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MODEL OF CONNECTION BETWEEN INFLATION AND INTEREST RATE BASED ON THE POLISH FINANCIAL MARKET

Abstract. The paper presents some empirical models of financial markets, which describe international interest rates and exchange rates. The main emphasis is placed on the model based on J. A. Frenkel's model of exchange rates, which presents the theory in detail and gives some practical applications.

The paper includes Fisher-Chow test of the stability of financial markets, shown for a large number of observations and concerning simulation models for small changes in interest rates and exchange rates, which can be used to estimate future interest rates.

1. ECONOMETRIC MODEL FOR FORECASTING INTEREST RATES AND EXCHANGE RATES

The models of financial markets analysed in the paper deal with international interest rates and exchange rates. The model introduced below is a version of Frankel's model. On the basis of this model, which applies parity of interest rates, it is possible to propose the following relation between the spot and forward exchange rates (Milo, Gontar, 1994):

$$\log S_t = a + b_1 \log F_{t-1} + b_2 \log F_{t-2} + b_3 \log F_{t-3} + b_4 \log F_{t-4} + e_t,$$

$$\log S_t = b_1 \log S_t + c_1 \log F_{t-1} + c_2 \log F_{t-2} + c_3 \log F_{t-3} + c_4 \log F_{t-4} + \log e_t,$$

where:

S_t – the spot rate in period t,
F_{t-1} – the forward rate in period $t-1$,
F_{t-2} – the forward rate in period $t-2$,

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F_{t-3} – the forward rate in period $t-3$,

F_{t-4} – the forward rate in period $t-4$,

a, b_1, b_2, b_3, c_1, c_2, c_3, c_4 – are parameters,

e_t – is the error in period t.

This model holds good for such currencies as the U.S. dollar, Canadian dollar, pound sterling, and Deutschmark. That is the main cause of forecasting spot and forward rates.

Using Frankel's model as a basis, it is possible to establish a model for forecasting future interest rates. The forecasted interest rate is dependent on the interest rate from previous period and the change in both the inflation rate and the exchange rate. The above – mentioned parameters are described with the use of the following symbols:

S_t – interest rate in period t,

S_{t-1} – interest rate in period $t-1$,

W_t – exchange rate in period t,

I_t – inflation rate in period t,

$\Delta I_t = I_t - I_{t-1}$ – difference between inflation rate in period t and in period $t-1$,

$\Delta W_t = W_t - W_{t-1}$ difference between exchange rate in period t and in period $t-1$.

In the simplest case, assuming that $I_t = 0$, we get:

$$S_t = S_{t-1}(1 + \Delta W_t) \tag{1}$$

and assuming that $\Delta W_t = 0$, then

$$S_t = S_{t-1}(1 + \Delta I_t) \tag{2}$$

More complicated and more real – life situations can be expressed as follows:

$$S_t = S_{t-1}(1 + a \cdot \Delta W_t) \tag{3}$$

if it is assumed that $\Delta I_t = 0$ and

$$S_t = S_{t-1}(1 + b \cdot \Delta I_t) \tag{4}$$

if it is assumed that $\Delta W_t = 0$.

On the basis of (3) and (4), the following dependence is derived:

$$S_t = S_{t-1}(1 + \alpha \cdot \Delta W_t)(1 + b \cdot \Delta I_t) \tag{5}$$

where: coefficients a, b make the above dependence (5) more elastic than firm equations (1), (2). Some elements of financial policy of the central bank or the government can be included in coefficients a, b. If it is assumed that ΔI_t and ΔW_t are insignificant, it is possible to reject component $ab\ \Delta W_t$ and get equation:

$$S_t = S_{t-1}(1 + a\Delta W_t + b\Delta I_t) \tag{6}$$

This equation has been chosen as a tool of calculation because it best suits the Polish economy. It is elastic and includes three major factors. An error factor can be added, but this would make calculations, more difficult and less clear.

