A C T A   U N I V E R S I T A T I S   L O D Z I E N S I S
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STUDYING DYNAMICS OF QUALITATIVE PHENOMENA BY THE USE OF INTERDEPENDENCE MEASURES

In social sciences one can distinguish a group of statistical investigations which are repeated "n" times periodically (e.g. quarterly or monthly). An example of this type of statistical-investigation can be a poll where the same, or slightly modified question, in asked several times. There are only two possible variante of answering this question i.e. "yes" or "no". Hence, the distribution of answers constitutes an n-variate 2 x 2 contingency table. Marginal distributions are successive n - 1, n - 2, ..., 2-variate contingency tables with 2^{n-1}, 2^{n-2}, ... cells respectively.

Statistical analysis of results contained in such an n-variate contingency table can be carried out in many ways. In this paper one of the possible approaches is proposed, i.e. an analysis of such marginal distributions which are 2 x 2 contingency tables. In n-variate contingency table there are $\binom{n}{2} = \frac{n(n-1)}{2}$ of such bivariate marginal distributions. Let us choose among them such an (n - 1)-element sequence of tables: $T_{1,2}$, $T_{2,3}$, ..., $T_{n-1,n}$ that the table $T_{1,2}$ contains the results of period 1 and 2 and $T_{2,3}$ - the results of period 2 and 3 and so on. The subsequent stage consists in computing the appropriate measures and indicators which are characteristic of the problem under investigation. In this case researchers use mainly indices and structu-

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re indicators for the analysis of dynamics of qualitative phenomena. They are both simple and convenient as far, as their interpretation is concerned. This paper presents some possible ways of the use of interdependence measures. However, these measures should have certain desired properties. The properties, for the general case of bivariate contingency table with "r" rows and "s" columns are presented below (S t ę p i e ń, 1983).

Let M denote the interior of the table i.e. r x s matrix

$$M = \begin{bmatrix} n_{11}, & n_{12} & \cdots & n_{1s} \\ n_{21}, & n_{22} & \cdots & n_{2s} \\ \cdots & \cdots & \cdots & \cdots \\ n_{r1}, & n_{r2} & \cdots & n_{rs} \end{bmatrix}$$

These measures properties will be formulated as follows:

1. A measure of dependence between two quality factors is the function of contingency table elements:

$w = w(M)$.

2. A measure has a full domain property when it is possible to compute it for each contingency table r x s ($r,s \geqslant 2$).

3. Invariance with respect to proportional transformations:

$w(kM) = w(M)$

for each real number k.

4. Invariance with respect to permutations of the table rows (columns):

$w(U_r M) = w(M)$, $w(MU_s) = w(M)$

for each U_r r x r (U_s s x s) permutation matrix.

5. Symmetry:

$w(M^T) = w(M)$.

6. Normalization:

$\min_M w(M) = 0$ and $\max_M w(M) = 1$

where w reaches 0 with independence of features and 1 with full dependence or

$\min_M w(M) = -1$ and $\max_M w(M) = 1$

where $w = \pm 1$ means full dependence and $w = 0$ - independence.

7. Invariance with respect to proportional transformations of rows (columns):

$w(W_r M) = w(M) \qquad w(MW_s) = w(M)$

for each diagonal nonsingular r x r (s x s) matrix.

Properties of certain measures, most widely known in the literature (B l a l o c, 1975, C r a m e r, 1958, K e n d a l l, S t u a r t, 1963, K u et al. 1971, S z u l c, 1963, Y u l e K e n d a l 1966) are presented in Table 1.

In order to analyse the degree of changes in opinion, such a measure should be chosen which satisfies all the properties except invariance with respect to permutations of rows (columns) of the table.

Among the normalized measures (property 6) we can use those of which the range of variability is the interval <-1, 1>. However, a simple transformation of this range into <0, 1> according to the formula:

$$w^* = \frac{w + 1}{2}$$

can be made measure w^* obtained in this way reaches 1 for full positive dependence (all persons asked answered twice "yes" or twice "no"), 0 when we deal with the full negative dependence (1^{st} question - "yes", 2^{nd} question - "no" or vice versa). The closer to 1 is the value of the calculated w^* coefficient the greater is the conformity of opinions in the compared periods. When the value of measure is close to zero, this means that conformity of opinions is very slight.

The average rate of changes in opinion, at a certain period of time can be calculated (analogously as for chain indices) by means of geometrical mean of interdependence coefficients:

$$\bar{w}^* = \sqrt[n-1]{w_{k_{1,2}} \, w_{k_{1,3}} \cdots w_{k_{n-1,n}}}$$

In order to illustrate applications of interdependence measures we shall use the following example. The purpose of investigation was to determine the degree of change in opinions expressed on a certain subject for a period of 1 year. Our investigation concerned a groups of 100 persons. The question asked at the end of each quarter of the year was answered either "yes" or "no". The results of this investigation are contained in Table 2 (for the sake of simplicity we used "+" for "yes" and "-" for "no").

