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MODELLING AND PROJECTION OF THE TRADE SHARES OF THE IN CMEA FOREIGN TRADE

1. Introduction

One of the main problems occurring in the process of building the system of national econometric models for a group of countries is the problem of linkage of these models into one consistent system. This function can be performed by the trade share matrix approach.

The purpose of this paper is the presentation and comparison of the application of alternative methods for the linkage of national econometric models through the trade share matrix approach in the CMEA model which has been built in the Institute of Econometrics and Statistics, University of Łódź [3].

2. Trade Share Matrix

The centerpiece of the trade share matrix approach is a trade share matrix X which permits the calculation of market shares.

Let X_{ij} (i, j = 1, ..., n) be the trade flow from country i to j. These elements can be arranged in an n x n matrix of trade flows, where n is the number of countries to be distinguished in the model (Table 1). If there are distinguished only countries (not regions), the value of diagonal elements X_{ii} will be equal zero.

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Table 1

Trade share matrix

Exports \ Imports	1	2	3	...	n	E_i
1	X_{11}	X_{12}	X_{13}	...	X_{1n}	E_1
2	X_{21}	X_{22}	X_{23}	...	X_{2n}	E_2
3	X_{31}	X_{32}	X_{33}	...	X_{3n}	E_3
⋮	⋮	⋮	⋮		⋮	⋮
n	X_{n1}	X_{n2}	X_{n3}	...	X_{nn}	E_n
M_j	M_1	M_2	M_3	...	M_n	X

The total imports M_j of country j are given by the column sums:

$$M_j = \sum_i X_{ij}, \ i, j = 1, \ldots, n \qquad (1)$$

and the total exports E_i of country i are found as the row sums:

$$E_i = \sum_j X_{ij}, \ i, j = 1, \ldots, n \qquad (2)$$

The world exports (imports) or the total exports (imports) of a given group of countries are given:

$$X = \sum_i E_i = \sum_j M_j = X, \ i, j = 1, \ldots, n \qquad (3)$$

Relation (3) is called the condition of consistency[1].

For the purposes of analysis and prediction of the structure of trade matrix X it is useful to define certain coefficients ([1], [5], [6], [7]).

[1] In fact, there exist statistical discrepancies between total exports and imports due to differences in valuation methods (fob, cif), differences in timing etc.

The matrix of import coefficients λ_{ij} is obtained by dividing each column by its column sum (imports):

$$\lambda_{ij} = \frac{X_{ij}}{M_j}, \quad \sum_i \lambda_{ij} = 1, \quad i, j = 1, \ldots, n \tag{4}$$

Thus λ_{ij} is the share of the total imports of country j that is supplied by country i. The matrix of export coefficients $\bar{\lambda}_{ij}$, describing the regional distribution of the exports of country i, is defined in the following way:

$$\bar{\lambda}_{ij} = \frac{X_{ij}}{E_i}, \quad \sum_j \bar{\lambda}_{ij} = 1, \quad i, j = 1, \ldots, n \tag{5}$$

For some purposes other coefficients seem to be interesting:

$$\beta_{ij} = \frac{X_{ij}}{X}, \quad \sum_i \sum_i \beta_{ij} = 1, \quad i, j = 1, \ldots, n \tag{6}$$

$$\delta_{ij} = \frac{\frac{X_{ij}}{X}}{\frac{E_i}{X} \cdot \frac{M_j}{X}}, \quad i, j = 1, \ldots, n \tag{7}$$

The β_{ij} coefficients describe the geographical structure of trade, and the δ_{ij} denote the relation of the X_{ij} to the total value of trade, weighted by the shares of total exports of country i and total imports of country j in the total value of trade X.

