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# INEQUALITY AND WELFARE EFFECTS OF CHANGES IN INCOME COMPONENTS IN POLAND

**Abstract.** Abbreviated social welfare functions, dependent on average income and a concentration coefficient, are a simple tool used for comparisons of income distributions. The social welfare function proposed by Sen is based on the Gini ratio which is considered to be the best synthetic measure of income inequality. This function can be generalized by introducing the parameter measuring the level of inequality aversion.

The aim of our work was to testify the influence of particular income sources on overall income inequality and social welfare. To do this, the Gini index decomposition by income components was used. It allowed examining how policy changes concerning income distributions can be assessed in terms of their effect upon both inequality and the level of social welfare. The calculations were based not only on family income but also on equivalent income taking into account family size.

**Key words:** the Gini index, social velfare, income concentration.

## I. INTRODUCTION

Household income is an economic category which is the aggregate of various components having different contribution to total income and its distribution.

The aim of our work was to analyze the influence of particular income components on overall income inequality and social welfare. To do this, the Gini index decomposition by income components was used. It allowed to examine how policy changes concerning income distributions can be assessed in terms of their effect upon both inequality and the level of social welfare. The calculations were based not only on family income but also on equivalent income taking into account family size.

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## II. MEASURING SOCIAL WELFARE

The social welfare function most often considered has a utilitarian form and is defined as the sum of individual utility functions of income $U(y)$:

$$W=\sum_{i=1}^{n}U(y_i) \tag{1}$$

Other approaches propose social welfare measures not aggregated from individual utility functions. Motivation for such measures is their simplicity and clear economic interpretation. The simplest example can be the following measure of social welfare:

$$W=Y_1+Y_2+...+Y_n \tag{2}$$

where:
$W$ – social welfare,
$Y_i$ – individual income in a $n$-element population.

In this case maximsing social welfare means maximising the total income of a society without regard to the level of inequality. Another extreme form of a social welfare function is the maximin function based on the works of John Rawls (1974):

$$W=\min(Y_1, Y_2,...,Y_n) \tag{3}$$

Here, maximising social welfare would mean maximising only the income of the poorest member of the population (extreme inequality aversion).

The abbreviated social welfare function proposed by Sen (1973) is a compromise between the above extreme approaches. It is a function of mean income $\mu$ and a measure of income concentration $I$:

$$S=\mu\ (1-I) \tag{4}$$

As a measure of income inequality various concentration measures can be used. Sen proposed the Gini ratio, while Foster (1996) suggested the Atkinson index based on "equally distributed equivalent" connected with the coefficient of inequality aversion. The Sen function can be also generalized to reflect different levels of inequality aversion. Such a generalized form can be used together with the Gini index which is considered to be the best synthetic measure of income inequality:

$$S(\mu G)=\mu(1-\varepsilon G),\ \varepsilon>0 \tag{5}$$

where: $\varepsilon$ – parameter of aversion to inequality (altruism).

The higher the value of $\varepsilon$ is, the greater reduction in mean income can be accepted by a society in return for a decreasing level of inequality.

## III. THE GINI INDEX OF CONCENTRATION

The Gini coefficient or index (Gini, 1912) is perhaps one of the most frequently used indicators of social and economic condition. This measure is understood by many economists and has been applied in several numerical studies and policy research. The Gini index can be used to measure the dispersion of a distribution of income, or consumption, or wealth or a distribution of any other kinds.

It can be expressed as a ratio of two regions defined by a line of equal shares and a Lorenz curve in a unit box (Gini, 1912; Lorenz, 1905), or a function of Gini mean difference (Gini, 1912), or the covariance between incomes and their ranks (Stuart, 1954, ; Lerman, Yitzhaki 1984, 1985).

The Gini coefficient can be expressed in terms of the area under the Lorenz curve, where the Lorenz curve relates the cumulative proportion of income units to the cumulative proportion of income received when the units are arranged in ascending order of their income:

$$G = 1 - 2\int_0^1 L(p)dp \tag{6}$$

where: $L(p)$ is the Lorenz curve.

