ACTA UNIVERSITATIS LODZIENSIS
FOLIA OECONOMICA 156, 2002

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HOMOSCEDASTICITY TESTS FOR THE LINEAR TREND

Abstract. In this paper we consider single parameter models of heteroscedasticity: linear, square, exponential, group. A significant predominance of the parametric tests over the peak tests is shown using the variability coefficient as the most natural measure of homoscedasticity and the summary Kendal statistic as a measure of a test power. Another suggestion is that it is worth using the Goldfeld–Quandt parametric test, when the growth in the variance is quite „smooth" (in other case – the classical *F*-test is better). Prevalence of the *F*-test over the peak test is much smaller.

Key words: *F*-test, peak test, Goldfeld–Quandt test, Kendal statistic homoscedasticity, heteroscedasticity, F_c statistic test power, quantiles.

1. INTRODUCTION

Goldfeld and Quandt (1965) presented two propositions of the homoscedasticity tests for the random component in a one equation econometric model. On the basis of the Monte Carlo experiments the authors stated, that power of both tests is satisfying. This conclusion however, must lead to some doubts. Power evaluations obviously show considerable predominance of the parametric test over the non-parametric test. On the other hand, power comparisons of the Goldfeld, Quandt and Welfe (1998) parametric test power (see also: Pagen, Ullah (1999), Charemza, Deodman (1997), with the Theil's (1971) Best Linear Unbiased Scalar (BLUS) test and the tests of Harvey and Philips (1974), performed by the latter on the basis of accurate calculations show, that the Goldfeld and Quandt test is as good as its two other competitors.

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The purpose of this paper is to provide a further contribution to the power evaluation of the Goldfeld–Quandt homoscedasticity test.

In order to achieve more precise results, the investigation was reduced to the linear trend models and was based on a higher number of samples namely 10,000.

2. GOLDFELD AND QUANDT HOMOSCEDASTICITY TEST

Consider a linear econometric model written in matrix notation as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\alpha} + \boldsymbol{\varepsilon} \tag{1}$$

where for the variance-covariance matrix of the random component ε, satisfying other classical assumptions, the heteroscedasticity

$$\mathbf{D}^2\,\boldsymbol{\varepsilon} = \Omega = \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \sigma_n^2 \end{bmatrix} \tag{2}$$

is allowed (see, e.g., Goldberg (1966)).

Suppose we want to verify the hypothesis

$$H_0 : \sigma_1^2 = \sigma_2^2 = \dots = \sigma_n^2 \tag{3}$$

against a somewhat obscure alternative hypothesis having monotone heteroscedasticity,

$$H_1 : \sigma_1^2 \leqslant \sigma_2^2 \leqslant \dots \leqslant \sigma_n^2 \tag{4}$$

Let

$$\mathbf{e} = \mathbf{y} - \mathbf{X}\boldsymbol{\alpha} \tag{5}$$

be a vector of residuals obtained from the model (1) fitted by the least squares estimate

$$\boldsymbol{\alpha} = (\mathbf{X}^{\mathrm{T}}\,\mathbf{X})^{-1}\,\mathbf{X}^{\mathrm{T}}\,\mathbf{y} \tag{6}$$

of $\boldsymbol{\alpha}$ (T denoting transposition).

2.1. THE PEAK TESTS

We say that the t-th residual ($t = 2, 3, ..., n$) makes the „peak", if $|e_t| > |e_u|$ for each $u = 1, ..., t-1$. The statistic of the non-parametric Goldfeld–Quandt test is the number of such peaks, i.e. the number of residuals for which the above inequalities hold, which can be written as

$$G = card\ \{t : 2 \leqslant t \leqslant n, |e_t| > |e_u| \text{ for any } u = 1, ..., t-1) \quad (7)$$

2.2. GOLDFELD–QUANDT PARAMETRIC TEST

Let us now present the matrix **X** and the vectors **y** and **e** in the form

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_C \\ \mathbf{X}_2 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_C \\ \mathbf{y}_2 \end{bmatrix}, \mathbf{e} = \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_C \\ \mathbf{e}_2 \end{bmatrix} \quad (8)$$

where $\mathbf{x}_1$, $\mathbf{y}_1$, $\mathbf{e}_1$ incorporate the first n_1 rows or elements of **X**, **y**, **e**, respectively, $\mathbf{x}_2$, $\mathbf{y}_2$, $\mathbf{e}_2$ the last n_2 such rows or columns, while $\mathbf{X}_C$, $\mathbf{y}_C$, $\mathbf{e}_C$ contain the remaining $(n - (n_1 + n_2)$ rows or columns, being then called the „central" observations.