Using equations (4) and (5) the following balance equations can be written[2].

$$E_i = \sum_j \lambda_{ij} M_j, \quad i, j = 1, \ldots, n \tag{8}$$

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[2] These equations perform the same role as the balance equations in the input-output model.

$$M_j = \sum_i \bar{\lambda}_{ij} E_i, \ i, j = 1, \ldots, n \tag{9}$$

$$X = \sum_i \sum_j \lambda_{ij} M_j = \sum_j \sum_i \bar{\lambda}_{ij} E_i, \ i, j = 1, \ldots, n, \tag{10}$$

These relations are used to construct export or import predictions.

In most international trade models (e.g. Project LINK [6], EPA [1]) the predictions of trade are constructed in two main stages:

1^0 at the first stage the specification of import equations is performed (at the level of construction of particular country models)

2^0 at the second stage, having the predicted imports obtained as described above and the matrix of import coefficients λ_{ij} the values of exports are determined by equation (6)[3].

3. Methods of Linkage of Models Through the Trade Share Matrix

The exports of individual country i can be determined for the period t by the following relation:

$$E_i^t = \sum_i \lambda_{ij}^t \cdot M_j^t, \ i, j = 1, \ldots, n \tag{11}$$

where the import coefficients λ_{ij} and imports must be given for this period. The data connected with the trade flows are usually lagged to the rest of the data. Hence, the matrix of import coefficients λ_{ij} is constructed for earlier period than the period t, defined further as a base period.

[3] Of course, it is also possible to determine imports having the export equations and export coefficients.

Two possible assumptions can be taken into account:

1^o λ_{ij} does not change over time,

2^o λ_{ij} changes over time.

In the first case, the exports in the period t are equal

$$E_i^t = \sum_i \lambda_{ij}^o \cdot M_j^t \text{ , } i, j = 1, \dots, n, \tag{12}$$

where

λ_{ij}^o - the import coefficients in the base period.

This method is called a "naive method" and is usually applied as a basis for comparisons.

In the second case (typical in practice) the problem of determining λ_{ij} in the period t occurs. Some methods of determination of the λ_{ij}^t by the direct prediction and estimation are presented below.

3.1. Birpoportional Methods of Constructing the Balanced Predictions

The iterative methods of constructing the balanced predictions assume that the import coefficients matrix in the base period $\lambda^o = [\lambda_{ij}^o]$, the vector of the total imports in the period t, M^t, the vector of the total exports in the period t, E^t are known. The obtained prediction of the matrix $\hat{\lambda}^t$ is the balanced prediction, i.e. satisfying the condition that the sum of the predictions of rows (columns) is equal to the prediction of its sum. This can be written as follows:

$$\begin{cases} E_i^t = \sum\limits_j \hat{\lambda}_{ij}^t \; M_j^t \\ \sum\limits_j \hat{\lambda}_{ij}^t = 1, \quad i, j = 1, \dots, n \\ \sum \sum \hat{\lambda}_{ij}^t \; M_j^t = X \end{cases} \tag{13}$$

Hence $\hat{\lambda}^t = \hat{\lambda}^t_{ij}$ satisfies the consistency condition (3). The main role among these methods is played by the set of methods based on the c l a s s i c a l R A S m e t h o d.

The modifications of the RAS method allow to apply the RAS to the part of coefficients only. There are distinguished "important" coefficients from the λ^o matrix and directly determined for the prediction period t. These "important" ones can be chosen using the different criteria, e.g. the coefficients indicating the significant changes over time [2]. The classical RAS method is applied to prediction of the rest of coefficients, which needs the earlier correction of the vectors M^t, E^t and the unit vector (the vector of sum of coefficients in columns) - to keep the consistency condition (3). The chosen "important" coefficients determine the trends and zeros matrix, $T = [t_{ij}]$, nxn. The number of the non-zero elements is equal to the number of chosen coefficients. The classical RAS method is applied to the matrix $\lambda'_{ij} = \lambda^o_{ij}$:

$$\lambda'_{ij} = \begin{cases} \lambda^o_{ij}, & t_{ij} = 0 \\ 0, & t_{ij} \neq 0 \end{cases} \qquad i, j = 1, \ldots, n \tag{14}$$

Finally, the prediction of the matrix $\hat{\lambda}^t = [\hat{\lambda}^t_{ij}]$ is obtained:

$$\hat{\lambda}^t_{ij} = \lambda'_{ij} + t_{ij}, \quad i, j = 1, \ldots, n \tag{15}$$

This method is called the t r e n d R A S m e t h o d.
The classical RAS method can be also applied as the method which satisfies the consistency condition of the predicted matrix $\hat{\lambda}$, obtained by the other methods, like the index or the probability methods.

