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AN ALGORITHM OF THE ESTIMATION METHODS OF FINAL FORM'S PARAMETERS OF SIMULTANEOUS LINEAR ECONOMETRIC MODELS

1. Introduction

We limit our consideration to the case of simultaneous linear econometric models with a maximum lag equal to 1. The estimators formulae used in the algorithm belong to the class of noniterative estimators of constrained least squares method. Let us assume that the empirical structural form of the simultaneous linear econometric model is given by

(1) $$B_o Y_t + B_1 Y_{t-1} + G_o X_t + G_1 X_{t-1} = V_t ,$$

where:

a) $$\underset{m\times m}{B_o} = \begin{bmatrix} 1 & b_{12}^{(o)} & \dots & b_{1m}^{(o)} \\ b_{21}^{(o)} & 1 & \dots & b_{2m}^{(o)} \\ \dots & \dots & \dots & \dots \\ b_{m1}^{(o)} & b_{m2}^{(o)} & \dots & 1 \end{bmatrix}, \quad \underset{m\times m}{B_1} = \begin{bmatrix} b_{11}^{(1)} & b_{12}^{(1)} & \dots & b_{1m}^{(1)} \\ b_{21}^{(1)} & b_{22}^{(1)} & \dots & b_{2m}^{(1)} \\ \dots & \dots & \dots & \dots \\ b_{m1}^{(1)} & b_{m2}^{(1)} & \dots & b_{mm}^{(1)} \end{bmatrix}$$

are the matrices of the estimators of the parameters standing at the endogenous variables in the periods t, t-1, respectively,

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b)

$$\underset{m\times n}{Y_t} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1t} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2t} & \cdots & Y_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Y_{m1} & Y_{m2} & \cdots & Y_{mt} & \cdots & Y_{mn} \end{bmatrix},$$

$$\underset{m\times n}{Y_{t-1}} = \begin{bmatrix} Y_{1,1-1} & Y_{1,2-1} & \cdots & Y_{1,t-1} & \cdots & Y_{1,n-1} \\ Y_{2,1-1} & Y_{2,2-2} & \cdots & Y_{2,t-1} & \cdots & Y_{2,n-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Y_{m,1-1} & Y_{m,2-1} & \cdots & Y_{m,t-1} & \cdots & Y_{m,n-1} \end{bmatrix}.$$

are the matrices of the observations on the endogenous variables in the periods t, t-1, respectively,

c)

$$\underset{m\times k}{G_o} = \begin{bmatrix} g_{11}^{(o)} & g_{12}^{(o)} & \cdots & g_{1k}^{(o)} \\ g_{21}^{(o)} & g_{22}^{(o)} & \cdots & g_{2k}^{(o)} \\ \cdot & \cdot & \cdot & \cdot \\ g_{m1}^{(o)} & g_{m2}^{(o)} & \cdots & g_{mk}^{(o)} \end{bmatrix}, \quad \underset{m\times k}{G_1} = \begin{bmatrix} g_{11}^{(1)} & g_{12}^{(1)} & \cdots & g_{1k}^{(1)} \\ g_{21}^{(1)} & g_{22}^{(1)} & \cdots & g_{2k}^{(1)} \\ \cdot & \cdot & \cdot & \cdot \\ g_{m1}^{(1)} & g_{m2}^{(1)} & \cdots & g_{mk}^{(1)} \end{bmatrix}$$

are the matrices of the estimators of the parameters standing at the exogenous variables in the periods t, t-1, respectively,

d)

$$\underset{k\times n}{X_t} = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1t} & \cdots & X_{1n} \\ X_{21} & X_{22} & \cdots & X_{2t} & \cdots & X_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{k1} & X_{k2} & \cdots & X_{kt} & \cdots & X_{kn} \end{bmatrix},$$

$$\underset{k\times n}{X_{t-1}} = \begin{bmatrix} X_{1,1-1} & X_{1,2-1} & \cdots & X_{1,t-1} & \cdots & X_{1,n-1} \\ X_{2,1-1} & X_{2,2-1} & \cdots & X_{2,t-1} & \cdots & X_{2,n-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{k,1-1} & X_{k,2-1} & \cdots & X_{k,t-1} & \cdots & X_{k,n-1} \end{bmatrix}$$

