A C T A U N I V E R S I T A T I S L O D Z I E N S I S
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AN APPROXIMATION OF EXPECTATION VALUE OF VECTOR-VALUED STATISTICS

1. INTRODUCTION

The expectation of vector statistics is an example of Volterra's (see V o l t e r r a [6]) function of function. Von M i s e s [5] has shown that Volterra's calculus functions of functions can be adapted to statistical functions of functions. It is not easy, however, to translate Volterra-von Mises's concepts of calculus into such manageable forms that can be used in calculating the expectation of vector statistics. This is why we propose a method of approximation of expectation that is not based on Volterra-von Mises ideas. This method would have a few versions depending on a way of calculating the ray r of ball $\mathcal{B}(x, r) = \{y: \|x\| \leqslant r\}$, where $\mathcal{B}(x, r)$ is used in determining the value of expression $\int_{\mathcal{B}(x,r)} f(x)\,dx$, and $f(x)$ denotes a given shape of density function of random vector X.

Denoting by $F(x) \equiv F_X(x)$ the distribution function of random vector X and using $dF(x) = f(x)\,dx$ one can define, for example, the following function of function used in statistics, econometrics, stochastic programming:

$$G_O(F(x)) \equiv \int_{R^n} x\,dF(x) \equiv \int_{R^n} x f(x)\,dx \equiv G_O(f(x)) \equiv G_O(f) \equiv E(X).$$

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$$G_1(F(x)) \equiv \int_{R^n} b(x)dF(x) \equiv \int_{R^n} b(x)f(x)dx \equiv G_1(f(x)) \equiv G_1(f) \equiv$$

$$\equiv \mathfrak{E}(X),$$

where $B(X) = B = \mathfrak{Z}^{-1}Z^{\tau}x$, $R^k \ni b(x) = \mathfrak{Z}^{-1}Z^{\tau}x$, $R^{kxk} \ni \mathfrak{Z} = Z^{\tau}Z$, $Z \in R^{nxk}$, R^{nxk} is the set of real n x k matrices, τ is the transposition sign and $b(x)$ is a sample value of B.

We will comment how to calculate $G_0(f)$, $G_1(f)$ in the case of the following shapes of f:

a0) $f_0 \equiv f_0(x) = (2\pi\sigma^2)^{n/2} \exp -(2\sigma^2)^{-1}\|x\|^2$;

(f_0 is the density of n-dimensional isotropic gaussian distribution of x; $\|x\|$ is the Euclidean norm of real vector x, $x \in R^n$);

a1) $f_1 \equiv f_1(x) = (2\pi)^{-1/2}(\det\Sigma)^{-1}\exp(-2^{-1} < \Sigma^{-1}(x - a), x - a >)$,

(f_1 is the density of n dimensional normal distribution of X with $\mathfrak{E}X \equiv a \equiv Z\beta$, $\beta \in R^k$, $X = Z\beta + \mathfrak{Z}$, $P_X = N_X(Z\beta, \Sigma)$, i.e. where $< \ldots >$ denotes a scalar product);

a2) $f_2 \equiv f_2(x) = \begin{cases} (2a)^{-n}, & x \in C = \{x: -a \leqslant x \leqslant a\}. \\ 0, & x \notin C, \end{cases}$ x, $a \in R^n$, $a = (a, \ldots, a)$,

(f_2 is the density function of n-dimensional uniform distribution);

a3) $f_3 \equiv f_3(x) = \dfrac{\Gamma(\frac{1}{2}(\nu + n))}{(\pi\nu)^{n/2}\Gamma(\frac{1}{2}\nu)\det^{1/2}\Sigma}(1 + \nu^{-1}x^{\tau}\Sigma^{-1}x)^{-\frac{1}{2}(\nu+n)}$,

(f_3 is the density of multivariate t-distribution).

In what follows we will use the following meaning put to the symbols A, B, X:

$A = \mathfrak{Z}^{-1}Z^{\tau}$, $B = AX$;

$X : (\mathcal{U}, \mathcal{F}) \to (R^n, \mathcal{F}_{R^n})$,

where $\mathcal{U}$ is a space of elementary events, $\mathcal{F}$ is a σ- field of Borel subsets of $\mathcal{U}$, $\mathcal{F}_{R^n}$ is a σ-field of Borel subsets of R^n;

$$G(f) \equiv G_1(f) = \int_{R^n} AXf(x)dx$$

is a function G of function f.