The i n d e x m e t h o d assumes that the vector of the total exports E and the vector of the total imports M are known in the base and t periods. The predicted elements of the matrix λ in the period t are obtained:

$$\hat{\lambda}_{ij}^{t(1)} = \begin{cases} \sqrt{a_i b_j} \cdot \lambda_{ij}^{o} & \text{version A} \\ \dfrac{a_i + b_j}{2} \cdot \lambda_{ij}^{o} & \text{version B} \end{cases} , \quad i, j = 1, \ldots, n \qquad (16)$$

where:

$$a_i = \frac{E_i^t}{E_i^o} \text{ - export index,}$$

$$b_j = \frac{M_j^t}{M_j^o} \text{ - import index.}$$

If the obtained matrix $\hat{\lambda}^{t(1)}$ does not satisfy the consistency condition, the classical RAS method is applied at the second stage as the corrected method.

The probability method assumes that each of the elements of trade matrix $X = [x_{ij}]$, is chosen with a certain probability from the total sum of the foreign trade:

$$p_{ij} = \frac{E_i}{X} \cdot \frac{M_j}{X}, \quad i, j = 1, \ldots, n. \qquad (17)$$

Simultaneously, this method guarantees the choice of the most probable matrix for the predicted period t[4]. The predicted trade flow $x_{ij}^{t(1)}$ is determined:

[4] This method can be applied for the short-term prediction under the assumption of the independency of origin and destination of trade flows what in a case of the intra CMEA trade causes the certain underestimation of "the expexted" trade flows.

$$X_{ij}^{t(1)} = \frac{E_i^t \, M_j^t \, X^o}{E_i^o \, M_j^o \, X^t} \cdot X_{ij}^o , \quad i, j = 1, \ldots, n. \tag{18}$$

The consistent matrix $\hat{X}^t = [\hat{X}_{ij}^t]$ is obtained using the classical RAS method.

3.2. Methods of Linear Programming

The method of linear programming needs the same set of information as the classical RAS method (4). The coefficients λ_{ij}^o must be adjusted that the $\hat{\lambda}_{ij}^t$ are consistent with the observed values of exports and imports in the period t, E_i^t and M_j^t, and simultaneously the λ_{ij}^t differ the least from the corresponding λ_{ij}^o:

$$\begin{cases} \sum_i \sum_j \left| \dfrac{\lambda_{ij}^o}{\lambda_{ij}^o} - 1 \right| = \min \\ \sum_j \hat{\lambda}_{ij}^t \, M_j^t = E_i^t , \quad i, j = 1, \ldots, n \\ \sum_i \hat{\lambda}_{ij}^t \, M_j^t = M_j^t \end{cases} \tag{19}$$

The objective function (19) can be expressed in the terms of the trade flows in the absolute values:

$$\sum_i \sum_j \left| \hat{X}_{ij}^t - X_{ij}^* \right| \frac{1}{X_{ij}^*} = \min, \quad i, j = 1, \ldots, n \tag{20}$$

where

$$X_{ij}^t = \hat{\lambda}_{ij}^t \, M_j^t \qquad i, j = 1, \ldots, n$$

$$X^*_{ij} = \lambda^o_{ij} M^t_j$$

with the conditions

$$\left.\begin{aligned}\sum_j \hat{X}^t_{ij} &= E^t_i \\ \sum_i \hat{X}^t_{ij} &= M^t_j\end{aligned}\right\}, \quad i, j = 1, \ldots, n.$$