are the matrices of the observations on the exogenous variables in the periods t, t-1, respectively,

e) m - the number of equations (together with the identities),

f) n - the number of observations on the endogenous and exogenous variables,

g) k - the number of exogenous variables,

h)

$$V_t_{m\times n} = \begin{bmatrix} v_{11} & v_{12} & \dots & v_{1t} & \dots & v_{1n} \\ v_{21} & v_{22} & \dots & v_{2t} & \dots & v_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ v_{m1} & v_{m2} & \dots & v_{mt} & \dots & v_{mn} \end{bmatrix}$$

is the matrix of residuals in the period t (if any equation is an identity then no residual will appear).

The algorithm additionally requires as the input information:

a) the parameter J which means the number of recursive repetitions (J is usually less or equal to n-1),

b) the matrices $X_{t-2}, \dots, X_{t-J}$ denote the matrices of observations on the exogenous variables in the period t-2,..., t-J (each of these matrices has a dimension kxn and it is formed in the same way as the matrix X_t or X_{t-1}). The final form corresponding to (1) is as follows:

$$(2) \qquad Y_t = D_o X_t + F_1 X_{t-1} + F_2 X_{t-2} + \dots + F_J X_{t-J} + W_t,$$

where:

D_o is the mxk matrix of the direct (impact) multipliers,

$F_1, F_2, \dots, F_J$ are the mxk matrices of the indirect (delay, lagged) multipliers,

W_t is the mxn matrix of the residuals of the final form.

2. A Description of the Algorithm

In describing the algorithm we will use two kinds of estimators for the parameters of the final form [1, 2, 3]. The first one uses the information about the exogenous variables in the recent and past periods and it does not use the information about the mutual relations between the direct and indirect multipliers and the parameters of the structural form of an econometric model. The other method just takes into account the above mentioned relationships. Below we shall give a description of the successive steps of the algorithm which includes both of these estimators.

The input data:

$m, n, k, J, t \in I$ (I - the set of integers),

$B_o, B_1, C_1 \in R^{mxm}$ (R - the set of mxm real matrices),

$Y_t, Y_{t-1} \in R^{mxn}$,

$G_o, G_1, D_o, D_1, F_o \in R^{mxk}$,

$X_t, X_{t-1} \in R^{kxn}$, $\underline{X} \in R^{k(J+1)xn}$, $\hat{\theta}, \tilde{\theta} \in R^{mxk(J+1)}$,

S1: Given the matrices $Y_t, X_t, X_{t-1}, \ldots, X_{t-J}$ form the matrix

$$\underline{X} = \begin{bmatrix} X_t \\ \cdots\cdots \\ X_{t-1} \\ \cdots\cdots \\ \vdots \\ \cdots\cdots \\ X_{t-J} \end{bmatrix},$$

where $\underline{X}$ is the $k(J+1)xn$ matrix whose blocks are matrices of observations on the exogenous variables in the periods t, t-1,..., t-J.

S2: Compute

$$\hat{\theta}' = \begin{cases} (\underline{X}\underline{X}')^{-1}\underline{X}Y' & \text{if } \det(\underline{X}\underline{X}') \neq 0, \text{ i.e. if } n \geqslant k(J+1) \\ (\underline{X}\underline{X}')^{+}\ \underline{X}Y' & \text{if } \det(\underline{X}\underline{X}') = 0, \text{ i.e. if } n < k(J+1) \end{cases}$$

(the symbol "+" denotes the Moore-Penrose generalized inverse matrix for the calculation of which one of the known algorithms should be used, and $Y := Y_t$).

The matrix $\hat{\theta}$ constitutes the first group of estimators for the parameters of the final form of a linear econometric model. It has a dimension $mxk(J+1)$ and its blocks are matrices of direct and indirect multipliers.