By using the values in Tab. 1 (S_0, S_t, ΔW_t, ΔI_t) for $t = 1, 2, 3, \ldots, m$ to calculate coefficients a, b and applying the least square method, the following function can be generated:

$$F(a, b) = \sum_{t=1}^{m} [S_{t-1}(1 + a\Delta W_t + b\Delta I_t) - S_t]^2.$$

Coefficients a, b should be chosen in such a way that the values of function $F(a, b)$ are smallest (G a j d a, 1994). Methods of mathematical analysis are used to calculate a and b and the minimum of differential function $F(a, b)$. In the extremum point, the partial differentials $\frac{\partial F}{\partial a}$, $\frac{\partial F}{\partial b}$ are equal to 0, as in equation (7):

$$\begin{cases} \sum_{t=1}^{m} [S_{t-1}(1 + a\Delta W_t + bI_t) - S_t]S_{t-1}\Delta W_t = 0, \\ \sum_{t=1}^{m} [S_{t-1}(1 + a\Delta W_t + bI_t) - S_t]S_{t-1}\Delta I_t = 0. \end{cases} \tag{7}$$

and from the above we get:

$$\begin{cases} a\sum_{t=1}^{m} (S_{t-1}\Delta W_t)^2 + b\sum_{t=1}^{m} (S_{t-1}^2\Delta W_t\Delta I_t) = \sum_{t=1}^{m} S_{t-1}\Delta W_t(S_t - S_{t-1}), \\ a\sum_{t=1}^{m} (S_{t-1}^2\Delta W_t\Delta I_t) + b\sum_{t=1}^{m} (S_{t-1}\Delta I_t)^2 = \sum_{t=1}^{m} S_{t-1}\Delta I_t(S_t - S_{t-1}). \end{cases} \tag{8}$$

On the basis of inequality $\sum_{t=1}^{m} X_t^2 \cdot \sum_{t=1}^{m} Y_t^2 > \left(\sum_{t=1}^{m} X_t Y_t\right)^2$ for disproportional sequences $(X_1, \ldots, X_m)$ and $(Y_1, \ldots, Y_m)$, it is possible to say, that system of equations (8) is Cramer's system of equations therefore, and has only one solution $\left(\text{if there is no such proportion } \frac{\Delta W_1}{\Delta I_1} = \frac{\Delta W_2}{\Delta I_2} = \ldots = \frac{\Delta W_m}{\Delta I_m}\right)$.

When we solve this system of equations we get the following:

$$\begin{cases} a = \dfrac{\sum\limits_{t=1}^{m} S_{t-1}\Delta W_t(S_t - S_{t-1}) \sum\limits_{t=1}^{m} (S_{t-1}\Delta I_t)^2 - \sum\limits_{t=1}^{m} S_{t-1} I_t(S_t - S_{t-1}) \sum\limits_{t=1}^{m} S_{t-1}^2 \Delta W_t \Delta I_t}{\sum\limits_{t=1}^{m} (S_{t-1}\Delta W_t)^2 \sum\limits_{t=1}^{m} (S_{t-1}\Delta I_t)^2 - \left(\sum\limits_{t=1}^{m} S_{t-1}^2 \Delta W_t \Delta I_t\right)^2} \\ b = \dfrac{\sum\limits_{t=1}^{m} (S_{t-1}\Delta I_t)^2 \sum\limits_{t=1}^{m} S_{t-1}\Delta I_t(S_t - S_{t-1}) - \sum\limits_{t=1}^{m} S_{t-1}^2 \Delta W_t \Delta I_t \sum\limits_{t=1}^{m} S_{t-1}\Delta W_t(S_t - S_{t-1})}{\sum\limits_{t=1}^{m} (S_{t-1}\Delta W_t)^2 \sum\limits_{t=1}^{m} (S_{t-1}\Delta I_t)^2 - \left(\sum\limits_{t=1}^{m} S_{t-1}^2 \Delta W_t \Delta I_t\right)^2} \end{cases} \tag{9}$$

Denominators in these equations are positive, because of indexes p, $q \in \{1, 2, ..., m\}$ for which $\Delta W_p \Delta I_q \neq \Delta W_q \Delta I_p$.