In this sense the Gini index is double the area between the Lorenz curve and the line of equal shares. The geometric approach can be related to the statistical approach via a concept called mean difference:

$$G = \frac{\Delta}{2\mu_y} \tag{7}$$

where: $\Delta$ is Gini mean difference that can be written as follows:

$$\Delta = \int_0^\infty \int_0^\infty | x - y | \, dF(x)dF(y) \tag{8}$$

where:
$x, y$ – income variables identically distributed,
$F(x), F(y)$ – cumulative distribution function.

Gini mean difference can be interpreted as the sum of all possible absolute differences in a population of income receivers. Integrating the formula (6) by parts we obtain:

$$G = 2\int_0^1 pL'(p)dp - 1 \tag{9}$$

Suppose now we transform the variables with the substitution of $p = F(y)$ where $F(y)$ is the cumulative distribution of income:

$$G = -1 + 2\int_0^\infty \frac{yF(y)f(y)dy}{\mu_y} \tag{10}$$

where:
$f(y)$ is the density function of income,
$\mu_y$ is mean income.

From the formula for covariance between two random variables $X$ and $Z$ we have $E(XZ) - E(X)E(Z)$. Then we let $X$ be income ($y$) and $Z$ be $F(y)$ we have:

$$\mathrm{cov}[y, F(y)] = \int_0^\infty yF(y)f(y)dy - \frac{\mu_y}{2} \tag{11}$$

Combining (10) and (11) we obtain the formula expressed in terms of co-variance between incomes and their ranks:

$$G = 2\frac{\mathrm{cov}[y, F(y)]}{\mu_y} \tag{12}$$

The formula given above will be a starting point to the Gini index decomposition by income components.

## IV. INEQUALITY EFFECTS OF CHANGES IN SOURCES OF INCOME

Let $y_1 \ldots y_k$ represent components of income. Then using the covariance formula given in (12) we can write:

$$G = 2\frac{\sum_k \mathrm{cov}[y_k, F(y)]}{\mu_y} \tag{13}$$

where: $\mathrm{cov}[y_k, F(y)]$ is the covariance of income component *k* with the cumulative distribution function of income.

Multiplying and dividing each component *k* by the covariance between income component $y_k$ and the cumulative distribution of that component $F(y_k)$ and $\mu_k$ we obtain:

$$G = \sum_{k=1}^{K} \frac{\mathrm{cov}[y_k, F(y)]}{\mathrm{cov}[y_k, F(y_k)]} \cdot \frac{2\,\mathrm{cov}[y_k, F(y)]}{\mu_k} \cdot \frac{\mu_k}{\mu} \tag{14}$$

$$G = \sum_{k=1}^{K} R_k G_k W_k$$

where:
$R_k$ – the Gini correlation between income component *k* and the total income, $-1 \le R_k \le 1$
$G_k$ – Gini index for a component *k*,
$W_k$ – component *k*'s share of the total income.
$R_k G_k W_k$ – component *k's* share of the total income.

Using the above decomposition of the Gini index we can examine how changes in particular income sources will affect overall inequality. To do this let us suppose that we have an exogenous change in each household's income component *j* by a factor *e*, such that $y_{j=}$ (1+e)$y_j$. Then (see: Stark, Taylor, Yitzaki, 1986):

$$\frac{\partial G}{\partial e} = W_j (R_j G_j - G) \tag{15}$$

and the proportional change will be given by:

$$\frac{\partial G / \partial e}{G} = \frac{S_j G_j R_j}{G} - S_j \tag{16}$$

It can be easily noticed that an increase in component *j* will decrease total inequality *G* when $R_j$ is negative or zero. That means that there is not positive correlation between $y_j$ and *y*- the rankings of households according to total income and selected income component are not similar.

## V. INFLUENCE OF CHANGES IN INCOME COMPONENTS ON SOCIAL WELFARE

On the basis of the abbreviated social welfare function proposed by Sen (5) and using the transformation of income component $j$ by a factor $e$ we obtain:

$$\frac{\partial S}{\partial e}=\frac{\partial \mu}{\partial e}(1-\varepsilon G)-\varepsilon\mu\frac{\partial G}{\partial e} \tag{17}$$

Hence, using the formula (15):

$$\frac{\partial S}{\partial e}=\mu_j(1-\varepsilon R_j G_j) \tag{18}$$

The proportional changes in Sen social welfare function can be written as follows:

$$\frac{\partial S/\partial e}{S}=W_j\frac{1-\varepsilon R_j G_j}{1-\varepsilon G} \tag{19}$$

Using these derivatives we can evaluate the influence of proportional changes in income components not only on overall inequality measured by $G$ but on the level of social welfare as well. All the parameters of equation (19) can be estimated from the data except for inequality aversion $\varepsilon$.. Given $\varepsilon=1$ equal weighs are attached to equity and mean income. The higher is the value of this parameter the higher weight is attached to equity- society prefers smaller inequality rather than higher income.

