^2}{\sigma_X^2} \tag{5}$$

where σ_X^2 is the variance of process X_t and σ_{Xi}^2's are the variances of part of process X_t on section P_i.

Let us write the processes X_t and Y_t in the spectral form

$$X_t = \int_{-\pi}^{\pi} e^{i\omega t} dZ_X(\omega) \tag{7}$$

and

$$Y_t = \int_{-\pi}^{\pi} e^{i\omega t} dZ_Y(\omega), \tag{8}$$

and let

$$dZ_Y(\omega) = \alpha(\omega) \times dZ_X(\omega).$$

Then we obtain the following relationship between α and $\alpha(\omega)$:

$$\alpha^2 = \int_{-\pi}^{\pi} \alpha^2(\omega) \times \frac{f_X(\omega)}{K_X(0)}\, d(\omega) \tag{9}$$

where $f_X(\omega)$ is a spectral density of process X_t, and $K_X(0)$ is the variance of process X_t.

The function $\alpha(\omega)$ is the transfer function of an unknown filter. When $\alpha(\omega)$ is a constant function, then $\alpha(\omega)$ equal to the parameter α.

The knowledge of parameters α_i in formula (4) is very important. In relationship between processes Y_t and X_t some of frequencies plays more significant role then others. Several frequencies correspond to a long-term, middle-term or short-term changes respectively. Applying ideal filter L (see Figure 12), we can study relationship between LY_t and LX_t in section of frequencies P_i.

Consider, for example, relationship between the production of yarn (Y_t) and the employment (X_t) in the spinning factory "Merinotex" in Toruń. The production was measured by kg´s of yarn and the employment by man-hours. These processes had been observed from January 1975 to June 1987 monthly. Since we had long time series, it was possible to apply linear filters and estimation of spectral density function Figure 1 shows the observed production and Figure 2 shows the observed employment. Variance of the process X_t is: $V(X) = 5873.6$. The basic model is as follows: $Y_t = b + a \times X_t$.

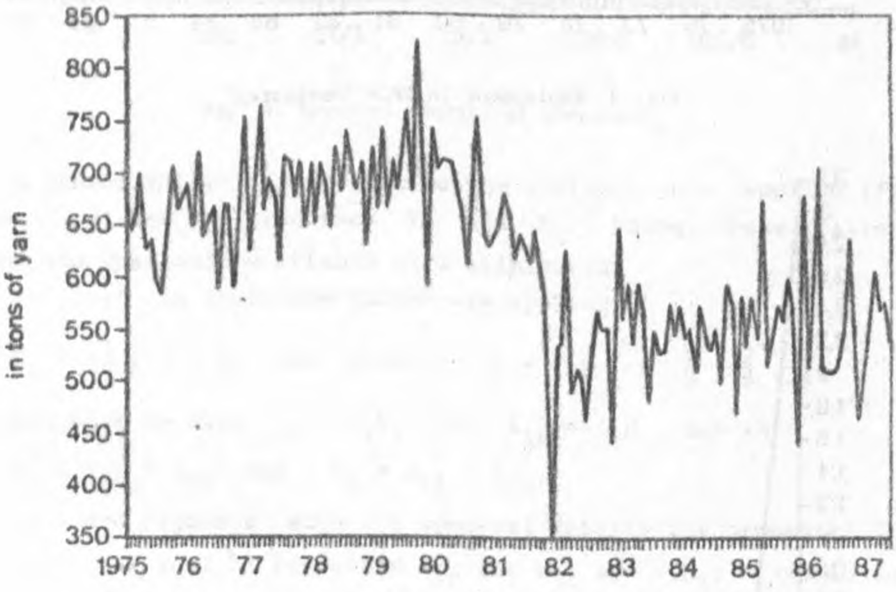

Fig. 1. Production of yarn in TPCz "Merinotex"

The following equation gives an estimated form of the model:

$$Y_t = \underset{(29.08)}{38.151} + \underset{(0.04665)}{0.94003} \times X_t, \text{ where}$$

$R^2 = 0.75463$ D-W = 1.9704.