Due to economic and financial reasons it is possible to have such assumptions that denominators in system of equation are positive. Another mathematical problem is whether a, b calculated a, b for equations (9) for function $F(a, b)$ achieve minimum.

From mathematical analysis it follows that it is enough to fulfil such an inequation

$$W(a, b) = \frac{\partial^2 F}{\partial a^2} \cdot \frac{\partial^2 F}{\partial b^2} - \left(\frac{\partial^2 F}{\partial a \, \partial b}\right)^2 > 0$$

using a and b from (9) we can write:

$$\frac{\partial^2 F}{\partial a^2} = 2 \sum_{t=1}^{m} (S_{t-1}\Delta W_t)^2,$$

$$\frac{\partial^2 F}{\partial b^2} = 2 \sum_{t=1}^{m} (S_{t-1}\Delta I_t)^2,$$

$$\frac{\partial^2 F}{\partial a \, \partial b} = 2 \sum_{t=1}^{m} S_{t-1}^2 \Delta W_t \Delta I_t.$$

For a, b from system of equations (9) is

$$W(a, b) = 4 \cdot \left[\sum_{t=1}^{m} (S_{t-1}\Delta W_t)^2 \cdot \sum_{t=1}^{m} (S_{t-1}\Delta I_t)^2 - \left\{\sum_{t=1}^{m} S_{t-1}^2 \Delta W_t \Delta I_t\right\}^2\right] > 0.$$

Table 1

T	S_t	ΔI_t	ΔW_t
0	0.316	–	–
1	0.317	-0.003	-0.010
2	0.319	0.003	-0.022
3	0.286	-0.022	-0.010
4	0.267	-0.009	0.018
5	0.268	-0.003	-0.012
6	0.247	-0.016	-0.010
7	0.232	-0.02	-0.007
8	0.228	-0.003	-0.002
9	0.190	-0.036	0.020
10	0.185	-0.009	-0.015
11	0.165	-0.016	-0.010
12	0.151	-0.016	-0.008

Source: Author's calculations based on empirical data from: Report on inflation rate published by and Report about monetary policy National Bank of Poland Warsaw.

From the system of equation (9), the following values are derived:

$$a = 0.268965, \quad b = 4.6359.$$

This means that for

$$S_t = S_{t-1}(1 + 0{,}26\Delta W_t + 4{,}63\Delta I_t) \tag{10}$$

function $F(a, b)$ has the smallest value.

After comparing on S_t given in Tab. 1 with values S_t' calculated on the basis of equation (10) we have:

Table 2

S_t and square difference of S_t and S'_t needed for receiving value of error, based on data given in Tab. 1

T	S_t	S'_t	$S_t - S'_t$	$(S_t - S'_t)^2$
1	0.317	0.319	0.002	0.00004
2	0.319	0.323	−0.004	0.000018
3	0.286	0.287	−0.001	0.0000017
4	0.267	0.273	−0.006	0.000032
5	0.268	0.264	0.004	0.000015
6	0.247	0.249	−0.002	0.0000034
7	0.232	0.225	0.007	0.000055
8	0.228	0.229	−0.0009	0.0000008
9	0.19	0.189	0.001	0.0000016
10	0.185	0.183	0.002	0.0000047
11	0.165	0.172	−0.007	0.000046
12	0.151	0.153	−0.002	0.0000044

Source: Authors' calculations.

From Tab. 2 we get:

$$\sum_{t=2}^{m} (S_t - S'_t)^2 = 0.0002226 \tag{11}$$

The result given in (11) shows that model (6) is a good model for searching dependencies between S_t from S_{t-1}, ΔW_t and ΔI_t. Changes in the inflation rate I_t are of greater importance and have a bigger influence on S_t than changes in the exchange rate W_t. Therefore, the difference between model (1), (2) and model (6) is obvious.

