

Marek Kaluszka*, Alina Kondratiuk-Janyska**

ON DURATION-DISPERSION STRATEGIES FOR PORTFOLIO IMMUNIZATION

Summary. This paper deals with new immunization strategies for a noncallable and default-free bond portfolio. This approach refers to the Fong and Vasicek (1984), the Nawalkha and Chambers (1996), the Balbás and Ibáñez (1998), and the Balbás, Ibáñez and López (2002) studies among others and relies on minimizing a single-risk measure which is a linear combination of the duration gap and the dispersion of portfolio payments.

Keywords: Bond portfolio, immunization, duration, M^2 , M^A .

1. INTRODUCTION

Management of interest rate risk, the control of changes in value of a stream of future cash flows as a result of changes in interest rates is an important issue for an investor. Therefore many researchers have examined the immunization problem for a bond portfolio in a situation where the investor is in debt and has to pay it off in a fixed horizon date. For simplicity, we consider the case where the liability stream consists of a single negative cash flow at some specified future date. Multiple liabilities can be handled as an extension of the single liability case by separately immunizing each of liability cash flow. The investor knows, in advance, the sum of money which he owes. An ideal situation is when the portfolio present value is equal to the discounted worth of investor's liability at the present moment and does not fall below the target value (the terminal value of the portfolio under the scenario of no change in the interest rate) at prespecified time. Early work on immunization was based upon the Macaulay definition of duration (1938) and it was shown independently by Hicks (1939), Samuelson (1945) and Redington (1952) that if the Macaulay

*Assistant Professor in the Institute of Mathematics, Technical University of Łódź.

**Assistant in the Institute of Mathematics, Technical University of Łódź.

duration of assets and liabilities are equal, the portfolio is protected against a local parallel change in the yield curve. Fisher and Weil (1971) formalized the traditional theory of immunization defining the conditions under which the value of an investment in a bond portfolio is hedged against any parallel shifts in the forward rates. The main result of this theory is that immunization is achieved if the Fisher-Weil duration of the portfolio is equal to the length of the investment horizon. A generalization of the Fisher and Weil results (1971) can be found in Montrucchio and Peccati (1991). They proved that if the set K consists of all shocks k such that the function $t \rightarrow \exp\left(\int_t^m k(s) ds\right)$ is convex, then every duration-matching portfolio is immunized. The classical theory of immunization is treated in details in Fabozzi (1993) and Panjer (1998) among others.

Immunization strategies have been also developed for alternative models of interest rate behaviours. Cox et al. (1979), Khang (1979), Bierwag and Kauffman (1977), Bierwag (1987), Chambers et al. (1988), Prisman and Schores (1988), Crack and Nawalkha (2000) and others assumed various models of interest rate movements and implied different measures of duration, which if they are equal to the holding period length, then immunization is achieved. Rządkowski and Zaremba (2000) generalized bond portfolio immunization for an additive term structure model developing a definition of duration. See also Prisman (1986), Shiu (1987), Reitano (1991, 1992), Zaremba (1998), and Zaremba and Smoleński (2000ab). The comprehensive treatment of the present state of the art can be found in Nawalkha and Chambers (1999) and Jackowicz (1999). But above approach and also all mentioned earlier have serious limitation. They imply arbitrage opportunities that are inconsistent with equilibrium – it is in contradiction to the rules of modern finance theory.

To overcome the main drawback of the traditional theory, Fong and Vasicek (1984), Nawalkha and Chambers (1996), Balbás and Ibáñez (1998) and Balbás et al. (2002) examined the effect of an arbitrary interest change on a default-free, noncallable bond portfolio. They considered shocks in a more general context i.e. they worked with differentiable functions with bounded derivative (cf. Fong and Vasicek, 1984) or with bounded functions (see Balbás and Ibáñez, 1998 and Balbás, Ibáñez and López, 2002) and found that the classically immunized portfolio (Fong and Vasicek, 1984; Balbás and Ibáñez, 1998; Balbás, Ibáñez and López, 2002) or any portfolio (Nawalkha and Chambers, 1996) had negative lower bound depending upon the different dispersion measures thereby immunization strategies were advocated. The comprehensive treatment of the present state of the art can be found in and Nawalkha and Chambers (1999).