The concept underlying the Goldfeld–Quandt test consists in the application of the classical F-test against group heteroscedasticity, and the exclusion of $n_C + n_1 - n_2$ „central" observations. Let

$$\mathbf{e}_1 = \mathbf{y}_1 - \mathbf{X}_1\boldsymbol{\alpha}_1 \text{ where } \boldsymbol{\alpha} = (\mathbf{X}_1^T\ \mathbf{X}_1)^{-1}\ \mathbf{X}_1^T\ \mathbf{y}_1 \quad (9)$$

and

$$\mathbf{e}_2 = \mathbf{y}_2 - \mathbf{X}_2\boldsymbol{\alpha}_2 \text{ where } \boldsymbol{\alpha}_2 = (\mathbf{X}_2^T\ \mathbf{X}_2)^{-1}\ \mathbf{X}_2^T\ \mathbf{y}_2 \quad (10)$$

The statistic

$$F_C = \frac{\mathbf{e}_1^T\ \mathbf{e}_1\ /\ (n_1 - k)}{\mathbf{e}_2^T\ \mathbf{e}_2\ /\ (n_2 - k)} \quad (11)$$

where k is the rank of the matrix **X**, assumed to be equal to the ranks of $\mathbf{X}_1$ and $\mathbf{X}_2$ (here equal to their number of columns), has under (3) a central F distribution with n_1-k and n_2-k degrees of freedom.

3. DEFINING THE PROBLEM

The objective of the paper is evaluation of a further evaluation of the power of the Goldfeld–Quandt tests, and particularly, the peak tests. We base our examination on the results of the Monte-Carlo experiments. In the paper the following single parameter types of heteroscedasticity will be considered (see, e.g. A. Tomaszewicz (1987)):

(a) linear heteroscedasticity

$$\sigma_t^2 = \sigma_0^2 \, \Phi_1 \, (\beta, t) = \sigma_0^2 \left(1 + \beta \frac{t}{n}\right) \tag{12}$$

(b) square heteroscedasticity

$$\sigma_t^2 = \sigma_0^2 \, \Phi_K \, (\beta, t) = \sigma_0^2 \left[1 + \beta \left(\frac{t-1}{n}\right)^2\right] \tag{13}$$

(c) exponential heteroscedasticity

$$\begin{aligned} \sigma_t^2 &= \sigma_0^2 \, \Phi_K \, (\beta, t) = \sigma_0^2 \\ \sigma_t^2 &= \sigma_0^2 \, \Phi_W \, (\beta, t) = \sigma_0^2 \, e^{\beta t / n} \end{aligned} \tag{14}$$

(d) group heteroscedasticity

$$\sigma_t^2 = \sigma_0^2 \, \Phi_G \, (\beta, t) = \sigma_0^2 \begin{cases} 1 & \text{for } t \leqslant n / 2 \\ 1 + \beta & \text{for } t > n / 2 \end{cases} \tag{15}$$

Parameter β must be selected so that $\sigma_t^2 > 0$ for $t = 1, 2, ..., n$.

To compare different models, one needs a common measure of homoscedasticity. As the most natural measure the coefficient of variability (variattion) can be chosen i.e. the coefficient

$$v = \frac{\sqrt{\frac{1}{n} \sum_t (\sigma_t^2 - \bar{\sigma}^2)^2}}{\bar{\sigma}^2} \tag{16}$$

where $\bar{\sigma}^2 = \frac{1}{n} \sum_t \bar{\sigma}_t^2$

Note that expressing the mean squared deviation of the set $\sigma_1^2, \sigma_2^2, ..., \sigma_n^2$ in the units of their mean, this measure is independent of the proportionality coefficient σ_0^2.

As it has already been stated, we confine ourselves to the linear trend model (1), where the matrix X is of the form

$$\mathbf{X} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & n \end{bmatrix}^T \tag{17}$$

Goldfeld and Quandt (1965) suggest, on the basis of the obtained results, that the number of the rejected observations n_c should amount approximately to 30% of their total number (with at the same time the postulate of $n_1 = n_2$). This conclusion cannot, however, be considered as general since the optimal number of n_c depends both on the structure of matrix $\mathbf{X}$, and on the type of heteroscedasticity.