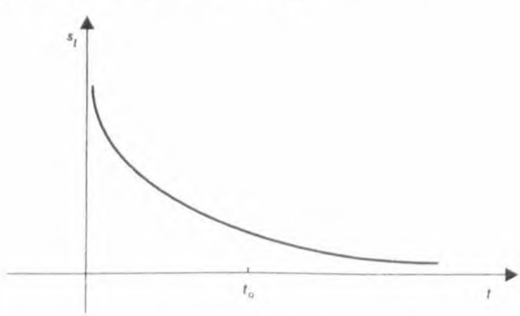

Fig. 1. Presentation of model (6), on the basis of empirical data

2. SIMULATION MODEL FOR STABILITY OF INTEREST RATES

One of the methods commonly used as a simulation model is the Fisher-Chow test. This test enables to predict the forward rate for a given series of dates.
Fisher-Chow test takes the form (M i l o, G o n t a r, 1994):

$$F^{m}_{n-k} = \frac{\frac{ss - ss_1}{m}}{\frac{ss_1}{n-k}},$$

where:

ss – is the sum of squares for the entire series of the interest rate,
ss_1 – is the sum of squares for the first half of series of interest rate,
m – is the number of observations excluded when calculating ss_1,
n – is the number of observations included when calculating ss_1,
k – is the number of parameters.

If $0 < F^{m}_{n-k} < 1$ the interest rate has a desirable (decreasing) tendency. This model helps to simulate stability of interest rate. The result for this example shows $F^{m}_{n-k} = 0.679$.

3. SOME ASPECTS OF RISK PREMIUMS: CAPITAL ASSET PRICING MODEL AND ARBITRAGE PRICING THEORY

Recognition that greater systematic risk makes an asset less desirable can be used to understand the capital asset pricing model (CAPM). The CAPM is useful because it provides an explanation for the magnitude of an asset's risk premium; the difference between the assets expected return and the risk-free interest rate.

An asset contributes risk to a well-diversified portfolio in the amount of its systematic risk as measured by beta. When an asset has a high beta, meaning that it has a large amount of systematic risk, and is therefore less desirable, we would expect that investors would be willing to hold this asset only if it yielded a higher expected return. This is exactly what the CAPM shows in the equation (M i s h k i n, 1995).

$$\text{Risk premium } = R^{e} - R_{f} = \beta(R^{e}_{m} - R_{f}).$$

where:

R^e – expected return on the assets,

R_f – risk-free interest rate,

β – beta of the asset,

R^e_m – expected return for the market portfolio.

The CAPM equation provides the common sense result that when an assets beta is zero, with means that it has no systematic risk, its risk premium will be zero. If its beta is 1.0, i.e. it has the same systematic risk as the entire market, it will have the same risk premium as the market, $R^e_m - R_f$. If the asset has an even higher beta, e.g. 2.0, its risk premium will be greater than that of the market. For instance, if the expected return on the market is 7% and the risk-free rate is Z, the risk premium for the market is 5%. The asset with the beta of 1.5 would then be expected to have a risk premium of 7.5% ($= 1.5 \times 5\%$).

Although the capital assets pricing model has proved to be useful in the real life applications, it assumes that there is only one source of systematic risk that is found in the market portfolio. However, an alternative theory, the arbitrage pricing theory (APT), takes the view that there are several sources of risk in the economy that cannot be eliminated by diversification. These sources of risk can be considered as related to economy wide factors such as inflation and aggregate output. Instead of calculating a single beta, like the CAPM, arbitrage-pricing theory calculates many betas by estimating the sensitivity of an asset's return to changes in each factor. The arbitrage pricing theory equation is (Mishkin, 1995) as follows:

$$\text{Risk premium} = R^e - R_f = \beta_1(R^e_{factor} - R_f) + \beta_2(R^2_{factor2} - R_f) + \ldots + \beta_1(R^e_{factor3} - R_f).$$

Thus the arbitrage pricing theory thus indicates that the risk premium for an asset is related to the risk premium for each factor, and that as the asset's sensitivity to each factor increases, its risk premium will increase as well.

It is still uncertain which of these theories provides a better explanation of risk premiums. Both agree that an asset has a higher risk premium when it has a higher systematic risk, and both are considered to be valuable tools for explaining risk premiums.

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