Fig. 2. Employment in TPCz "Merinotex"

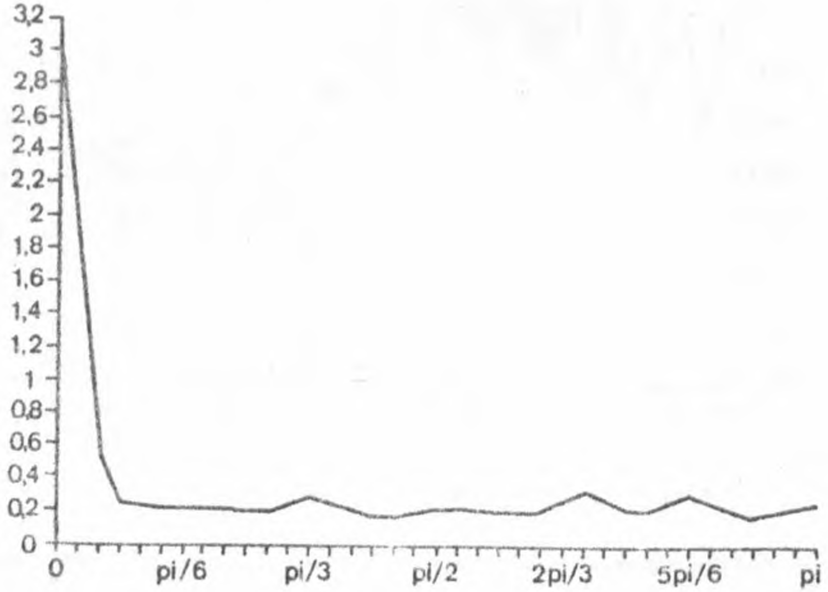

Fig. 3. Spectral density of process Y_t

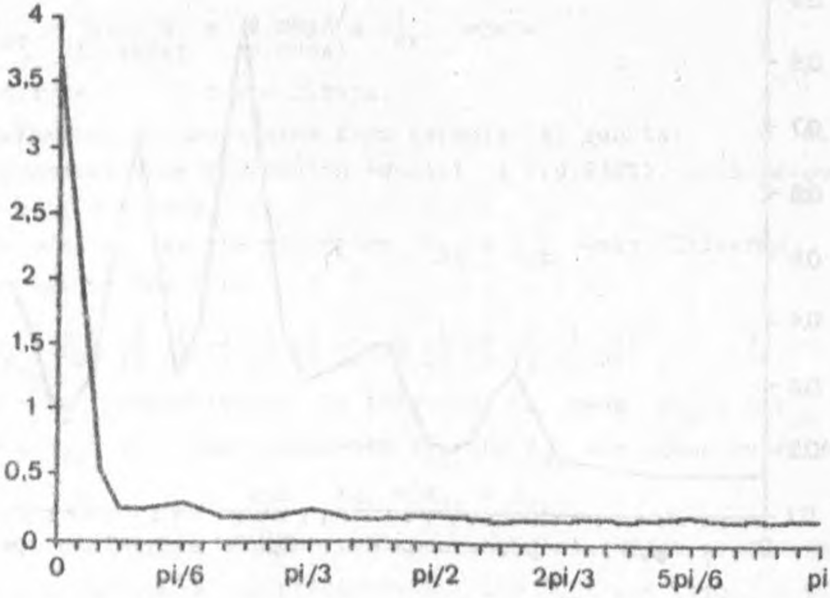

Fig. 4. Spectral density of process X_t

A linear filter (symmetric moving average) was applied (Figures 3, 4) to the processes Y_t and X_t. Using these filters trend and seasonality effects were eliminated.

As first the following filter was applied

$$L_1 = [\tfrac{1}{9}\ \tfrac{2}{9}\ \tfrac{3}{9}\ \tfrac{2}{9}\ \tfrac{1}{9}] \quad \text{and then } 1 - L_1 = [-\tfrac{1}{9}\ \ -\tfrac{2}{9}\ \ \tfrac{6}{9}\ \ -\tfrac{2}{9}\ \ -\tfrac{1}{9}]$$

In this case we have $Y_{1t} = L_1 Y_t$ and $X_{1t} = L_1 X_t$, and then

$$Y_t = Y_{1t} + Y_{2t} \quad \text{and} \quad X_t = X_{1t} + X_{2t}.$$

Figure 5 and Figure 6 show the spectral density for processes Y_{2t} and X_{2t}. Variance of processes X_{1t} and X_{2t} are $V(X_1) = 4647.6$ and $V(X_2) = 866.1$.

