A C T A U N I V E R S I T A T I S L O D Z I E N S I S
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LOG-PERIODICITY AND DYNAMICS OF OPEN PENSION FUNDS

Abstract. The aim of the paper is to show that the methods of statistical physics proposed as a description of market dynamics may be used to Open Pension Funds dynamics. The theoretical concept of log-periodic function is introduced. Analysis of value dynamics of Open Pension Funds is carried out.

Key words: critical point, self-similarity, log-periodic function, Open Pension Funds.

I. DISCRETE SCALE INVARIANCE

In recent years many papers have appeared dealing with the applications of methods of statistical physics in the stock market. One of the problems discussed, is an attempt to understand the financial crashes by invoking the concepts of the theory of complex systems and critical phenomena.

In 1996 Feigenbaum and Freund [4] and Sornette, Johansen and Bouchaud [5] have independently proposed a picture of crashes as critical points in a hierarchical system with discrete scaling. When system is at critical point, it becomes scale invariant, meaning that it becomes statistically similar to itself when considered at different magnifications. The signature of self-similarity is power law, related to the scale-invariance of the system in the critical point.

The discrete scale invariance is the symmetry of a system which manifests itself so that an observable near a critical point $\Psi(x)$ as a function of control parameter x is scale invariant under the change $x \to \lambda x$ for specific choices of the magnification factor λ, i.e., a number $\mu(\lambda)$ exists such that

$$\Psi(x) = \mu(\lambda)\Psi(\lambda x). \tag{1}$$

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One of the solutions of (1) is power law $\Psi(x) = x^{\alpha}$, with $\alpha = -\frac{\log \mu}{\log \lambda}$. The factor λ belongs to an infinite but countable set of values $\lambda_1, \lambda_2, \ldots$ that can be written as $\lambda_n = \lambda^n$. λ is the fundamental scaling ratio determining the period of the resulting log-periodicity.

The most general solution of (1)

$$\Psi(x) = x^{\alpha} P\left(\frac{\ln x}{\ln \lambda}\right) \tag{2}$$

where $P\left(\frac{\ln x}{\ln \lambda}\right)$ is an arbitrary periodic function of period 1 in the argument, hence the name log-periodicity. In the stock market, x represents a distance to the critical point t_c

$$x = |t - t_c| . \tag{3}$$

If we have three consecutive minima or maxima x_n of the log-periodic oscillations, we will determine the linear scale from the relation

$$\lambda = \frac{x_{n+1} - x_n}{x_{n+2} - x_{n+1}} . \tag{4}$$

The equations (3) and (4) for $t < t_c$ can be used to determine t_c

$$t_c = \frac{t_{n+1}^2 - t_{n+2} t_n}{2t_{n+1} - t_n - t_{n+2}} . \tag{5}$$

Several potential sources can be listed for discrete scaling: hierarchical structure of investors, investors with a specific time horizon or periodic events like market closure and opening, quarterly publication of reports, etc. The application of log-periodicity functions to describe financial market is very interesting because it potentially offers a tool for predictions. This method assumes that a pattern of log-periodic oscillations, which characterize particular small crashes, leads to a more serious crash which potentially may occur a year or two later. The hierarchical structure on the stock market may be distored by many external factors, so the searches for the long time precursors of crashes have to be taken with some reservatish. In our calculations we look for the precursors of crashes during two years before a crash.

Due to periodicity, the function (2) may be expanded in Fourier series

$$P\left(\frac{\ln x}{\ln \lambda}\right) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(\omega x) + b_n \sin(\omega x)\right). \tag{6}$$

Specifically, for financial bubbles prior to large crashes, we shall see that the first order representation of eq. (6)

$$I(t) = a + b(t_c - t)^{\alpha} + c(t_c - t)^{\alpha}\cos(\omega \ln(t_c - t) + \varphi) \tag{7}$$

captures well the behaviour of the market price $I(t)$ prior to a crash or large correction at a time $\approx t_c$. A detailed analysis of description of stock market with the use of log-periodicity is introduced in works [1]–[3], [6].