The aim of the paper is to present a new strategy of immunization based on a single duration-dispersion risk measure (see section 3). As a by-product, we generalize the Fong-Vasicek, the Balbás-Ibáñez and the Nawalkha-Chambers inequalities that give lower limits on the change in the end of horizon value of the duration-matching portfolios.

2. PRELIMINARY NOTATIONS

Denote by $[0, T]$ the time interval with $t = 0$ the present moment. Let m be the investor planning horizon, $0 < m < T$. We write $q = (q_1, q_2, \dots, q_n)$ for the investor portfolio. This vector gives us the number of i th bond units q_i that the investor bought at time 0. The coupons paid before m will be reinvested by purchasing the considered n bonds. We assume $q_i \geq 0$ to exclude short position from the analysis.

Let c_{it} denote the time- m present value of payment of i th bond due at time t provided the current instantaneous forward rate is $g(t)$. By $V_i(k)$ we denote the time- m present value of i th bond if the shock $k = k(t)$ on the instantaneous forward rate takes place. We assume the additive model of shocks although the others can be treated in a similar way. Obviously,

$$V_i(k) = \sum_t c_{it} \exp\left(\int_t^m k(s) ds\right) \quad (1)$$

Here and below, the summation is over all t . Denote by $V(q, k)$ the value of the portfolio q at time m under the assumption the shock $k = k(t)$ appeared, i.e.

$$V(q, k) = \sum_{i=1}^n q_i V_i(k) = L \sum_t c(t, q) \exp\left(\int_t^m k(s) ds\right) \quad (2)$$

where L denotes a liability due at time m and $c(t, q) = \frac{1}{L} \sum_{i=1}^n q_i c_{it}$. Let P_i stand for the market price of the i th bond at time zero and let C denote the total amount of investment at $t = 0$. Clearly,

$$L = C \exp\left(\int_0^m g(t) dt\right).$$

Definition. A portfolio $q = (q_1, \dots, q_n)$ is called a **feasible portfolio** if

$$\sum_{i=1}^n q_i P_i = C \text{ and } q_i \geq 0 \text{ for every } i.$$

Denote by $V(q, K)$ the guaranteed value by portfolio q , that is,

$$V(q, K) = \inf_{k \in K} V(q, k) \quad (3)$$

where K means a class of feasible shocks. We say that a feasible portfolio q is immunized if $V(q, K) \geq L$.

Definition. A feasible portfolio q is called the **duration-matching portfolio** if the Fisher-Weil duration of portfolio q is equal to the investment horizon

length, i.e. $D(q) = m$, where $D(q) = \sum_t t c(t, q)$.

3. DURATION-DISPERSION PORTFOLIOS

In empirical immunization studies, duration-matching portfolios often work as well as more complex immunizing strategies. During the 1980s duration has explained 80% to 90% of the return variance for government bonds (see e.g. Ilmanen, 1991). It means that parallel movements play significant role in shocks behaviour. However, such duration-matching portfolios are not unique. How should one of them be chosen? Which portfolios produce returns with the least deviation from the promised return?

In the pioneering work, Fong and Vasicek (1984) proposed the following wider class of shocks with an arbitrary type of interest rate change, including parallel shifts:

$$K_{FV} = \left\{ k; \frac{dk(t)}{dt} \leq \lambda, 0 \leq t \leq T \right\} \quad \text{with } \lambda > 0 \quad (4)$$

They proved that if the short sale is forbidden and if q is a duration-matching portfolio, then

$$V(q, K_{FV}) \geq L \left(1 - \frac{\lambda}{2} M^2 \right) \quad (5)$$

where $M^2 = \sum_t (t - m)^2 c(t, q)$ is a dispersion measure. They concluded that the problem of immunization should be formulated as follows

P1: find a duration-matching portfolio which minimizes M^2 .