Before we start analysing the power of the Goldfeld and Quandt tests, we establish the optimal number n_c which is a function of n_1, provided that $n_1 = n_2 : n_c = n - 2n_1$.

Under the general alternative hypothesis (4), without postulating a heteroscedasticity model, the determination of the number of observations n_1 (and hence n_c) maximizing the power of the heteroscedasticity test is impossible. Moreover, one can state that for the group heteroscedasticity (15), if n_1 in (11) is known, the test is most powerful for $n_2 = n - n_1$, i.e. for $n_c = 0$.

As the Goldfeld–Quandt tests are recommended to verify general alternative hypotheses (4), when the heteroscedasticity model is unknown, defining n_1 should be a sort of compromise between various forms of the model. In our examination we assumed the following heuristic procedure based on subjective premises.

For:

- four types of heteroscedasticity (12), (13), (14), (15)
- $n = 10, 20, 30, 40, 50, 60$,
- 6 values of the variability coefficient.

In each of them the empirical power of the Goldfeld–Quandt parametric test was determined i.e. the quantity $\lambda\ (n, a, n_1, v, \beta)$.

Depending on:

- the number of the observations considered, $n_1 = n_2$,
- significance level $a = 0.10, 0.05, 0.01$.

As a measure of the test power we have adopted the following summary Kendal statistic

$$\Psi\ (n, n_1) = \sum_{a} \sum_{k=3}^{n/2} \sum_{v} \sum_{\beta} sign\ (\lambda(n, a, n_1, v, \beta) - \lambda(n, a, k, v, \beta)) \tag{18}$$

We prefer the non-parametric measure as being less sensitive to outliers. For a given n, the series

$$\Psi(n_1) = \Psi(n, n_1) \tag{19}$$

were then smoothed using a parabola

$$\Psi(n, n_1) = a_0(n) + a_1(n)n_1 + a_2(n)n_1^2 \tag{20}$$

(according to the least squares criterion). As the optimum value of n_1

$$n_1^*(n) = -\frac{a_1(n)}{2a_2(n)} \tag{21}$$

was assumed in a natural way, i.e. the value for which the left-hand side in (20) reaches the maximum.

Supposing that the series $n_1^*(n)$ should be „smooth" enough, as the optimum n_1 were assumed not the values $n_1^*(n)$, but their approximations also obtained by using OLS, and by using the parabola

$$n_1^{**}(n) / n = b_0 + b_1 n + b_2 n^2$$

with the additional condition

$$b_0 = 5\text{–}10b_1 + 100b_2,$$

equivalent to

$$n_1^{**}(10) = 5$$

The calculated values $n_1^*(n)$, $n_1^*(n) / n$, $n_1^{**}(n) / n$, are presented in columns 2–4 of Table 1. Its column 6 gives the number $\hat{n}_1(n)$ equal to the value of $n_1^{**}(n)$ rounded to the next integer. Numbers $\hat{n}_1(n)$ are further considered as the optimum values of n_1 for the Goldfeld–Quandt parametric test. Estimation of the error $n_1(n)$ is quite difficult. We repeated the described experiment to get an idea of its size. The values of n'_1 obtained in the second experiment are shown in column 7 of Table 1.

Table 1

Optimal values $n_1(n_c)$ (test F_c)

n	$n_1^*(n)$	$n_1^*(n)/n$	$n_1^{**}(n)/n$	Difference	$\hat{n}_1$	n'_1	$n'_1 - \hat{n}_1$
10	5.04	0.5038	0.5000	0.00385	5	5	0
20	7.54	0.3772	0.4356	–0.05837	9	9	0
30	11.86	0.3953	0.3903	0.00495	12	12	0
40	14.55	0.3638	0.3641	–0.00027	14	15	1
50	18.54	0.3508	0.3570	0.01284	18	18	0
60	20.82	0.3470	0.3689	–0.02196	22	22	0

Sources: the author's own elaboration.