The following models were constructed for the filtering data:

$$Y_{1t} = b_1 + a_1 \times X_{1t} \quad \text{and} \quad Y_{2t} = b_2 + a_2 \times X_{2t}$$

Their estimated form is:

$$Y_{1t} = \underset{(16.9188)}{21.9918} + \underset{(0.0272)}{0.9658} \times X_{1t}, \text{ where}$$

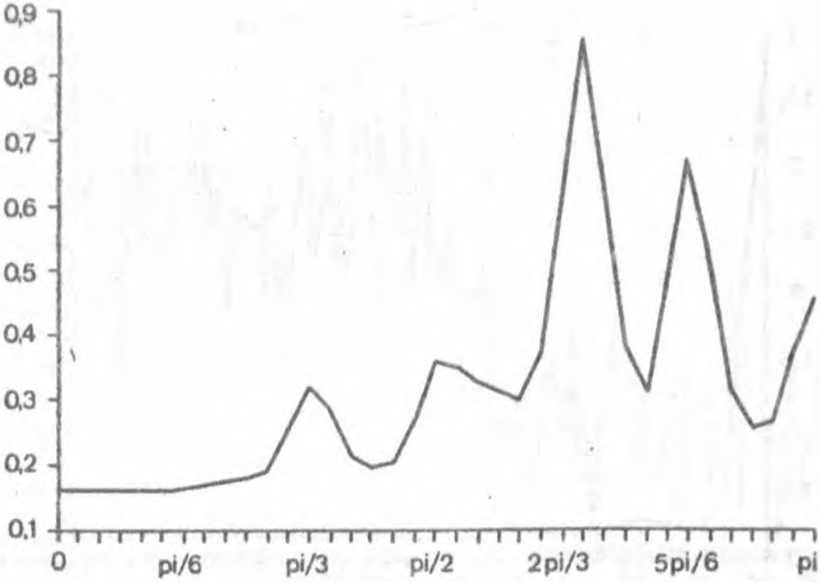

Fig. 5. Spectral density of process Y_{2t}

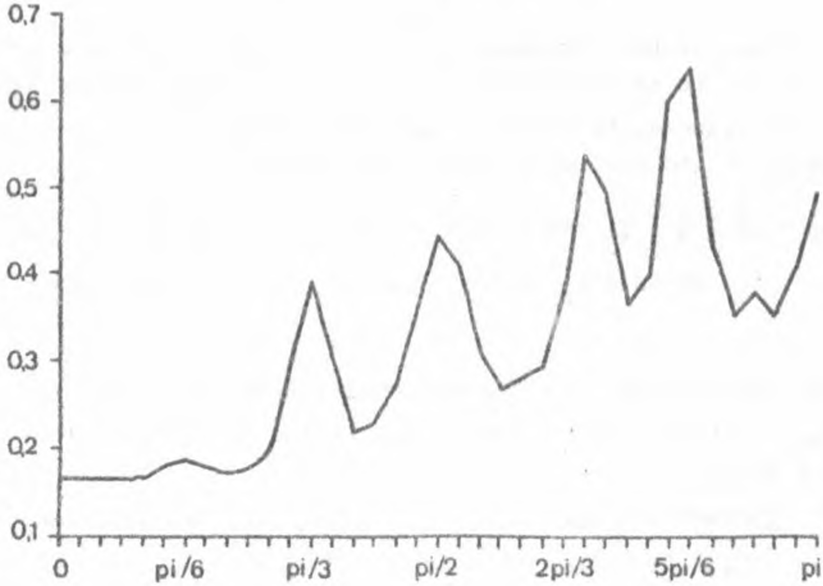

Fig. 6. Spectral density of process X_{2t}

$R^2 = 0.9052$ D-W = 0.2495;

$$Y_{2t} = \underset{(2.6608)}{0.2326} + \underset{(0.0904)}{0.8984} \times X_{2t}, \text{ where}$$

$R^2 = 0.4279$ D-W = 2.9536.