S3: Given B_o, B_1, G_o, G_1 compute (under the condition that $\det B_o \neq 0$)

a) $C_1 = -B_o^{-1}B_1$,

b) $D_o = -B_o^{-1}G_c$ (the matrix of direct multipliers),

c) $D_1 = -B_o^{-1}G_1$.

S4: For $p = 1, 2, 3, \dots, J$ calculate.

$F_p = C_1^{p-1}(C_1D_o + D_1)$ (the matrices of indirect multipliers).

S5: Form

$$K' = \begin{bmatrix} D_o'C_1' \\ \cdots \\ F_1'C_1' \\ \cdots \\ F_2'C_1' \\ \cdots \\ \vdots \\ \cdots \\ F_J'C_1' \end{bmatrix}$$

S6: Compute

$$\tilde{\theta}' = \begin{cases} (\underline{X}\underline{X}')^{-1}\underline{X}Y' - (\underline{X}\underline{X}')^{-1}\underline{X}Y'C_1(C_1C_1')^{+}C_1 + (\underline{X}\underline{X}')^{-1}\underline{X}\underline{X}'K'(C_1C_1')^{+}C_1 \\ \text{if } \det(XX') \neq 0 \text{ and } \det(C_1C_1') \neq 0 \text{ or } \det(C_1C_1') = 0 \text{ i.e. } n \geq k(J+1) \\ (\underline{X}\underline{X}')^{+}\underline{X}Y' - (\underline{X}\underline{X}')^{+}\underline{X}Y'C_1(C_1C_1')^{+}C_1 + (\underline{X}\underline{X}')^{+}\underline{X}\underline{X}'K'(C_1C_1')^{+}C_1 \\ \text{if } \det(\underline{X}\underline{X}') = 0 \text{ and } \det(C_1C_1') = 0 \text{ or } \det(C_1C_1') = 0 \text{ i.e } n < k(J+1) \end{cases}$$

or in the simple way $\tilde{\theta}'$ can be expressed as

$$\theta' = \begin{cases} K'(C_1')^{-1}, & \text{if } n \geq k(J+1) \text{ and } \det(C_1C_1') \neq 0 \\ \hat{\theta} - \hat{\theta}'C_1'(C_1C_1')^{+}C_1 + (\underline{X}\underline{X}')^{+}\underline{X}\underline{X}'K'(C_1C_1')^{+}C_1, & \text{if } n < k(J+1). \end{cases}$$

The matrix $\tilde{\theta}$ has the same dimension and structure as the matrix $\hat{\theta}$ and it constitutes the second group of estimators for the parameters of the final form of an econometric model.

We remind that the matrix θ of the parameters of the final form, for which estimators are the matrices $\hat{\theta}$ and $\tilde{\theta}$, has the following structure

$$\theta = \left[D_o \vdots F_1 \vdots F_2 \vdots \dots \vdots F_J \right],$$

and its blocks are interpreted as the direct (D_o) and indirect multipliers ($F_1, \dots, F_J$). It should be also noted that in practice we are forced to use the generalized inverse matrix for the calculation of $\hat{\theta}$ or $\tilde{\theta}$ because of the relation $n < k(J+1)$ (which frequently occurs in the econometric models) and in the special form of identities included into the model.

The above described algorithm is available at the Institute of Econometrics and Statistics, University of Łódź.

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ALGORYTM ESTYMACJI PARAMETRÓW POSTACI KOŃCOWEJ W LINIOWYCH WSPÓŁZALEŻNYCH MODELACH EKONOMETRYCZNYCH

W ostatnich latach powstało kilka prac na temat metod estymacji parametrów postaci końcowej liniowych modeli ekonometrycznych [1, 2, 3], jednakże żadna z nich nie zawiera algorytmu pozwalającego na obliczenie wartości estymatorów parametrów tej postaci. Głównym celem tego artykułu jest prezentacja takiego właśnie algorytmu. Praca składa się z dwóch części. Pierwsza jest wstępem i zawiera informacje wejściowe do algorytmu estymującego parametry postaci końcowej. Druga natomiast przedstawia opis algorytmu.