The purpose of this paper is to describe an approximation method of values of G and to comment how it should be used for densities other than the densities f_o, f_1. We will also comment on the use of approximated values of G for two different densities in assesing a sensivity of expectation of B on the change of density from, for instance, f_o, into f_2.

The proposed method seems to be attractive from numerical point of view. This does not mean that it cannot be refined or improved. It has, moreover, serious limitations. It can not be used, for example, in the case when $A \equiv A(x) \equiv (Z + cI)^{-1}Z^{\tau}$

$$R_+ \ni c = k\frac{x^{\tau}H_o x}{x^{\tau}H_1 x}, \quad H_o = I - Z(Z^{\tau}Z)^{-1}Z^{\tau}, \quad H_1 = Z(Z^{\tau}Z)^{-2}Z^{\tau}.$$

In the next section we will describe a few wariants of a method of approximation for $G(f_i) \equiv G_1(f_i)$ valid for any, nonnegative scalar locally integrable in R^n, function $f(x)$ fulfilling the condition $\lim_{||x|| \to \infty} ||x||^{\alpha} f(x) = 0$, $\alpha > n + 1$.

2. APPROXIMATION OF G(f)

There are known exact analytical expressions of $G_o(f_i)$.

$G(f_i) \equiv G_1(f_i)$ if $f_i \equiv f_i(x)$, $i = \overline{0, 1}$.

There is no exact expression for $G(f_i)$ if A is more complex (for example if $A \equiv A(x) = (Z + cI)^{-1}Z^{\tau}$ and $G(f_i) = \int_{R^n} A(x)f_i(x)dx$) or/and $i > 1$. In such cases there is a need for an approximation of values of $G(f_i)$. In this paper we will limit our attention to the case when A is not a function of x, i.e if $A \equiv A \in R^{kxn}$. We represent A as:

$$A = \begin{pmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_k \end{pmatrix}, \quad a_i \in R^{1xn}, \quad i = \overline{1, k}.$$

Let f be a nonnegative real scalar function defined on R^n and locally integrable on any bounded subset of R^n, and let it fulfill the condition

$$c1) \quad \lim_{\|x\| \to \infty} \|x\|^{\alpha} \, f(x) = 0, \quad \alpha > n+1.$$

A set of such functions f is a linear space $\mathcal{L}_\alpha$ on which it is easy to define such an operator G that

$$\mathcal{L}_\alpha \ni f \to G(f) \in R^k, \tag{1}$$

where

$$G(f) = \Big(\int_{R^n} < a_1, x > f(x) \, dx, \ \ldots,$$

$$\int_{R^n} < a_k, x > f(x) \, dx \Big)^{\tau} \tag{2}$$

and

$$G_1(f) = \int_{R^n} < a_1, x > f(x) \, dx, \quad 1 = \overline{1, k},$$

$$f \equiv f_0, f_1, f_2, f_3, \ldots,$$

From (c1) it follows that for any $\varepsilon > 0$, $f \in \mathcal{L}_\alpha$ there exists such positive real number $m_\varepsilon(f) \geq 0$ that

$$\|x\| > m_\varepsilon(f) \Longrightarrow \|x\|^{\alpha} \, |f(x)| < \varepsilon$$

holds.

In order to find a ray $m_\varepsilon(f)$ of a ball $B(x, m_\varepsilon(f))$ one should:

- to fix ε (for example $\varepsilon = 10^{-3}$);
- to fix f (for example $f \equiv f_0$);

- to solve in x the inequality $\|x\|^{\alpha} \cdot |f(x)| < \varepsilon$, i.e. find x^* that flufills for a given ε, f the equation $\|x\|^{\alpha} f(x) = \varepsilon$ (for example $\|x\|^{10} f_0(x) = 10^{-3}$, $\alpha = n + 1 = 9 + 1$);

- to set $m_{\varepsilon}(f) := \|x^*\|$.