The parameter α calculated from formula (5) equals: $\alpha = 0.926$. The parameter from regression equals: a = 0.94003, with standard error s(a) = 0.0466.

In similar way the processes Y_{2t} i X_{2t} were filtered. New filter was of the form:

$$L_2 = [\frac{1}{24}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{24}]$$

and $1 - L_2$ respectively. In this case we have $Y_{3t} = L_2Y_{2t}$ and $X_{3t} = L_2X_{2t}$, and than processes Y_{2t} and X_{2t} are given by formulas

$$Y_{2t} = Y_{3t} + Y_{4t} \quad \text{and} \quad X_{2t} = X_{3t} + X_{4t}.$$

Figure 7 and Figure 8 show the spectral density for processes Y_{3t} and X_{3t}. Variances of processes X_{3t} and X_{4t} are $V(X_3) = 0.9843$ and $V(X_4) = 863.21$. The estimated models for processes Y_{3t} and Y_{4t} are:

$$Y_{3t} = \underset{(0.1149)}{0.0134} + \underset{(0.1159)}{0.66127} \times X_{3t}, \text{ where}$$

$R^2 = 0.1978$ D-W = 1.6975;

$$Y_{4t} = \underset{(2.6677)}{0.2223} + \underset{(0.0908)}{0.89616} \times X_{4t}, \text{ where}$$

$R^2 = 0.4246$ D-W = 2.9518.

The parameter α calculated from formula (5) is: $\alpha_2 = 0.895$. The parameter from regression is: $a_2 = 0.8984$ with standard error $s(a_2) = 0.0904$.

Let´s say, that filters which were applied are not ideal (see Figure 9 and Figure 10).

Recursive filters are applied in the further part of this study. They have better characteristics then filters used above. We use the recursive filter of the first order:

$$Y_{Lt} = (1 - \beta) \times Y_t + \beta \times Y_{Lt-1}$$

and

$$X_{Lt} = (1 - \beta) \times X_t + \beta \times X_{Lt-1},$$

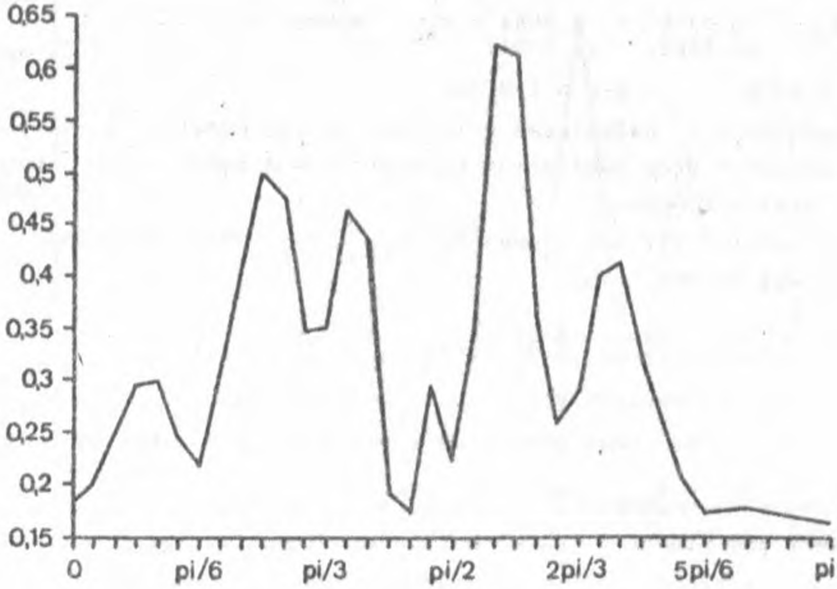

Fig. 7. Spectral density of process Y_{3t}

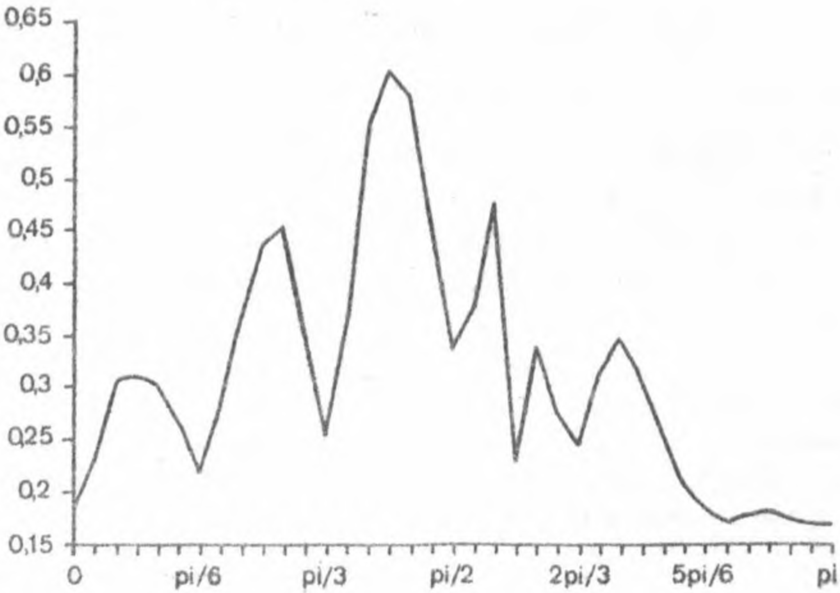

Fig. 8. Spectral density of process X_{3t}

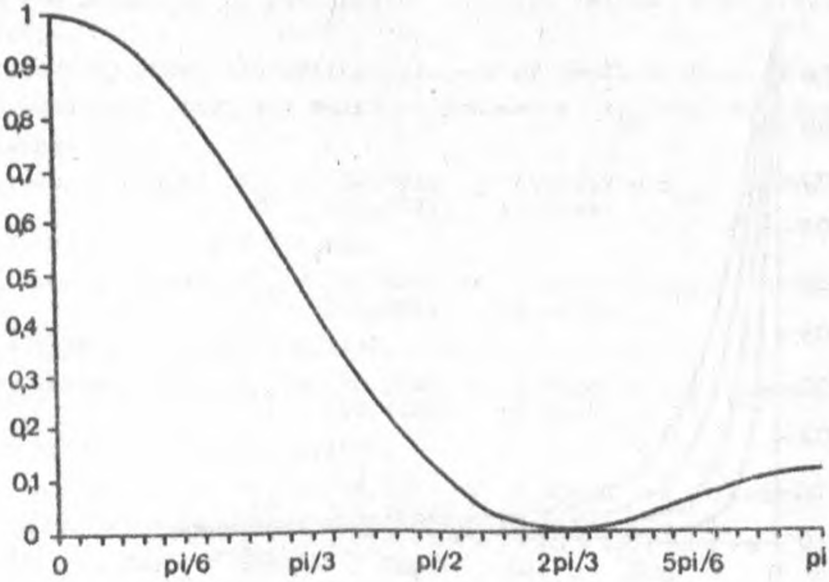

Fig. 9. Transfer function of filter L_1

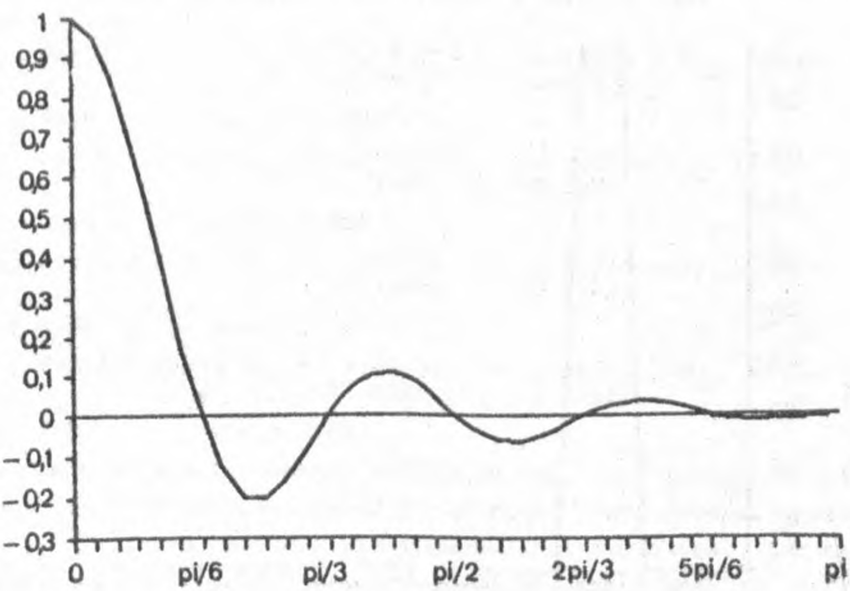

Fig. 10. Transfer function of filter L_2

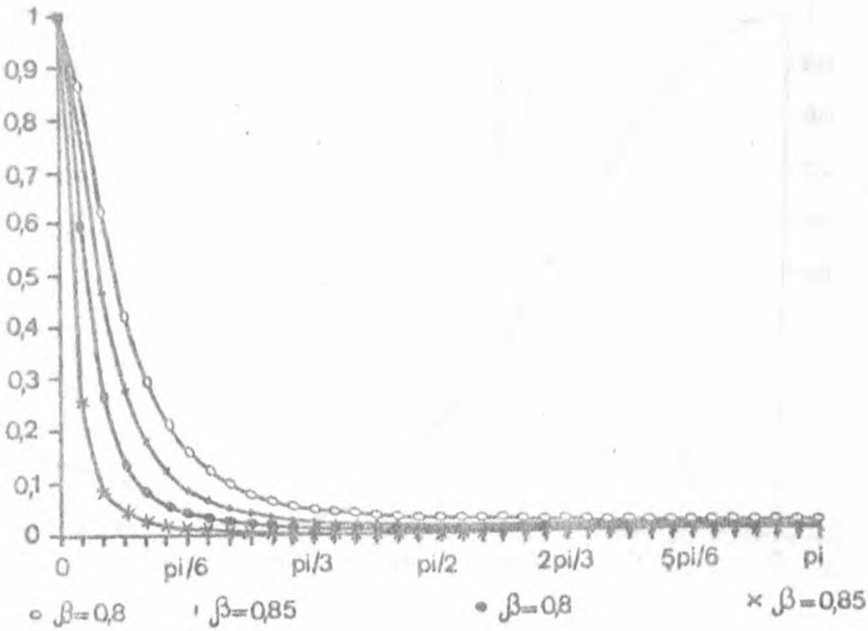