The form of (20) can be solved by the classical simplex algorithm with certain reformulations based on the introduction of active variables X^+_{ij} and X^-_{ij} which satisfy the relation:

$$\hat{X}^t_{ij} = X^*_{ij} + X^+_{ij} - X^-_{ij}, \quad i, j = 1, \ldots, n \tag{21}$$

where $X^+_{ij} \geqslant 0$ and $X^-_{ij} \geqslant 0$.
Then the problem can be written:

$$\sum_i \sum_j \left| X^+_{ij} + X^-_{ij} \right| \frac{1}{X^*_{ij}} = \min, \quad i, j = 1, \ldots, n \tag{22}$$

with the constraints

$$\left.\begin{aligned}\sum_j \left| X^+_{ij} + X^-_{ij} \right| &= E_i - \sum_j X^*_{ij} \\ \sum_i \left| X^+_{ij} + X^-_{ij} \right| &= M_j - \sum_i X^*_{ij}\end{aligned}\right\}, \quad i, j = 1, \ldots, n.$$

In the process of computations it turned out that the algorithm did not guarantee the positive signs of the trade flows and the following constraints were added:

$$\hat{X}^t_{ij} = X^*_{ij} + X^+_{ij} - X^-_{ij} \geqslant 0, \quad i, j = 1, \ldots, n. \tag{23}$$

The possibility of introducing the additional information by the new constraints is the feature of the method. However, this method transmits the influence of the "fitness" of the coefficients of the higher values being usually more stable. This method can be modified through the changes of the coefficients of the objective function with the introduction of r_{ij} elements (i, j = 1, ..., n) reflecting the stability of trade flows. The new objective function has the form:

$$\sum_i \sum_j (X^+_{ij} + X^-_{ij}) r_{ij} = \min, \quad i, j = 1, \ldots, n \tag{24}$$

with the same set of constraints.
These r_{ij} have been constructed in the following way:

$$r_{ij} = \frac{X^0_{ij}}{X^{-1}_{ij}}, \quad i, j = 1, \ldots, n \tag{25}$$

where

X^{-1}_{ij} - trade flow from country i to country j in the previous period preceding the base period.

3.3. Methods for the Estimation of Import Coefficients

The next group of methods of the imports coefficients determination are the methods based on the direct estimation of these coefficients or the trade flows. T a p l i n [11] assumes that λ_{ij} is determined by the relation of export and import prices and the elasticity of country i share in the imports of country j is constant and the same for all countries exporting to country j:

$$\lambda_{ij} = A_{ij}(PE_i/\tilde{PM}_j)^{-\varepsilon_j}, \quad i, j = 1, \ldots, n \tag{26}$$

where:

A_{ij} - constant term,
PE_i - export price of country i,

$\tilde{PM}_j$ - the average import price of country j,

ε_j - constant elasticity of imports of country j.

M o r i g u c h i [9] introduced additionally the relation between the export capacities of country i and the scale of imports of country j:

$$\lambda_{ij} = A_{ij} \, (PE_i/PCOM_{ij})^{\beta_{ij}} \, (SX_i/M_j)^{\gamma_{ij}} \qquad i,j = 1, \ldots, n \qquad (27)$$

where:

SX_i - export capacities of country i,

$PCOM_{ij}$ - export price of country i on the j import market.

M a r w a h [8] proposed an approach, where λ_{ij} was determined by the relative export prices and the relation between the total exports of country i to the world exports.

Interesting methods, applied in the LINK Projects are the following methods satisfying the consistency condition:

- the LES-type method developed by K l e i n and J o h n s o n [6],
- the Hickman-Lau method [5].