Since (3) is equivalent to the implication

$$\|x\| > m_{\varepsilon}(f) \Rightarrow \|x\| \cdot |f(x)| \leqslant \frac{\varepsilon}{\|x\|^{\alpha-1}},$$

then, due to

$$\left| \int_{R^n} \langle a_1, x \rangle f(x)\, dx \right| \leqslant \int_{R^n} |\langle a_1, x \rangle f(x)|\, dx \leqslant$$

$$\leqslant \int_{R^n} \|a_{\iota}\| \cdot \|x\| \cdot |f(x)|\, dx \qquad (5)$$

Denoting the upper bound $\int_{R^n} \|a_1\| \cdot \|x\| \cdot |f(x)|\, dx$ of $G_{(1)}(f)$ by $\tilde{G}_{(1)}(f)$ we can write for $1 = \overline{1, k}$

$$\tilde{G}_{(1)}(f) = \int_{x \leqslant m_{\varepsilon}(f)} \|a_1\| \cdot \|x\| \cdot |f(x)|\, dx +$$

$$+ \int_{\|x\| > m_{\varepsilon}(f)} \|a_{\iota}\| \cdot \|x\| \cdot |f(x)|\, dx,$$

Hence

$$\tilde{G}_{(1)}(f) \leqslant m_{\varepsilon}(f) \|a_1\| \int_{\|x\| \leqslant m_{\varepsilon}(f)} |f(x)|\, dx +$$

$$+ \varepsilon \|a_1\| \int_{\|x\| > m_{\varepsilon}(f)} \frac{1}{\|x\|^{\alpha} - 1}\, dx \qquad (5)$$

By the theorem about the change of coordinates one can denote a system of spherical polar coordinates by Φ and the Jacobian of system Φ by $J(\Phi)$. The determinant of $J(\Phi)$ is equal

$$\det J(\Phi) = r^{n-1} \cos\theta_2 \cos^2\theta_3, \ldots, \cos^{n-1}\theta_n.$$

Hence the upper bound of $\tilde{G}_{(1)}(f)$ in (5), denoted here by $\tilde{\tilde{G}}_{(1)}(f)$, is equal

$$\tilde{\tilde{G}}_{(1)}(f) = m_\varepsilon(f)\ \|a_1\| \int\limits_{\|x\| \leq m_\varepsilon(f)} |f(x)|\,dx +$$

$$+\ \varepsilon\ \|a_1\| \int\limits_{m_\varepsilon(f)}^{\infty} \int\limits_{0}^{2\pi} \int\limits_{-\pi/2}^{\pi/2} \cdots \int\limits_{-\pi/2}^{\pi/2} \frac{r^{n-1}}{r^{\alpha-1}} \cdot \cos\theta_2, \ \ldots, \qquad (6)$$

$$\cos^{n-1}\theta_n \cdot\ d\,\theta_{n-1}, \ \ldots, d\,\theta_2\ d\theta_1 dr$$

denoting

$$\varkappa = \pi \cdot \varkappa_1\ \varkappa_2, \ \ldots, \ \varkappa_{n-2}; \qquad \varkappa_i = \int\limits_0^{\pi/2} \cos^i\theta\ d\,\theta,$$

$$i = \overline{1,\ n-2},$$

and using properties of $\varkappa_i$ we obtain

$$\varkappa = \frac{\pi^{\,E\left(\frac{n}{2}\right)}\ 2^{\,E\left(\frac{n+1}{2}\right)-\,n\,+\,1}}{(n-2)\,!\,!}$$

where $E(q)$ denotes integer part of number q and where $(n-2)!! = (n-2)(n-4), \ldots, 4.2$.

Thus

$$\tilde{\tilde{G}}_{(1)}(f) = m_\varepsilon(f)\ \|a_1\| \int\limits_{\|x\| \leq m_\varepsilon(f)} |f(x)|\,d\,x +$$

$$+\ \frac{2^{n-1}\varepsilon\ \ \|a_i\|\ (m_\varepsilon(f))^{n-\alpha+1}}{\alpha - n + 1} \qquad (7)$$

From (7) it is seen that in order to determine a value of 1-th component $\tilde{\tilde{G}}_{(1)}(f)$ of vector-valued approximate $\tilde{\tilde{G}}(f)$ one has to assign, some numerical values, to $m_\varepsilon(f)$, $\|a_1\|$, n, α, ε, $\varkappa$ and to calculate the value $\int\limits_{\|x\| \leq m_\varepsilon(f)} |f(x)|\,dx$ by using one of known algorithms of numerical integration in the ball $B\,(0,\ m_\varepsilon\,(f))$ with the center 0 and radius $m_\varepsilon(f)$.