The approach is not free from a critique. Bierwag et al. (1993) and others examined the theoretical and empirical properties of M^2 in designing

duration-hedged portfolios. They found that minimum M^2 portfolios fail to hedge as effectively as portfolios including a bond maturing on the horizon date.

The class of shocks considered by Balbás and Ibáñez (1998) is of the form

$$K_{BI} = \{k; |k(t_2) - k(t_1)| \leq \lambda, 0 \leq t_1 < t_2 \leq T\} \quad (6)$$

Balbás and Ibáñez showed that for any duration-matching portfolio q

$$V(q, K_{BI}) \geq L \left(1 - \frac{\lambda}{2} \tilde{N}\right) \quad (7)$$

where $\tilde{N} = \sum_t |t - m| c(t, q)$ (cf. Balbás, Ibáñez, 1998, the formula (16) with $L = RC$). As a consequence they proposed to

P2: find a duration-matching portfolio which minimizes \tilde{N} .

With an example Balbás and Ibáñez (1998) showed that the duration-matching portfolio with minimal \tilde{N} can include a maturity matching bond (cf. the empirical results of Bierwag, Fooladi and Roberts, 1993). Balbás and Ibáñez (1998) took into account many considerations about the possible shocks on the interest rates to minimize the \tilde{N} measure.

We propose new strategies of immunization of a bond portfolio based on a single-risk measure model. Our approach is close to that of Nawalkha and Chambers (1996) because they also focused on a single-risk-measure immunization model. Define a functional A which measures the average value of shock. We assume that the functional A is equivariant, i.e. $A(k + c) = A(k) + c$ for all feasible shocks k and real c . We also assume that $A(0) = 0$. Examples will be given below. Define the following class of shocks

$$K(W, a) = \left\{ k; \int_m^t (k(s) - A(k)) ds \leq W(t), 0 \leq t \leq T, |A(k)| \leq a \right\} \quad (8)$$

Herein $0 \leq a \leq \infty$ and W is a nonnegative and convex function such that $W(m) = 0$. Observe that the class $K(W, a)$ includes all flat shocks not greater than a . Throughout the paper, $\infty \cdot 0 = 0$.

Theorem 1. For every feasible portfolio q

$$V(q, K(W, a)) \geq L \exp(-a|m - D(q)| - M^W) \quad (9)$$

where $M^W = \sum_t W(t) c(t, q)$.

Proof. Observe that

$$\begin{aligned} V(q, K(W, a)) &= L \inf_{k \in K(W, a)} \int_0^T \exp \left[\int_t^m k(s) ds \right] dC(t, q) = \\ &= L \inf_{k \in K(W, a)} \int_0^T \exp \left[A(k)(m-t) + \int_t^m (k(s) - A(k)) ds \right] dC(t, q). \end{aligned}$$

Since the short position is excluded, $t \rightarrow C(t, q) = \sum_{s \leq t} c(s, q)$ is the distribution function of a probability measure on $[0, T]$ for each q . By the Jensen inequality (see e.g. Durrett, 1996, p.14) with the distribution function $t \rightarrow C(t, q)$, we get

$$\begin{aligned} V(q, K(W, a)) &\geq L \inf_{k \in K(W, a)} \exp \left[A(k) \int_0^T (m-t) dC(t, q) + \int_0^T \int_t^m (k(s) - A(k)) ds dC(t, q) \right] = \\ &= L \inf_{k \in K(W, a)} \exp \left[A(k)(m - D(q)) - \int_0^T \int_0^t (k(s) - A(k)) ds dC(t, q) \right]. \end{aligned}$$

By the definition of $K(W, a)$

$$\begin{aligned} V(q, K(W, a)) &\geq L \exp \left[\inf_{k \in K(W, a)} [A(k)(m - D(q))] - \int_0^T W(t) dC(t, q) \right] = \\ &= L \exp (-a|m - D(q)| - M^W). \end{aligned}$$

The proof is complete. \square

As a consequence of theorem 1 we propose a new strategy of immunization:

P3: find a feasible portfolio which minimizes $a|m - D(q)| + M^W$.