The method of the LES-type is based on the following export function:

$$E_i = \alpha_i PE_i + \beta_i \sum_j \lambda^o_{ij} M_j - \sigma_i PCOM_i + \delta_i TREND, \qquad i, j = 1, \ldots, n \qquad (28)$$

where:

λ^o_{ij} - import coefficient in the base period;

$PCOM_i$ - export price of country i; $PCOM_i = \sum_j \bar{\lambda}^o_{ij} \sum_{k \neq j} \lambda^o_{kj} PE_k$,

where $\bar{\lambda}^o_{ij}$ - export coefficients of country i in the base period.

Equation (24) has a similar structure to the expenditure function in the linear expenditure system (LES). Thus, if the parameters of equation (24) satisfy the conditions:

$$\begin{cases} \sum_i \alpha_i PE_i^t = \sum_i \sigma_i PCOM_i^t \\ \sum_i \delta_i = 0, \qquad i, j = 1, \ldots, n \\ \sum_i \beta_i \lambda_{ij}^o = 1 \end{cases} \tag{29}$$

then $\sum_i E_i^t = \sum_j M_j^t$.

J o h n s o n and K l e i n [6] proposed the application of the RAS method to modify λ_{ij}^o to satisfy the condition:

$$E_i = \sum \lambda_{ij} M_j, \qquad i, j = 1, \ldots, n \tag{30}$$

because the LES-type method concerns the total export functions only.

H i c k m a n and L a u [5] introduced the method based on the CES-type function. The index of imports M_j^* is determined in the following way:

$$M_j^* = \left[\sum_i b_{ij} X_{ij}^{-\rho_j}\right]^{-1/\rho_j}, \qquad i, j = 1, \ldots, n \tag{31}$$

where

b_{ij} - constant parameters;

$\rho_j = (1/\sigma_j - 1)$, where σ_j - elasticity of substitution of imports of country j.

The import demand function can be written as:

$$X_{ij} = \lambda_{ij}^{*o} PE_{ij}^{-\sigma_j} \left[\sum_{k=1}^{n} \lambda_{kj}^{*} PE_{kj}^{-\sigma_j}\right]^{-1} M_j, \quad i, j = 1, \ldots, n \tag{32}$$

or in the linear approximation:

$$X_{ij} = \lambda^{o}_{ij}M_j - \sigma_j X^{o}_{ij}\ (PE_{ij} - \tilde{PM}_j), \quad i, j = 1, \ldots, n \qquad (33)$$

where

$$\tilde{PM}_j = \sum \lambda^{o}_{ij}\ PE_{ij}.$$

The Hickman-Lau method satisfies the consistency condition as well.

4. The Comparison of the Alternative Methods of the Import Coefficients Prediction

The analysis of the chosen methods of the import coefficients prediction has been based on the data concerning the commodity groups of the intra-CMEA trade flows[5].

There have been distinguished seven European CMEA countries: Bulgaria, Czechoslovakia, GDR, Poland, Romania, Hungary, the Soviet Union, and the four commodity groups, according to the foreign trade statistics of the CMEA countries:

- fuels, raw materials and materials (I),
- machinery and equipment (II),
- consumer goods of the industrial origin (III),
- food and raw materials for food production (IV).

The statistical data of the trade flows have been available in current prices only. The intra-CMEA trade flows used in the computations have been based on the exporters statistics assuming the better accuracy of flows[6].

The trade share matrix of 1976 was used as the base matrix for the projection. The projections were constructed for the year 1977 (the latest year of the available data) and compared with the empirical trade matrices for this year.

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[5] At the actual stage of research the methods based on the direct estimation have not been tested because of the lack of the foreign trade price indices in the statistics of the CMEA countries. Some research has been presently undertaken concerning the specification problem without the introduction of the price variables.

[6] See comments in 1.

The matrices have been projected with the application of the following methods:

- classical RAS,
- index method (version A),
- index method (version B),
- probability method,
- linear programming method,
- modified algorithm of the linear programming.