## VI. APPLICATION

The methods mentioned above were applied to the income data coming from the Household Budgets Survey conducted by Polish Central Statistical Office in 2005. All the calculations were made not only for family income but also for eqivalent income which was obtained by means of the OECD eqivalence scale. The results are presented in tables and in figures.

Tables 1 and 2 present the measures describing the influence of particular income components on overall inequality and social welfare. Inequality was measured by the Gini ratio while social welfare was measured by the abbreviated social welfare function proposed by Sen. The columns 4 and 5 show contributions of component $j$ to the Gini coefficient while the column 8 presents proportional marginal effects of changes in particular income components on overall inequality *G*. The last three columns present the marginal effect of changes in income components on social welfare for different values of inequality aversion parameter.

Figure 1 and 2 describe the structure of total family income and equivalent income from the point of view of income components. Figures 3 and 4 present the structure of the Gini coefficient by sources of income showing the contribution of particular components to overall Gini coefficient.

The last two figures show the structure of marginal welfare effects connected with proportional changes in income components.

## VII. CONCLUSIONS

The main sources of income concentration in Poland are wages and salaries-$I_j$=0.6088. That means that this income source contributes to the Gini index in 60,88%. Is is connected with high positive correlation with the total income. Moreover, this component's share in household income is relatively high. Income from *social services* is negatively correlated with family income what results in negative contribution to overall inequality ($I_j$=–0.0207). Thus, the increase of inequality within this income source by 1% reduces overall inequality by 0,2% (see column 8). Similar situation can be observed for income from *social insurance* (retirees' pensions, old-age pensions etc.). For this component the Gini correlation is positive but very small, and the share in total income is high.

Taking into consideration the influence of various income components on social welfare it can be easily noticed that *wages and salaries* and *social insurance* are the two main sources of social welfare growth. When wages and salaries grow up by 1% Sen social welfare function increases by 0,38 % assuming that $\varepsilon$ =1. For *social insurance* this value is similar (see: column 10). Inequality aversion level $\varepsilon$ =2 (see: column 11) results in higher positive influence on social welfare for these income sources for which the level of Gini index is relatively low, while for sources with extremely high Gini the impact on overall welfare is negative (*self employment, property income*). The income increase within *social insurance* by 1% leads to an increase of welfare by 0.7%. It is the result of the highest negative impact of this income source on overall inequality (column 5). For inequality aversion $\varepsilon$ = 0.5 the main sources of social welfare growth will be the components with high income shares regardless the high level of concentration as in the case of *wages and salaries*.

Table 1. Inequality and welfare effects of changes in sources of income

| Income source | $R_j$ | $G_j$ | $W_j$ | $R_jG_jW_j$ | $\frac{R_jG_jW_j}{G}=I_j$ | $G(I_j-W_j)$ | $I_j/W_j$ | $I_j-W_j$ | $W_j\frac{1-R_jG_j\cdot 0,5}{1-0,5G}$ | $W_j\frac{1-R_jG_j}{1-G}$ | $W_j\frac{1-2R_jG_j}{1-2G}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Wages and salaries | 0.7064 | 0.6424 | 0.4579 | 0.2078 | 0.6088 | 0.0515 | 1.3295 | 0.1509 | 0.4268 | 0.3797 | 0.1333 |
| Self-employment | 0.6821 | 0.9480 | 0.0827 | 0.0535 | 0.1566 | 0.0252 | 1.8943 | 0.0739 | 0.0675 | 0.0444 | –0.0764 |
| Property income | 0.8003 | 0.9996 | 0.0007 | 0.0005 | 0.0015 | 0.0003 | 2.3435 | 0.0009 | 0.0005 | 0.0002 | –0.0012 |
| Social insurance | 0.1171 | 0.6448 | 0.2625 | 0.0198 | 0.0581 | –0.0698 | 0.2212 | –0.2044 | 0.3046 | 0.3684 | 0.7023 |
| Social services | –0.0276 | 0.8104 | 0.0569 | –0.0013 | –0.0037 | –0.0207 | –0.0655 | –0.0607 | 0.0694 | 0.0884 | 0.1875 |
| Other social transfers | 0.2732 | 0.8282 | 0.0634 | 0.0144 | 0.0421 | 0.0073 | 0.6629 | –0.0214 | 0.0678 | 0.0745 | 0.1095 |
| Farm produce | 0.6487 | 0.9500 | 0.0726 | 0.0447 | 0.1310 | 0.0200 | 1.8054 | 0.0584 | 0.0605 | 0.0423 | –0.0532 |
| Other disposable income | 0.5823 | 0.9950 | 0.0033 | 0.0019 | 0.0056 | 0.0008 | 1.6974 | 0.0023 | 0.0028 | 0.0021 | –0.0017 |