II. VALUE DYNAMICS OF OPEN PENSION FUNDS

Due to the fact that Open Pension Funds (OPF) invest in stock market, we postulate that the value dynamics of OPF could be described by the log-periodicity function. The price fluctuation of stock market should influence the unit of value of this OPF, to some extent depending on the portfolio structure of a particular OPF. The portfolio structure of OPF is given in Table 1.

It follows from the portfolio structure that the Polsat fund has the highest percentage of assets invested in stock market. Under analysis of nonlinear regression we obtain the parameters of equation (7) of Polsat (Table 2). For the analysis we have taken three consecutive minima of unit value of the log-periodic oscillations during 03.2006–09.2007. The calculation shows that the linear scale λ is equal 1,54 and the critical time fell on 20.06.2008. It was the beginning of the period of sharp decline of unit value. In next three months the unit value of Polsat fell by 15%.

An important aspect in the analysis of price dynamics is effectiveness of Open Pension Funds. The rate of return of individual OPF is presented in Table 3. The most effective funds are: Polsat, ING and Generali so the value dynamics for them are very similar. The Pocztylion was the last in the OPF ranking and it invests 26,92% assets in stock market.

Table 1. The portfolio structure of OPF in December 2009.

OPF	Assets [million PLN]	Bond and Note [%]	Deposits [%]	Stocks [%]	National Investment Funds [%]	Others [%]
AEGON	7275.10	63.13	2.41	28.18	0.00	2.21
Allianz	5052.91	62.00	1.44	31.43	0.00	2.05
Amplico OPF	13702.80	53.88	6.75	29.24	0.04	0.16
Aviva	45188.50	61.42	2.03	29.36	0.39	1.19
AXA	9251.35	66.72	0.44	28.05	0.28	0.98
Bankowy	4911.80	64.75	1.86	29.66	0.00	2.05
Generali	7724.89	74.92	1.94	20.40	0.45	1.48
ING	43245.30	61.06	1.26	31.86	0.05	0.00
Nordea	6905.62	51.17	1.51	29.42	0.39	6.02
Pekao	2841.07	59.96	0.82	32.61	0.34	3.19
Pocztylion	3477.32	68.99	1.21	26.92	0.15	1.45
Polsat	1689.55	60.68	1.30	36.73	0.00	0.18
PZU Złota Jesień	24751.30	63.01	1.39	30.60	0.14	3.18
Warta	2612.59	62.76	1.46	29.91	0.11	3.46

Source: http://www.money.pl

Table 2. The parameters of log-periodic function of Polsat.

Parameter	Parameter Estimate	Standard Error Estimate	T-statistic	PValue
a	3.96557	0.00260405	1522.85	0
b	-0.0572758	0.0127952	-4.47635	0.0000116595
c	0.0439197	0.00646101	6.79766	2.21814×10^{-9}
α	0.531841	0.0327903	16.2195	0
φ	21.3439	0.139499	153.004	0

Source: Own calculations.

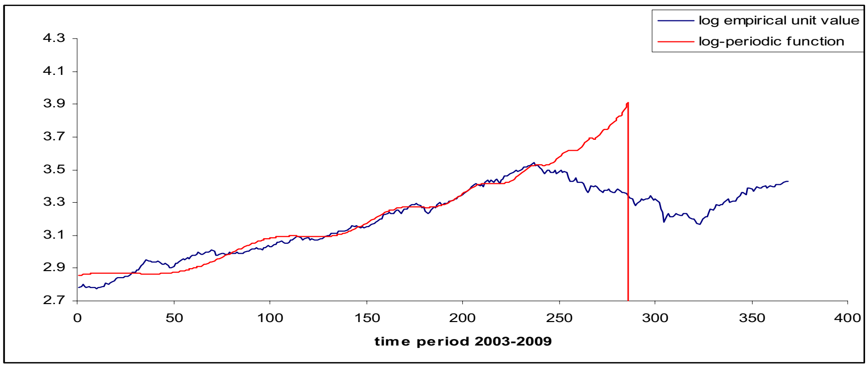

Figure 1. Time series of the logarithm of unit value of Polsat in period 2003–2009 and corresponding log-periodic function

Source: Own calculations.