4. EVALUATION OF THE TESTS POWER

The scope of the experiment, which formed the basis for evaluating the power of the tests was the following. For:

- four types of heteroscedasticity (12)–(15),
- 6 values of ν (see Table 2 and Table 3),
- 6 sizes of the sample n = 10, 20, 30, 40, 50, 60,

10 000 samples were drawn for each. In each of them the following values were calculated:

Table 2

Power of the tests: peak, Goldfeld–Quandt and F in the case of linear and square heteroscedasticity for $\alpha = 0.05$

n	ν	Linear peak test	Heteroscedasticity test		Square peak test	Heteroscedasticity test		ν
			Goldfeld –Quandt	F		Goldfeld –Quandt	F	
1	2	3	4	5	6	7	8	9
10	0.0	0.0515	0.0550	0.0550	0.0515	0.0850	0.0850	0.0
	0.1	0.0735	0.0800	0.0800	0.0735	0.0800	0.0800	0.1
	0.2	0.1102	0.1160	0.1160	0.1457	0.1840	0.1840	0.3
	0.3	0.1290	0.1770	0.1770	0.1765	0.3780	0.3780	0.5
	0.4	0.1982	0.2780	0.2780	0.2796	0.6540	0.6540	0.7
	0.5	0.2182	0.3690	0.3690	0.2855	0.8660	0.8660	0.9
20	0.0	0.0610	0.0640	0.0610	0.0610	0.0640	0.0610	0.0
	0.1	0.0765	0.1070	0.1110	0.0712	0.1050	0.1100	0.1
	0.2	0.1288	0.2280	0.2270	0.1826	0.3760	0.3910	0.3
	0.3	0.2238	0.4050	0.4200	0.3298	0.7700	0.7730	0.5
	0.4	0.3218	0.6180	0.6510	0.4849	0.9590	0.9670	0.7
	0.5	0.4744	0.7690	0.8040	0.5487	0.9940	0.9990	0.9
30	0.0	0.0521	0.0420	0.0420	0.0521	0.0420	0.0420	0.0
	0.1	0.1026	0.1490	0.1490	0.9010	0.1480	0.1400	0.1
	0.2	0.1605	0.3540	0.3550	0.2074	0.5910	0.6110	0.3
	0.3	0.2989	0.5810	0.6160	0.4461	0.9180	0.9430	0.5
	0.4	0.4447	0.7610	0.8210	0.6582	0.9930	0.9990	0.7
	0.5	0.6157	0.9040	0.9540	0.6936	1.0000	1.0000	0.9
40	0.0	0.0520	0.0430	0.0460	0.0520	0.0430	0.0460	0.0
	0.1	0.0918	0.1720	0.1790	0.0845	0.1660	0.1670	0.1
	0.2	0.1952	0.4040	0.4350	0.2508	0.6670	0.6990	0.3
	0.3	0.3330	0.6890	0.7640	0.5053	0.9800	0.9840	0.5
	0.4	0.5025	0.8820	0.9320	0.7285	1.0000	1.0000	0.7
	0.5	0.6709	0.9630	0.9860	0.7746	1.0000	1.0000	0.9
50	0.0	0.0467	0.0530	0.0610	0.0467	0.0530	0.0610	0.0
	0.1	0.1160	0.2070	0.2350	0.1080	0.2060	0.2200	0.1

Table 2 (cd.)

1	2	3	4	5	6	7	8	9
	0.2	0.2165	0.4880	0.5310	0.2790	0.7920	0.8040	0.3
	0.3	0.3817	0.7800	0.8840	0.5503	0.9910	0.9980	0.5
	0.4	0.5825	0.9350	0.9660	0.7988	1.0000	1.0000	0.7
	0.5	0.7768	0.9840	1.0000	0.8518	1.0000	1.0000	0.9
60	0.0	0.0488	0.0430	0.0490	0.0488	0.0430	0.0490	0.0
	0.1	0.1311	0.2350	0.2580	0.1216	0.2220	0.2410	0.1
	0.2	0.2660	0.6050	0.6230	0.3215	0.8640	0.8930	0.3
	0.3	0.4157	0.8440	0.8980	0.6059	0.9980	0.9990	0.5
	0.4	0.6333	0.9630	0.9860	0.8539	1.0000	1.0000	0.7
	0.5	0.8308	0.9980	0.9980	0.8989	1.0000	1.0000	0.9

Sources: the author's own elaboration.