Source: Author's calculations.

Table 2. Inequality and welfare effects of changes in sources of equivalent income

| Income source | $R_j$ | $G_j$ | $W_j$ | $R_jG_jW_j$ | $\frac{R_jG_jW_j}{G}=I_j$ | $G(I_j-W_j)$ | $I_j/W_j$ | $I_j-W_j$ | $W_j\frac{1-R_jG_j\cdot 0,5}{1-0,5G}$ | $W_j\frac{1-R_jG_j}{1-G}$ | $W_j\frac{1-2R_jG_j}{1-2G}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Wages and salaries | 0,6242 | 0,6501 | 0,4368 | 0,1772 | 0,5672 | 0,0407 | 1,2985 | 0,1304 | 0,4126 | 0,3775 | 0,2195 |
| Self-employment | 0,6525 | 0,9492 | 0,0792 | 0,0491 | 0,1570 | 0,0243 | 1,9821 | 0,0778 | 0,0648 | 0,0439 | –0,0505 |
| Property income | 0,8637 | 0,9995 | 0,0007 | 0,0006 | 0,0018 | 0,0004 | 2,7624 | 0,0012 | 0,0004 | 0,0001 | –0,0013 |
| Social insurance | 0,2315 | 0,6551 | 0,2982 | 0,0452 | 0,0147 | –0,0480 | 0,4853 | –0,1535 | 0,3266 | 0,3680 | 0,5540 |
| Social services | –0,2339 | 0,8086 | 0,0523 | –0,0099 | –0,0317 | –0,0263 | –0,6053 | –0,0840 | 0,0679 | 0,0905 | 0,1923 |
| Other social transfers | 0,3312 | 0,8367 | 0,0693 | 0,0192 | 0,0614 | –0,0025 | 0,8856 | –0,0079 | 0,0708 | 0,0729 | 0,0825 |
| Farm produce | 0,5113 | 0,9471 | 0,0601 | 0,0291 | 0,0930 | 0,0103 | 1,5487 | 0,0330 | 0,0540 | 0,0451 | 0,0051 |
| Other disposable income | 0,6002 | 0,9949 | 0,0034 | 0,0020 | 0,0065 | 0,0010 | 1,9109 | 0,0031 | 0,0028 | 0,0020 | –0,0018 |