Table 3. The rate of return of OPF period 1999–2009

OPF	Rate of return [%]
Polsat	+201.70
ING	+192.10
Generali	+190.10
WARTA	+178.80
Nordea	+178.80
PZU Złota Jesień	+177.90
AXA	+173.70
AEGON	+171.70
AVIVA CU	+171.40
Bankowy	+165.50
Amplico AIG	+165.00
Pekao	+162.80
Allianz	+159.10
Pocztylion	+156.90

Source: http:// www.money.pl

In Pocztylion case, the linear scale is equal 1,5 and the critical time fell on 25.07.2008.

All parameters of log-periodic function are statistically significant in both cases. The fit degree the log-periodic function to empirical data is greater for Polsat and is equal 94,55%; for Pocztylion the fit degree is 90%.

Table 4. The parameters of log-periodic function of Pocztylion.

Parameter	Parameter Estimate	Standard Error Estimate	T-statistic	PValue
a	3.53983	0.00252221	1403.46	0
b	–0.011607	0.0022543	–5.14881	5.32686×10^{-7}
c	–0.0561473	0.0107131	–5.24099	1.26888×10^{-6}
α	0.761385	0.0307785	24.7375	0
φ	0.941505	0.190437	4.94392	4.1404×10^{-6}

If we compare value dynamics of OPF with index value of WIG20, we will see that the consecutive minima of unit value are almost in the same time (Figure 3). It follows that unit value of OPF reacts to market fluctuation very strongly and the impairment of OPF values follows few days after the impairment of stock prices.

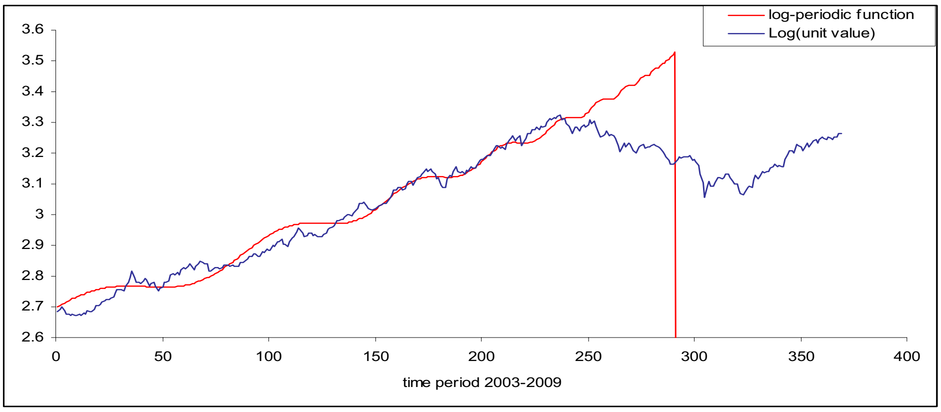

Figure 2. Time series of the logarithm of unit value of Pocztylion in period 2003–2009 and corresponding log-periodic function

Source: Own calculation.

Figure 3. Time series of unit value Polsat, Pocztylion and price of WIG20 in period 2003–2009

Source: own calculations.

III. CONCLUSION

The analysis presented above provides arguments for the existence of the log-periodic component in OPF dynamics and self-similarity on various time scale. Furthermore, the scaling factor – may very well be a constant close to1,5 for the given financial crash. We must remember that the self-similarity appears only when system is close to the critical point, so we can predict some crash only if it happens one or two years later. The form of the log-periodic modulation is non-robust on some external factors such as wars or political events.

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LOG-PERIODYCZNOŚĆ A DYNAMIKA OTWARTYCH FUNDUSZY EMERYTALNYCH

W pracy przedstawiona jest metoda opisu dynamiki Otwartych Funduszy Emerytalnych, zaczerpnięta z fizyki statystycznej. W modelu tym zakłada się, że układ znajdujący się w pobliżu punktu krytycznego (krachu finansowego) daje się opisać funkcją niezmienniczą na skalowanie. Jednym z rozwiązań równania symetrii skalowania jest iloczyn funkcji potęgowej i periodycznej.