Observe that the measure M^W is nonnegative (by the convexity of W and the Jensen inequality). Given a strictly convex function W , M^W is equal to zero if and only if there is one payment (at time m). Therefore M^W can be treated as a dispersion measure of the stream of portfolio payments. In other words, our strategy relies on minimizing a linear combination of the absolute value of the duration gap $|m - D(q)|$ and the dispersion measure M^W . It is clear that the larger a is, the smaller duration gap of the portfolio should be. Clearly, if $a = \infty$ then the strategy P3 is equivalent to the following one

P3': choose a duration-matching portfolio which minimizes M^W .

We now introduce a more specific class of shocks which are included in $K(W, a)$ for an appropriate chosen function W .

Example 1. Define

$$K_a(w) = \{k; k(t_2) - k(t_1) \leq w(t_2 - t_1), 0 \leq t_1 < t_2 \leq T, |k(m)| \leq a\} \quad (10)$$

where $0 \leq a \leq \infty$, $w = w(t)$ is a nondecreasing and nonnegative function such that $w(0) = 0$. Observe that $K_a(w)$ includes all parallel shocks not greater than a . It is easy to check that $K_a(w) \subset K(W, a)$, where $K(W, a)$ is defined by (8) with $A(k) = k(m)$ and $W(t) = \int_0^{t-m} w(s) ds$. From theorem 1 we obtain that

$$V(q, K_a(w)) \geq L \exp\left(-a|m - D(q)| - \sum_t c(t, q) \int_0^{t-m} w(s) ds\right) \quad (11)$$

for every feasible portfolio q . Our strategy is as follows

P4: find a feasible portfolio which minimizes $a|m - D(q)| + \sum_t c(t, q) \int_0^{t-m} w(s) ds$.

If $w = 0$ and if the portfolio duration is equal to the length of the planning horizon, then $V(q, K_a(w)) \geq L$. Thus the target value L is a lower bound of the terminal value of the portfolio regardless of any shifts in interest rates. Another reasonable choice of the function w seems to be $w(t) = \lambda t^p$ with $\lambda > 0$ for any p such that $0 < p < 1/2$ since Brownian paths are Hölder continuous with exponent p for any $0 < p < 1/2$ (see e.g. Durrett, 1996, p. 379). This leads to the problem

$$\begin{aligned} \text{minimize } a\left|m - \sum_{i=1}^n q_i D_i\right| + \sum_{i=1}^n q_i M_i \text{ subject to } \sum_{i=1}^n q_i P_i = C, q_i \geq 0, \\ i \geq 1, \end{aligned}$$

where $D_i = \frac{1}{L} \sum_t t c_{it}$ and $M_i = \frac{\lambda}{(p+1)L} \sum_t |t-m|^{p+1} c_{it}$ is the duration and the dispersion measure of i th bond, respectively. This case was considered in details by Balbás et al. (2002) with conclusion that the appropriate dispersion measures are those that $0 \leq p \leq 1$. An interesting alternative to the power

dispersion function is $w(t) = \lambda \sqrt{t \ln\left(\frac{T}{t}\right)}$ due to the following property of Brownian paths:

$$|B(t+h) - B(t)| \leq C \sqrt{h \ln\left(\frac{T}{h}\right)} \text{ almost surely,}$$

where $B(t)$ is a Brownian motion, C is a random variable and $0 \leq t \leq t+h \leq T$.

As we will see in examples 2–4, theorem 1 not only does a new strategy of immunization provide but also extends the results of Fong and Vasicek (1984) and Balbás and Ibáñez (1998).