Fig. 11. Transfer functions of recursive filters

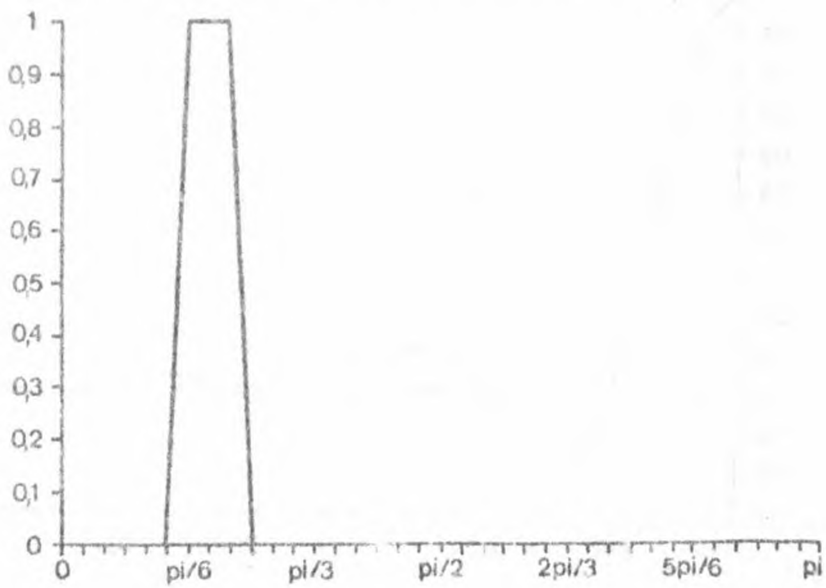

Fig. 12. Transfer function for ideal filter

where the parameter β was applied for four values: 0.8, 0.85, 0.9 and 0.95.

Figure 11 shows the characteristics of these filters. The estimated equations for all pairs of processes Y_{Lt} and X_{Lt} are the following:

1. For $\beta = 0.8$: $Y_{Lt} = \underset{(12.4727)}{28.7942} + \underset{(0.0199)}{0.95259} \times X_{Lt}$, where

$R^2 = 0.9394$ D-W = 0.262.