Source: Author's calculations

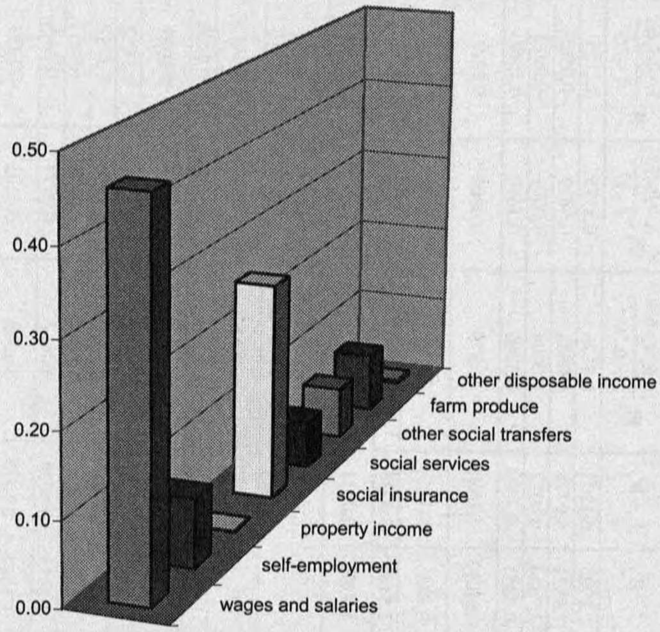


Fig.1. Structure of household income in 2005

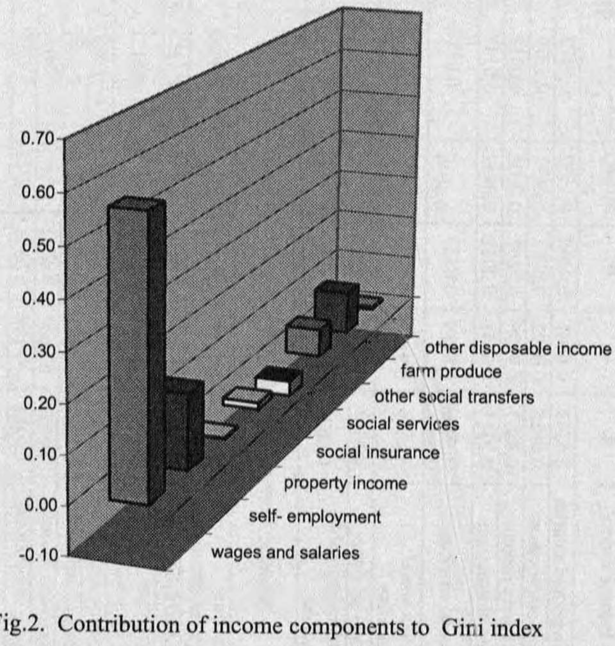


Fig.2. Contribution of income components to Gini index

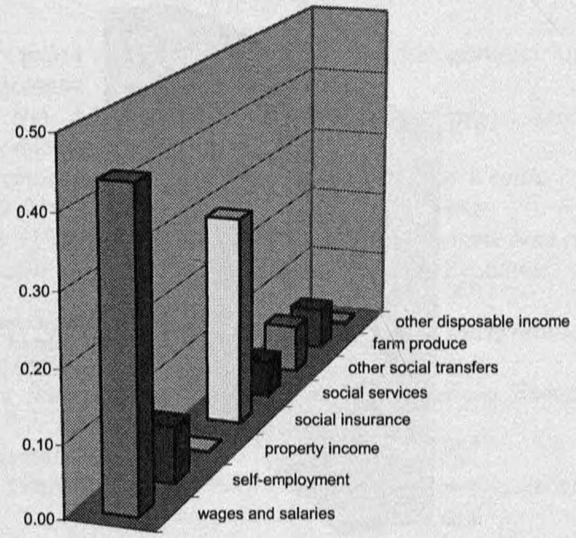


Fig. 3. Structure of equivalent income in 2005

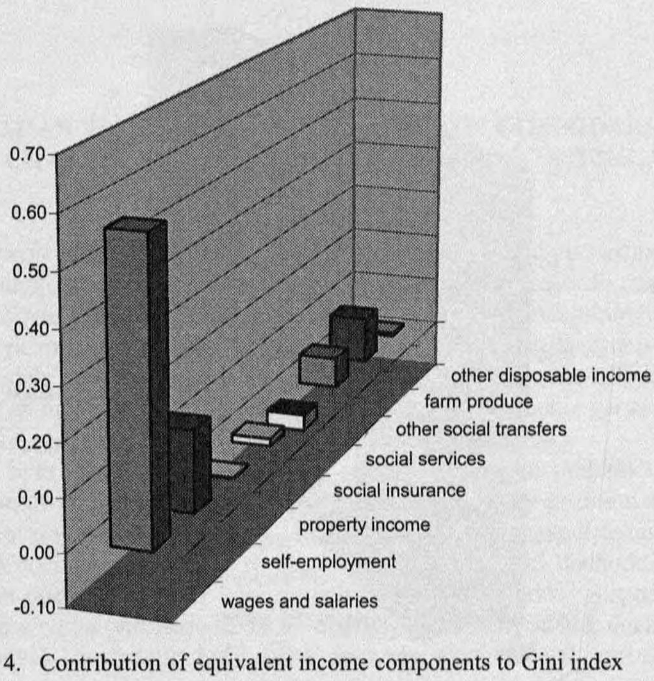


Fig. 4. Contribution of equivalent income components to Gini index

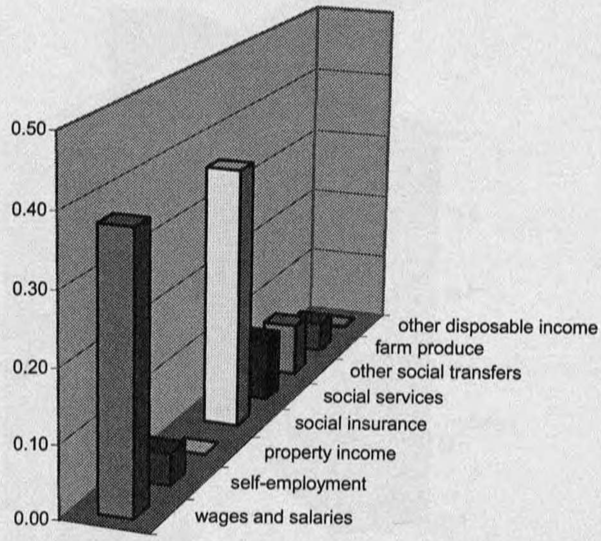


Fig. 5. Social welfare effects of changes in income components

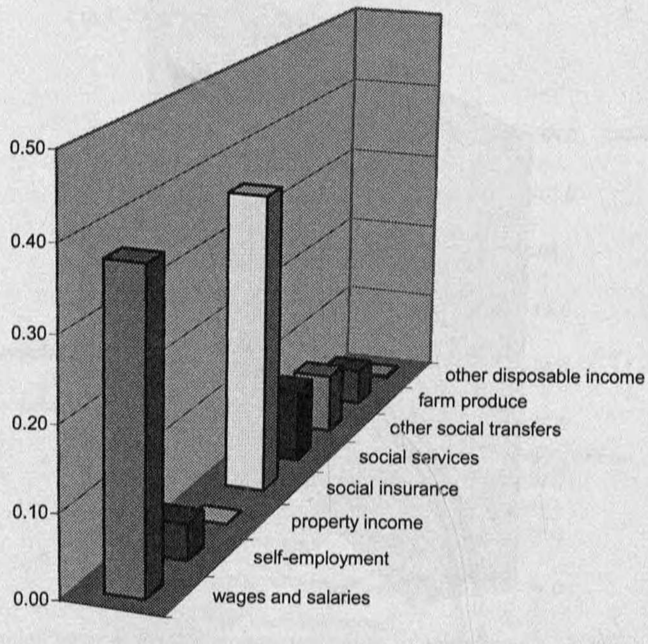


Fig. 6. Social welfare effects of changes in eguivalent income components

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## WPŁYW ZMIAN W SKŁADNIKACH DOCHODÓW GOSPODARSTW DOMOWYCH W POLSCE NA NIERÓWNOMIERNOŚĆ ROZKŁADU I DOBROBYT

Skrócone funkcje dobrobytu uzależnione od średniego poziomu dochodów oraz miary nierównomierności rozkładu stanowią proste narzędzie służące do analizy i porównywania dobrobytu społecznego. Jedną z takich funkcji jest funkcja dobrobytu społecznego Sena wykorzystująca jako miarę nierównomierności współczynnik Giniego, uznawany przez wielu statystyków i ekonomistów za najlepszą syntetyczną miarę koncentracji. Funkcja ta może być zmodyfikowana poprzez uwzględnienie parametru określającego stopień awersji do nierównomierności.

Celem pracy było zbadanie wpływu zmian w poszczególnych składnikach dochodów na zmiany nierównomierności całego rozkładu oraz na zmiany poziomu dobrobytu. Do tego celu wykorzystana została między innymi dekompozycja współczynnika Giniego według źródeł dochodu. Pozwoliło to na ocenę które składniki dochodu mają największy wpływ na poziom i zmiany koncentracji rozkładu dochodów gospodarstw domowych w Polsce a co za tym idzie także na dobrobyt społeczny. Obok dochodów gospodarstw domowych rozważane były także dochody ekwiwalentne uwzględniające efekty skali związane z rozmiarem gospodarstwa domowego.