Example 2. We now give an extension of the Fong and Vasicek result. Let $A(k) := k(m)$ and put

$$K_{FV}^*(a) = \left\{ k; \int_m^t (k(s) - k(m)) ds \leq \frac{\lambda}{2} (t - m)^2, 0 \leq t \leq T, \text{ with } |k(m)| \leq a \right\} \quad (12)$$

Of course, $K_{FV}^*(a) = K\left(\frac{\lambda}{2}(t - m)^2, a\right)$, where $K(W, a)$ is defined by (8). From theorem 1 it follows that for every feasible portfolio q

$$V(q, K_{FV}^*(a)) \geq L \exp\left(-a|m - D(q)| - \frac{\lambda}{2} M^2\right) \quad (13)$$

and for every duration-matching portfolio q

$$V(q, K_{FV}^*(\infty)) \geq L \exp\left(-\frac{\lambda}{2} M^2\right) \quad (14)$$

Observe that $K_{FV} \subset K_{FV}^*(\infty)$, where K_{FV} is defined by (4). In fact, for every $k \in K_{FV}$

$$k(s) - k(m) = \int_m^s k'(t) dt \leq \lambda(s - m) \text{ if } s \geq m \text{ and } k(s) - k(m) \geq \lambda(s - m) \text{ if } s < m.$$

Hence $\int_m^t (k(s) - k(m)) ds \leq \frac{\lambda}{2} (t - m)^2$ for every t so $k \in K_{FV}^*(\infty) = K\left(\frac{\lambda}{2}(t - m)^2, \infty\right)$. Since $e^x \geq 1 + x$, the bound (14) is an improvement of the Fong-Vasicek one (see (5)).

Example 3. We now give an extension of the Balbás-Ibáñez inequality. Put $A(k) = \frac{1}{2}(\inf_{0 \leq t \leq T} k(t) + \sup_{0 \leq t \leq T} k(t))$. Define

$$K_{BI}^* = \left\{ k; \int_m^t (k(s) - A(k)) ds \leq \frac{\lambda}{2} |m - t|, 0 \leq t \leq T \right\} \quad (15)$$

Clearly, $K_{BI}^* = K\left(\frac{\lambda}{2}|m - t|, \infty\right)$. Recall that K_{BI} is defined by (6). For every $k \in K_{BI}$, $k(s) \leq k^*(s)$ if $s \geq m$ and $k(s) \geq k^*(s)$ otherwise, where $k^*(s) = A(k) + \frac{\lambda}{2}$

for $s \geq m$ and $k^*(s) = A(k) - \frac{\lambda}{2}$ for $s < m$. Hence $\int_t^m k(s) ds \geq A(k)(t-m) - \frac{\lambda}{2}|t-m|$ for every t . As a consequence we get $K_{BI} \subset K_{BI}^*$. From theorem 1 it follows that for every duration-matching portfolio q

$$V(q, K_{BI}^*) \geq L \exp\left(-\frac{\lambda}{2} \tilde{N}\right) \quad (16)$$

Since $K_{BI} \subset K_{BI}^*$ and $e^x \geq 1+x$, the inequality (16) is an improvement of the Balbás-Ibáñez inequality (7).

Example 4. Consider the following class of shocks

$$K_{conv}(W, a) = \left\{ k; t \rightarrow \int_t^m k(s) ds + W(t) \text{ is convex on } [0, T], |k(m)| \leq a \right\}$$

with W being a given convex and differentiable function such that $W'(m) = 0$ (cf. Montrucchio, Peccati, 1991). Observe that $K_{conv}(W, a)$ includes all parallel shocks not greater than a . By convexity of $t \rightarrow \int_t^m k(s) ds + W(t)$

$$\int_t^m k(s) ds + W(t) \geq W(m) - k(m)(m-t) = W(m) - A(k)(m-t),$$

$$0 \leq t \leq T,$$

with $A(k) = k(m)$ for every $k \in K_{conv}(W, a)$. Hence $K_{conv}(W, a) \subset K(W - W(m), a)$ with $A(k) = k(m)$ (see (8)). Theorem 1 implies that if q is a feasible portfolio, then

$$V(q, K_{conv}(W, a)) \geq L \exp\left(-a|m - D(q)| - \sum_t (W(t) - W(m))c(t, q)\right) \quad (17)$$