Table 3

Power of the tests: peak, Goldfeld–Quandt and F in the case exponential and group heteroscedasticity for $a = 0.05$

n	v	Exponential peak test	Heteroscedasticity test		Group peak test	Heteroscedasticity test		v
			Goldfeld –Quandt	F		Goldfeld –Quandt	F	
1	2	3	4	5	6	7	8	9
10	0.0	0.0515	0.0550	0.0550	0.0515	0.0550	0.0550	0.0
	0.1	0.0728	0.0820	0.0820	0.0963	0.1430	0.1430	0.2
	0.3	0.1487	0.1890	0.1890	0.1410	0.3400	0.3400	0.4
	0.5	0.1824	0.3710	0.3710	0.1954	0.6550	0.6550	0.6
	0.7	0.2733	0.6060	0.6060	0.2325	0.9590	0.9590	0.8
	0.9	0.2810	0.8020	0.8020	0.3648	1.0000	1.0000	1.0
20	0.0	0.0610	0.0640	0.0610	0.0610	0.0640	0.0610	0.0
	0.1	0.0765	0.1070	0.1100	0.1021	0.2520	0.2420	0.2
	0.3	0.2005	0.3830	0.4010	0.2036	0.7290	0.6680	0.4
	0.5	0.3517	0.7550	0.7660	0.2658	0.9840	0.9640	0.6
	0.7	0.4744	0.9440	0.9500	0.2967	1.0000	1.0000	0.8
	0.9	0.5532	0.9920	0.9950	0.4207	1.0000	1.0000	1.8
30	0.0	0.0521	0.0420	0.0420	0.0521	0.0420	0.0420	0.0
	0.1	0.0993	0.1490	0.1440	0.1251	0.3790	0.3190	0.2
	0.3	0.2425	0.5940	0.6220	0.2126	0.9080	0.8300	0.4
	0.5	0.4839	0.9120	0.9400	0.3201	0.9990	0.9970	0.6
	0.7	0.6693	0.9900	0.9940	0.3649	1.0000	1.0000	0.8
	0.9	0.7388	1.0000	1.0000	0.4317	1.0000	1.0000	1.0
40	0.0	0.0520	0.0430	0.0460	0.0520	0.0430	0.0460	0.0
	0.1	0.0888	0.1710	0.1790	0.1253	0.5020	0.4130	0.2
	0.3	0.2929	0.6680	0.7140	0.2535	0.9570	0.8820	0.4

Table 3 (cd.)

1	2	3	4	5	6	7	8	9
	0.5	0.5597	0.9750	0.9840	0.3542	1.0000	0.9990	0.6
	0.7	0.7315	0.9980	0.9990	0.3830	1.0000	1.0000	0.8
	0.9	0.8136	1.0000	1.0000	0.4616	1.0000	1.0000	1.0
50	0.0	0.0467	0.0530	0.0610	0.0467	0.0530	0.0610	0.0
	0.1	0.1160	0.2090	0.2340	0.1485	0.6300	0.4800	0.2
	0.3	0.3312	0.8020	0.8260	0.3032	0.9910	0.9450	0.4
	0.5	0.6234	0.9900	0.9980	0.3654	1.0000	1.0000	0.6
	0.7	0.8027	1.0000	1.0000	0.4049	1.0000	1.0000	0.8
	0.9	0.8751	1.0000	1.0000	0.4446	1.0000	1.0000	1.0
60	0,0	0.0488	0.0430	0.0490	0.0488	0.0430	0.0490	0.0
	0,1	0.1301	0.2340	0.2570	0.1848	0.7010	0.5559	0.2
	0.3	0.3883	0.8650	0.9030	0.3034	0.9960	0.9780	0.4
	0.5	0.6612	0.9970	0.9990	0.4041	1.0000	1.0000	0.6
	0.7	0.8533	1.0000	1.0000	0.4487	1.0000	1.0000	0.8
	0.9	0.9255	1.0000	1.0000	0.4582	1.0000	1.0000	1.0

Sources: the author's own elaboration.

– the value of the statistic G of the Goldfeld–Quandt non-parametric test,

– the value of the F_c statistic of the Goldfeld–Quandt parametric test (for $n_1 = n_2 = \hat{n}$ – column 6, Table 1),

– the value of the statistic F for the classical test against the group heteroscedasticity (i.e. for a test based on the statistic (11) with $n_1 + n_2 = n$, i.e. $n_c = 0$).

The values of the statistics of the above listed tests were compared with the critical values for the levels of significance $\alpha = 0.10, 0.05, 0.01$. The results are presented for the value of 0.05.