2. For $\beta = 0.85$: $Y_{Lt} = \underset{(11.5011)}{30.4933} + \underset{(0.0183)}{0.94917} \times X_{Lt}$, where

$R^2 = 0.9479$ D-W = 0.1737.

3. For $\beta = 0.9$: $Y_{Lt} = \underset{(10.4465)}{36.6993} + \underset{(0.0165)}{0.93801} \times X_{Lt}$, where

$R^2 = 0.9562$ D-W = 0.0965,

4. For $\beta = 0.95$: $Y_{Lt} = \underset{(8.9624)}{58.4378} + \underset{(0.0139)}{0.90096} \times X_{Lt}$, where

$R^2 = 0.966$ D-W = 0.0389.

These equations can be treated as a description of relation between trend effects on Y_t and X_t processes. Relationships between residual processes (after elimination of trend effects) are the following:

1. For $\beta = 0.8$: $Y_{Lt} = \underset{(2.7812)}{0.346019} + \underset{(0.0791)}{0.84045} \times X_{Lt}$, where

$R^2 = 0.4408$ D-W = 2.269.

2. For $\beta = 0.85$: $Y_{Lt} = \underset{(2.9416)}{0.581237} + \underset{(0.0775)}{0.8484} \times X_{Lt}$, where

$R^2 = 0.4558$ D-W = 2.207.

3. For $\beta = 0.9$: $Y_{Lt} = \underset{(3.1523)}{1.181461} + \underset{(0.0749)}{0.86943} \times X_{Lt}$, where

$R^2 = 0.4846$ D-W = 2.127.

4. For $\beta = 0.95$: $Y_{Lt} = \underset{(3.6298)}{3.491321} + \underset{(0.0711)}{0.91873} \times X_{Lt}$, where

$R^2 = 0.5382$ D-W = 2.013.

Looking at the parameters estimates of the above equations one has to observe a variability of them. For several sections of frequencies we have different parameters estimates. In order to determine variability of parameters estimates precise filters are needed. For this purpose the high order digital recursive filters are particularly appriopriate.

Józef Stawicki

ZMIENNOŚĆ PARAMETRÓW W LINIOWYM MODELU EKONOMETRYCZNYM JAKO EFEKT FILTRACJI PROCESÓW ENDOGENICZNEGO I EGZOGENICZNEGO

W referacie przedstawiono problem zmienności parametrów modelu ekonometrycznego postaci:

$$Y_t = \sum_{i=1}^{N} \alpha_i \cdot X_{it} + \varepsilon_t \tag{1}$$

Niech procesy Y_t i X_{it} będą dane w postaci spektralnej

$$Y_t = \int_{-\pi}^{\pi} e^{i\omega t} \, dZ_y \omega \tag{2}$$

$$X_{it} = \int_{-\pi}^{\pi} e^{i\omega t} \, dZ_{X_i}(\omega) \tag{3}$$

Niech Y_t^L i X_{it}^L będą przefiltrowanymi procesami za pomocą filtru liniowego postaci:

$$L(u) = \sum_{s=-q}^{p} l_s \, u^s \tag{4}$$

Ekonometryczny model w tym przypadku ma postać następującą:

$$Y_t^L = \sum_{i=1}^{N} \alpha_i^L \cdot X_{it}^L + \varepsilon_t^L \tag{5}$$

Niech

$$Y_t = \sum_{j=1}^{K} Y_i^j \tag{6}$$

$$X_{it} = \sum_{j=1}^{K} X_{it}^j \tag{7}$$

Parametry z modelu (1) dają się wyrazić poprzez parametry modeli (5) za pomocą formuły:

$$\alpha^2 = \Sigma (\alpha_i^j)^2 \frac{\sigma^2_{X_i^j}}{\sigma^2_{X_i}} \tag{8}$$

gdzie $\sigma^2_{X_i}$ i $\sigma^2_{X_i^j}$ są wariancjami procesu X_i oraz składowych tego procesu X_i^j.

W referacie przedstawiono empiryczny przykład modelu produkcji w zależności od zatrudnienia w TPCz "Merinotex" w Toruniu dla danych miesięcznych.