The accuracy of the predictions has been measured by the computation of the prediction errors S_{ij} for all coefficients:

$$S_{ij} = \frac{\left|\lambda^{t}_{ij} - \hat{\lambda}^{t}_{ij}\right|}{\lambda^{t}_{tj}}, \quad i, j = 1, \ldots, n \tag{34}$$

and the mean prediction error S for the matrix

$$S^{\cdot} = \frac{\sum\limits_{i}\sum\limits_{j} S_{ij}}{n^2}, \quad i, j = 1, \ldots, n. \tag{35}$$

The mean prediction errors of the four commodity groups and the tested methods are presented in Table 2.

The comparison of the applied methods indicates that the obtained results, from the point of view of the accuracy of prediction, measured by the mean prediction error, does not differ much. In group I - fuels, raw materials and materials - the mean prediction error (except the linear programming method) is about 10%, in group II - machinery and equipment - about 6%, in group III - consumer goods of the industrial origin - about 7%, and in group IV - food and raw materials for food production - about 9%. The least mean prediction error in group II is caused by the fact that in the years 1976-1977 the import coefficients in this group were more stable than in other groups, e.g. in group I the annual changes of the coefficients were equal 100%.

Table 2

Comparison of the prediction methods of the elements of the trade share matrix for the four commodity groups in the intra-CMEA foreign trade

Method	Mean prediction errors for the commodity groups			
	fuels, raw materials and materials (I)	machinery and equipment (II)	consumer goods of the industrial origin (III)	food and raw materials for food production (IV)
RAS	0.1076	0.0598	0.0718	0.0937
Index method				
version A	0.1076	0.0598	0.0718	0.0937
version B	0.1078	0.0598	0.0717	0.0916
Probability method	0.1075	0.0598	0.0719	0.0935
Linear programming	0.1258	-	-	-
Linear programming (modified algorithm)	0.1501	-	-	-

Source: The author's calculations.

In group I (fuels, raw materials and materials) relatively the "best" results have been obtained by the application of the probability method. The linear programming method and the modified algorithm of the linear programming had been tested, but the results have been unsatisfactory (the mean prediction errors 12--15%). The modified algorithm, in spite of expectations, has produced the worse results than the classical version of the algorithm. "The adjusted" influence of this method has been concentrated on the least stable trade flows. The high value of the mean prediction error seems to be caused by the lack of constraints which would allow to exist the zero values of flows in the set of the constraints.

The results obtained for group II (machinery and equipment), as the group of relatively stable coefficients, are the same for all of the tested methods.

In group III - (consumer goods of the industrial origin), relatively the best fitted projections have been obtained by the index method, version B.

The results of computations, presenting the mean prediction errors for the methods of projection of the matrix coefficients of the intra CMEA trade allow to apply the foreign trade flows model to determine the exports of the countries in the aggregation for the four commodity groups, substituting the stochastic equations in this model. The introduction of the foreign trade flows model allows to observe and analyze the multilateral connections in the CMEA region, which can be important for the process of construction of the forecasting and simulation scenarios.

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MODELOWANE I PROGNOZOWANIE MACIERZY UDZIAŁÓW HANDLU W WYMIANIE WZAJEMNEJ KRAJÓW RWPG

Celem artykułu jest prezentacja i porównanie zastosowania alternatywnych metod łączenia ekonometrycznych modeli poprzez macierz udziałów handlu w ekonometrycznym modelu krajów RWPG.

Omówione zostały teoretyczne podstawy metody RAS, programowania liniowego oraz specyfikacji równań w zakresie współczynników importowych.

W macierzy handlu wzajemnego wyróżniono cztery grupy wyrobów (klasyfikacja RWPG). Na podstawie metod biproporcjonalnych oraz programowania matematycznego skonstruowano prognozy macierzy współczynników. Otrzymane rezultaty zanalizowano pod kątem ich przydatności do prognozowania macierzy udziałów handlu wzajemnego krajów RWPG, opierając się na uzyskanych błędach prognoz.