Table 1

Properties of interdependence measures

Measure	Domain restraints	Invariance with respetto proportional transformations	Invariance with respetto permutations of		Symmetry	Invariance with respetto proportional transformations of		Range	
			rows	columns		rows	columns	from	to
1	2	3	4	5	6	7	8	9	10
$D = \frac{ed - bc}{N}$	x	-	-	-	+	-	-	0	x
$Q = \frac{ad - bc}{ad + bc}$	x	+	-	-	+	+	+	-1	1
$Y = \frac{\sqrt{ad} - \sqrt{bc}}{\sqrt{ad} + \sqrt{bc}}$	x	+	-	-	+	+	+	-1	1
$\phi = \frac{\lvert ad - bc \rvert}{(a+b)(a+c)(b+d)(c+d)}$	x	+	+	+	+	-	-	0	1
$W = \frac{(a+d) - (b+c)}{(a+d) + (b+c)}$	x	+	-	-	+	-	-	-1	1
$R = \frac{ad}{bc}$	bc > 0	+	-	-	+	+	+	0	$+\infty$
$L_R = \ln \frac{ad}{bc}$	bc > 0	+	-	-	+	+	+	$-\infty$	$+\infty$

Table 1 (contd)

1	2	3	4	5	6	7	8	9	10
$D^* = \frac{a}{a+c} - \frac{b}{b+d}$	$a + c > 0$ $b + d > 0$	+	-	-	-	-	+	-1	1
$R_c = \frac{a\,(b+d)}{b\,(a+c)}$	$b > 0$ $a + c > 0$	+	-	-	-	+	+	0	$+\infty$
$r = \cos \frac{\sqrt{bc}}{\sqrt{ad} + \sqrt{bc}}\,\pi$	x	+	-	-	+	+	+	-1	1

S o u r c e: The author´s calculations.

Table 2

Changes in opinion expressing on a certain subject in a period of four quarters

III quarter / IV quarter			+		-	
			I quarter		II quarter	
			+	-	+	-
+	I quarter	+	19	10	1	3
		-	-	7	2	7
-	II quarter	+	9	2	5	8
		-	1	1	10	15

Source: The author's calculations.

Tables 3, 4 and 5 contain information concerning I^{st} and II^{nd}, II^{nd} and III^{rd}, III^{rd} and IV^{th} quarters, respectively.

In Table 6 calculated structure indicators and chain indices per cents were specified. The structure indicators show what changes in opinion occurred. In follows from them that opinion in the IV^{th} quarter did not change when compared with the III^{rd} one. The same kind of conclusions can be drawn on the basis of chain indices observations. This, however, is not confirmed by the observations contained in Tables 3, 4 and 5, because e.g. in the compared quarters III^{rd} and IV^{th} so many as 26 persons changed their opinions into the opposite ones.

What we can do now is to show in what way, on the basis of interdependence measures, the above conclusions can be supplemented. We use a well-known Yule's interdependence coefficient (Yule, Kendall, 1966). It satisfied the desired properties after correcting it according to the formula:

$$Q^* = \frac{Q + 1}{2}$$

and calculating values for subsequent comparable quarters we get

$$Q^*_{I,II} = 0{,}77, \quad Q^*_{II,III} = 0{,}89, \quad Q^*_{III,IV} = 0{,}80.$$

It turns out that degree of changes in opinion in the III^{rd} and IV^{th} quarters is rather considerable.

Table 3

Changes in opinion expressing on a certain subject in the first and second quarter

II quarter \ I quarter	+	-	Σ
+	34	23	57
-	13	30	43
Σ	47	53	100

Source: The author's calculations.

Table 4

Changes in opinion expressing on a certain subject in the second and third quarter

III quarter \ II quarter	+	-	Σ
+	40	9	49
-	17	34	51
Σ	57	43	100

Source: The author's calculations.

Table 5

Changes in opinion expressing on a certain subject in the third and fourth quarter

IV quarter \ III quarter	+	-	Σ
+	36	13	49
-	13	38	51
Σ	49	51	100

Source: The author's calculations.

Table 6

Structure indicators and chain indices of changes in expressing opinions on a certain subject

Quarter	Structure indicators		Chain indices	
	+	-	+	-
I	47	53	-	-
II	37	43	121	81
III	49	51	86	115
IV	49	51	100	100

Source: The author's calculations.

The geometric mean of the above values is equal to

$$\bar{Q}^* = \sqrt[3]{0{,}77 \cdot 0{,}89 \cdot 0{,}80}$$

and its shows that with in the period of investigation (1 year) there occurred in opinion on the subject given. Therefore, the average degree of changes is also large.

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ZASTOSOWANIE MIERNIKÓW WSPÓŁZALEŻNOŚCI DO BADANIA DYNAMIKI ZJAWISK

W naukach społecznych do badania współzależności cech jakościowych stosowane są odpowiednie mierniki. W szczególności, gdy badane cechy są dychotomiczne, można dla pogłębienia analizy statystycznej używać ich (obok indeksów dynamiki) do określania dynamiki zjawisk. Może to mieć miejsce wówczas, gdy dokonujemy kilkakrotnej, co pewien czas, obserwacji statystycznej zjawiska, dla ustalonego, nie zmieniającego się zespołu jednostek statystycznych. Będzie to zatem analiza n-wymiarowej (2^n-polowej) tablicy kontyngencyjnej. Wybór odpowiedniego miernika zależy od posiadania przez niego pewnych własności, które winny mieć "dobre" mierniki.