Under the notation

$$\bar{\varepsilon} = \frac{\pi^{E\left(\frac{n}{2}\right)}_{2} \; E\left(\frac{n+1}{2}\right)}{(\alpha - n - 1)(n - 2)!!(m_{\varepsilon}(f))^{\alpha-n-1}} \cdot \varepsilon$$

we can write (7) as

$$\overset{\approx}{G}_{(1)}(f) = \|a_1\| \left(m_{\varepsilon}(f) \int_{\|x\| \leq m_{\varepsilon}(f)} |f(x)| \, d\, x + \bar{\varepsilon}\right) < \infty \tag{7a}$$

If we want to make the radius $m_{\varepsilon}(f)$ of integration ball $\mathfrak{B}(0, m_{\varepsilon}(f))$ to be dependent on the values of n and α, then we assign to ε the value ε_o^{α} satisfying

$$\varepsilon_o^{\alpha} \leq \left(\frac{\pi^{E\left(\frac{n}{2}\right)}_{2} \; E\left(\frac{n+1}{2}\right)}{(\alpha - n - 1)(n - 2)!!}\right)^{-1}$$

and replace (7a) with

$$\overset{\approx}{G}^{(\alpha)}_{(1)}(f) = \|a_1\| \left(m_{\varepsilon_o^{\alpha}}(f) \int_{\|x\| \leq m_{\varepsilon_o^{\alpha}}(f)} |f(x)| \, d\, x + \right.$$

$$\left. + \left(m_{\varepsilon_o^{\alpha}}(f)\right)^{n-\alpha+1}\right) \tag{7b}$$

Since

$$G(f) = \left(G_{(1)}(f), \ldots, G_{(1)}(f), \ldots, G_{(k)}(f)\right)^{\tau}$$

therefore, due to (7a), (7b) we can determine two approximates of vector valued function $G(f)$. These are

$$\overset{\approx}{G}(f) = \left(\overset{\approx}{G}_{(1)}(f), \ldots, \overset{\approx}{G}_{(1)}(f), \ldots, \overset{\approx}{G}_{(k)}(f)\right)^{\tau}.$$

or

$$\overset{\approx}{G}^{(\alpha)}(f) = \left(\overset{\approx}{G}^{(\alpha)}_{(1)}(f), \ldots, \overset{\approx}{G}^{(\alpha)}_{(1)}(f), \ldots, \overset{\approx}{G}^{(\alpha)}_{(k)}(f)\right)^{\tau}.$$

For the approximate $G(f)$ we have the inequality

$$\|G(f)\| \leqslant \|A\| \Big(m_{\varepsilon}(f) \int\limits_{\|x\| \leqslant m_{\varepsilon}(f)} |f(x)|\,dx + (m_{\varepsilon}(f))^{n-\alpha+1} \Big) \tag{8}$$

and for the approximate $\overset{\approx}{G}^{(\alpha)}(f)$ we have

$$\|G(f)\| \leqslant \|A\| \Big(m_{\varepsilon_o^{\alpha}}(f) \int\limits_{\|x\| \leqslant m_{\varepsilon_o^{\alpha}}(f)} |f(x)|\,dx + (m_{\varepsilon_o^{\alpha}}(f))^{n-\alpha+1} \Big) \tag{9}$$

In searching for a good upper bound of $G_{(1)}(f)$ or $G(f)$ up to now we have left aside a problem of fixing such a value of m that will minimise the range of $G_{(1)}(f)$ or $G(f)$. Because we treat A as functionally independent from x, the only functions that are easy to be minimized are

$$\Psi_{\varepsilon}(f) = \inf_{m_{\varepsilon}(f) \in M_{\varepsilon}} \Big\{ m_{\varepsilon}(f) \int\limits_{\|x\| \leqslant m_{\varepsilon}(f)} |f(x)|\,dx + (m_{\varepsilon}(f))^{n-\alpha+1} \Big\}$$

where $M_{\varepsilon} = \{m_{\varepsilon}(f) : (\|x\| > m_{\varepsilon}(f)) \Rightarrow (\|x\|^{\alpha} |f(x)| \leqslant \varepsilon)\}$

or

$$\Psi_{\varepsilon_o^{\alpha}}(f) = \inf_{m_{\varepsilon_o^{\alpha}} \in M_{\varepsilon_o^{\alpha}}} \Big\{ m_{\varepsilon_o^{\alpha}}(f) \int\limits_{\|x\| \leqslant m_{\varepsilon}^{\alpha}(f)} |f(x)|\,dx + (m_{\varepsilon_o^{\alpha}}(f))^{n-\alpha+1} \Big\}$$

where

$$M_{\varepsilon_o^{\alpha}} = \Big\{ m_{\varepsilon_o^{\alpha}}(f) : \big(\|x\| > m_{\varepsilon_o^{\alpha}}\big) \Rightarrow \big(\|x\|^{\alpha} |f(x)| < \varepsilon_o^{\alpha}\big) \Big\}.$$