Observe that if W is continuously differentiable, then

$$K_{conv}(W, a) = \{k; k(t_2) - k(t_1) \leq w(t_2) - w(t_1), 0 \leq t_1 < t_2 \leq T, |k(m)| \leq a\},$$

where $w(t) = W'(t)$. Hence, for a subadditive function w (i.e. $w(x+y) \leq w(x) + w(y)$ for all x, y), we have $K_{conv}(W, a) \subseteq K_a(w)$, with $K_a(w)$ defined by (10).

In Nawalkha and Chambers (1996) one can find the following result. Given $k_1, k_2 \in R$, they defined the class of shocks:

$$K_{NCh} = \{k; k_1 \leq k(t) \leq k_2, 0 \leq t \leq T\} \quad (18)$$

and proved that for any feasible portfolio q

$$V(q, K_{NCh}) \geq L(1 - k_3 M^A) \quad (19)$$

where $M^A = \sum_t |t - m| c(t, q)$ and $k_3 = \max\{|k_1|, |k_2|\}$. Observe that $M^A = \tilde{N}$.

Motivated by (19) they proposed to

P5: choose a feasible portfolio which minimizes M^A .

We now provide a modification of theorem 1 extending the result of Nawalkha and Chambers (1996). Letting A be a given real, define the class of shocks:

$$K_{NCh}(W, a) = \left\{ k; \int_m^t (k(s) - A) ds \leq W(t), 0 \leq t \leq T \right\} \quad (20)$$

where W is a nonnegative and convex function such that $W(m) = 0$.

Theorem 2. For every feasible portfolio q

$$V(q, K_A(W)) \geq L \exp(A(m - D(q)) - M^W) \quad (21)$$

where $M^W = \sum_t W(t) c(t, q)$.

Proof. The proof is extremely similar to that of theorem 1. \square

From theorem 2 we obtain the following strategy

P6: minimize $A(D(q) - m) + M^W$ over all feasible portfolios q .

Example 5. We give an improvement of the inequality (19). Define

$$K_{NCh}^*(A, B) = \left\{ k; \int_m^t (k(s) - A) ds \leq B|t - m|, 0 \leq t \leq T \right\}$$

with $A = \frac{1}{2}(k_1 + k_2)$ and $B = \frac{1}{2}(k_2 - k_1)$, where $k_1 < k_2$. By theorem 2, for every feasible portfolio q

$$V(q, K_{NCh}^*(A, B)) \geq L \exp(A(m - D(q)) - BM^A) \quad (22)$$

where $M^A = \sum_t |t - m| c(t, q)$. We proceed to show that (22) is an improvement of (19).

Since $A(m-t) - B|t-m| \geq -\max\{|k_1|, |k_2|\}|t-m|$ for all t , we have

$$V(q, K_{NCh}^*(A, B)) \geq L \exp(-k_3 M^A) \geq L(1 - k_3 M^A) \quad (23)$$

where $k_3 = \max\{|k_1|, |k_2|\}$. It is easy to check that $K_{NCh} \subset K_{NCh}^*(A, B)$ so from (23) we obtain the inequality (19).

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STRATEGIA UODPARNIANIA PORTFELA

Streszczenie

W artykule przedstawiono nową strategię uodparniania portfela, w skład którego wchodzi obligacje bez opcji zakupu przysługującej emitentowi (*noncallable*) i wolne od ryzyka niewykupienia (*default-free*). Strategia polega na minimalizacji miary, która jest liniową kombinacją luki duracyjnej i miary rozrzutu, przy różnych klasach zaburzeń chwilowej terminowej stopy procentowej (*instantaneous forward rate*). Ponadto otrzymano uogólnienia nierówności Fonga i Vasiceka (1984), Nawalkhai i Chambersa (1996) oraz Balbása i Ibáñeza (1998) na dolne ograniczenie zmiany wartości portfela w chwili rozliczenia.