A randomized version of the peak test was applied, on the basis of the approximation formulas for quantiles

$$g(n, \alpha) = \delta_{-2}(\alpha)n^{-2} + \delta_{-1}(\alpha)n^{-1} + \delta_0(\alpha) + \delta_1(\alpha)n + \delta_2(\alpha)n^2$$

For $\alpha = 0.05$ $g(n, 0.05) = 20.4n^{-2} - 16.027n^{-1} + 5.549 + 0.054n - 0.000348n^2$ presented in the paper by Tomaszewicz (1993). Critical values of the tests F and F_c are derived from the known tables of F-distribution.

The observed evaluations of the powers of the tests for all the listed cases are collected in Tables 2–3.

The obtained results prove significant predominance of the parametric tests over the peak tests. As far as the comparison of the F_c Goldfeld–Quandt test with the classical F test is concerned, the differences seem small: for the group heteroscedasticity the classical F test predominates, in other cases

– the F_c test. The gain in power when using the Goldfeld–Quandt test is the smaller, the more polarized the distribution of the variances $\sigma_1, ..., \sigma_n$ is, i.e. the more the heteroscedasticity model differs from the linear one. Differences in the power depend weakly on the choice of the significance level. Certainly, the tests power is higher when the significance level a increases. Nevertheless, when including different levels of significance in the experiment the shape of the test power curves remains similar.

One needs to pay attention to the fact that in the case of the group heteroscedasticity, the power of the peak test does not have to be an increasing function of the heteroscedasticity parameter ν. For large values of ν, the likelihood of satisfying the inequality

$$|e_t| < |e_u|$$

for all pairs $t = 1, ..., n_1$, $u = n_1 + 1, ..., n$ is close to 1. Hence, with the likelihood close to 1, the statistic G (see (7)), is the sum of two independent random variables

$$G_n = G_{n_1} + G_{n_2} + 1$$

(increased by 1, as e_{n+1} always makes the peak), whose distribution is the number of peaks in case of homoscedasticity (variances σ_t^2 are constant in each of the groups of observation $t = 1, ..., n_1$ and $t = n_1 + 1, ..., n$).

The power is distinctly increasing together with the rise in n (for $\nu > 0$). An exception here is the power of the peak tests for the group, where one can observe only a slight growth.

On the basis of the performed experiments two conclusions seem unquestionable.

1. The power of the peak tests is clearly smaller than the F test power, so the only argument (although quite weak) advocating its use is the simplicity of calculations.

2. Removal of the central calculations when using the F test results in a certain increase in power, the larger, the more uniform the growth of the variance of the random component is. In the case of a sudden increase (group heteroscedasticity) one can risk a considerable loss in power. Hence, it is worth using the Goldfeld–Quandt parametric test, when there are clear promises that the growth in a variance is quite „smooth". In other cases, it is better to maintain the classical F test.

However, our attention should be drawn to the fact, that the test was performed under conditions favourable for the F test, particularly with the assumption of normality of the random component distribution ε. Perhaps, if the distribution of ε is different from the normal one, or some other classical assumptions are not met, prevalence of the F test over the peak test, non-parametric in its very definition, is much smaller. This hypothesis is based on intuition only. Its confirmation or rejection needs some detailed research.

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TESTY HOMOSKEDASTYCZNOŚCI DLA MODELU LINIOWEGO

(Streszczenie)

W literaturze statystycznej i ekonometrycznej bardzo wyraźnie podkreśla się znaczenie i metody weryfikacji podstawowych założeń dotyczących modelu ekonometrycznego, chociaż w praktyce niezbyt często postulat ten jest realizowany. W szczególności chodzi tutaj o sprawdzenie założenia o homoskedastyczności. Przedmiotem rozważań będzie model liniowy

$$y = X\alpha + \varepsilon$$

dla którego spełnione są klasyczne założenia z wyjątkiem założenia o homoskedastyczności, tzn. że diagonalne elementy macierzy $D^2\varepsilon$ są sobie równe. Zamiast tego założenia postuluje się ogólniejsze

$$D^2\varepsilon = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \sigma_n^2 \end{bmatrix}$$

przyjmując założenie o braku autokorelacji, dopuszczając jednocześnie heteroskedastyczność.

W pracy rozważa się testy homoskedastyczności oparte na modyfikacji statystyki *F*, ilorazie wiarygodności oraz na resztach ortogonalnych. W szczególnosci prezentowana jest moc testów: Goldfelda–Quandta, szczytów oraz *F*.