One can formulate the following problems:

p1) find $m_\varepsilon^* = \arg \Psi_\varepsilon(f)$, where $\Psi_\varepsilon(f) \equiv \Psi_\varepsilon(f, m_\varepsilon(f))$;

p2) find $m_{\varepsilon_o^\alpha}^* = \arg \Psi_{\varepsilon_o^\alpha}(f)$, where $\Psi_{\varepsilon_o^\alpha}(f) \equiv \Psi_{\varepsilon_o^\alpha}(f, m_{\varepsilon_o^\alpha}(f))$.

Fixing $\alpha = n + 2$ and denoting

$$F_{m_\varepsilon} = \int\limits_{\|x\| \leqslant m_\varepsilon} |f(x)|\, dx, \qquad F_{m_o^\alpha} = \int\limits_{\|x\| \leqslant m_o^\alpha} |f(x)|\, dx$$

We can write down

$$\Psi_\varepsilon(f) = m_\varepsilon F_{m_\varepsilon} + m_\varepsilon^{-1}, \qquad \Psi_{\varepsilon_o^\alpha}(f) = m_{\varepsilon_o^\alpha} F_{m_{\varepsilon_o^\alpha}} + m_{\varepsilon_o^\alpha}^{-1},$$

and hence

$$m_\varepsilon^* = (F_{m_\varepsilon})^{-1/2}, \qquad \Psi_\varepsilon(f) = 2(F_{m_\varepsilon})^{1/2}$$

$$m_{\varepsilon_o^\alpha}^* = (F_{m_{\varepsilon_o^\alpha}})^{-1/2}, \qquad \Psi_{\varepsilon_o^\alpha}(f) = 2(F_{m_{\varepsilon_o^\alpha}})^{1/2}.$$

If $m_\varepsilon > (F_{m_\varepsilon})^{-1/2}$, then $m_\varepsilon^* = m_\varepsilon$ and $\Psi_\varepsilon(f) = m_\varepsilon F_{m_\varepsilon} + m_\varepsilon^{-1}$.

If $m_\varepsilon < (F_{m_\varepsilon})^{-1/2}$, then $m_\varepsilon^* = (F_{m_\varepsilon})^{-1/2}$ and $\Psi_\varepsilon(f) = 2(F_{m_\varepsilon})^{1/2}$.

If $m_{\varepsilon_o^\alpha} > (F_{m_{\varepsilon_o^\alpha}})^{-1/2}$, then $m_{\varepsilon_o^\alpha}^* = m_{\varepsilon_o^\alpha}$ and $\Psi_{\varepsilon_o^\alpha}(f) = m_{\varepsilon_o^\alpha} F_{m_{\varepsilon_o^\alpha}} + m_{\varepsilon_o^\alpha}^{-1}$.

If $m_{\varepsilon_o^\alpha} < (F_{m_{\varepsilon_o^\alpha}})^{-1/2}$, then $m_{\varepsilon_o^\alpha}^* = (F_{m_{\varepsilon_o^\alpha}})^{-1/2}$ and

$$\Psi_{\varepsilon_o^\alpha} f = 2 \left(F_{m_{\varepsilon_o^\alpha}} \right)^{1/2}.$$

Thus we can better approximate $G_1(f)$ or $G(f)$ if we use as values of integration ball radius the values m^*_ε or $m^*_{\varepsilon_o^\alpha}$.

For this pair of values the inequalities (8) and (9) will pass into the forms

$$\|G(f)\| \leqslant \|A\| \Psi_\varepsilon(f) \tag{8a}$$

$$\|G(f)\| \leqslant \|A\| \Psi_{\varepsilon_o^\alpha}(f) \tag{9a}$$

The norms in the above inequalities are Euclidean norms. In replacing these norms with the supremum norms one should remember about preserving the truth of implications analogues to the implications (4)-(9 a). In choosing α it should be remembered that the following inequalities are to be hold

i1) $\alpha > n + 1$ for $f_o(x)$, $f_1(x)$, $f_2(x)$;

i2) $\nu + n > \alpha > n + 1$ for $f_3(x)$.

3. APPLICATIONS

The approximation method from § 2 (described in a few versions in dependence on ways of calculating values of integration ball radius) enables calculations of values of vector-valued function $G(f)$. In statistics and econometrics $G(f)$ may denote expectation value of vector-valued estimator, predictor. For instance, $A = A_1, A_2$ are two versions of shapes of A and

j1) $A_1 X = B$, where $B = (Z^\tau Z)^{-1} Z^\tau X$, $A_1 = (Z^\tau Z)^{-1} Z^\tau$, $Z \in R^{nxk}$,

j2) $A_2 X = \hat{X}$, where $\hat{X} = ZB = Z(Z^\tau Z)^{-1} Z^\tau X$, $A_2 = Z(Z^\tau Z)^{-1} Z^\tau$.

The estimator B and predictor $\hat{X}$ are well known in the context of statistics and econometrics.

The described (in § 2) approximation method enables to calculate, for example,

k1) values of $\tilde{G}_1(f_i) = (\tilde{G}_{1,1}(f_i), \ldots, \tilde{G}_{1,l}(f_i), \ldots, G_{1,k}(f_i))^\tau$,

where $\tilde{G}_{1,l}(f_i) = \|a_l\| \int_{R^n} \|x\| \, |f_i(x)| \, dx$, $i \geqslant 0$ (see § 1-2);

k2) values of $\tilde{\tilde{G}}_1(f_i)$, $\tilde{\tilde{G}}_1^{(\alpha)}(f_i)$, $i \geqslant 0$ (see § 1-2);

k3) values of $\tilde{G}_1(f_{o,i})$ $\tilde{\tilde{G}}_1(f_{o,i})$, $\tilde{\tilde{G}}_1^{(\alpha)}(f_{o,i})$, $\tilde{G}_1^{(\alpha)}(f_{o,i})$, $i > 0$,

where $f_{o,i} \equiv f_o - f_i$, $i \neq 0$.

Notice 1. In calculating $m_\varepsilon(f_o - f_i) \equiv m_\varepsilon(f_{o,i})$ one can use inequality $m_\varepsilon(f_{o,i}) \leqslant m_{\varepsilon/2}(f_o) + m_{\varepsilon/2}(f_i)$.

Notice 2. Knowledge of values from (k3) helps us to evaluate an influence of density function shape's change on changes in values of moments of random vector functions where these moments, by definition, depend on this shape. Because the value

$$\|x\| \leqslant m_\varepsilon(f) \int |f_o(x) - f_i(x)| \, dx$$

cnaracterizes, in some way the distance between two probability measures $P_X^{(o)}$ and $P_X^{(i)}$, therefore, the described approximation method can be used in evaluating robustnees of moments on shapes changes of density functions.

Replacing f_o, f_j by characteristic functions of probability measures, i.e. $\varphi_X^{(o)}(t) \equiv \mathcal{E}_{(o)} \exp(i < t, X >) = \int_{R^n} e^{i<t,x>} P_X^{(o)}(dx)$,

$$\varphi_X^{(j)}(t) = \mathcal{E}_{(j)} \exp(i<t,X>) \equiv \int_{R^n} e^{i<t,x>} P_X^{(j)}(dx), \quad j \neq 0,$$

and using known formulas of regaining moments from $\varphi_X^{(t)}$ we can extend the list of possible usages to the class of non-continous probability distributions.

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APROKSYMACJA WARTOŚCI OCZEKIWANEJ WEKTOROWYCH STATYSTYK

Celem artykułu jest opis pewnej numerycznej metody aproksymacji pierwszego momentu wektorialnych statystyk i analiza warunków jej stosowalności w badaniach odpornościowych.

Poszczególne wersje metody różnią się w sposobie określania promienia "m" wielowymiarowej kuli całkowania związanego z całką $\int_{||x||\leqslant m} f(x)\,dx$, gdzie $f(x)$ oznacza zadany kształt gęstości wektora losowego X.

Podano sugestie zastosowań opisanej metody.