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HYPOTHESIS TESTING IN THE CASE OF INSUFFICIENT OBSERVATIONS – IDENTIFICATION OF OWN CRITICAL VALUES

Abstract. Economic transformation of the Central and Eastern European Countries and ongoing process of the European integration are important sources of structural changes in the concerned economies and therefore shorten available time series. Similarly, empirical studies focused on the economies in the European Union wrestle with short time series.

Consequently, time series modelling and results robustness of econometric analysis used for European macroeconomic time series are limited by the sample size. Number of observations is important not only for the basic time series modelling and subsequent forecasting, but especially for the verification of theoretical assumptions. Thus, quantitative methods usually used for hypothesis testing are in the case of insufficient observations often completed by qualitative analysis which encompass certain amount of subjectivity.

One of the factors influencing the stability of statistical hypothesis tests is the sample size. For large samples estimates behave usually asymptotically and have quite good stability. On the other hand, if the sample size is small (i.e. less then 30) the stability of statistical tests is lower. Moreover, for small samples the probability of wrongly rejected null hypothesis, so-called Type I Error, is even higher. In other words, inadequate power of a statistical hypothesis test determinates the possible application of econometric methods on short macroeconomic time series.

One of the possible solutions is the usage of a Monte Carlo simulation and detection of own critical values. Critical values are identified on the base of simulation of time series with suitable length with respect to the predefined model. The authors have applied this methodology only on the analysis of macroeconomic time series regression (using cointegration) without any further limitation in its usage. The basic assumption of the analysis is the robustness of the hypothesis test about time series stationarity. However, the Dickey-Fuller test applied on short time series leads to wrong rejection of the hypothesis about stationarity and thus rejects the existence of real regression and wrongly rejects the hypothesis about spurious regression.

Methodology presented and described in this paper is demonstrated on the causality between interbanking interest rates and selected retail banking interest rates in the Czech Republic.

Key words: Monte Carlo simulation, ADF test, cointegration, time series analysis, interest rates.

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1. INTRODUCTION

Researchers in many disciplines encounter the problem of short time series. Restrictions connected with short time series are apparent mainly in the field of macroeconomics. Insufficient number of observations decreases the results' robustness of quantitative analysis not only in the area of forecasting but also in the area of regression analysis. A typical example of short time series are macroeconomic indicators of post-transformational economies. There are only 56 quarterly observations available for the Czech Republic and when omitting the previous data influenced by structural changes and the monetary crisis in the years 1997 and 1998 then we obtain only 32 observations for the period 1999 to 2006. Of course, in some analyses of foreigner economies it is not unusual to work with time series over a time period of approximately 40 years. In the long run are all abnormalities "averaged" and therefore the results are more accurate.

Typical for macroeconomic data are structural changes. For example the abandonment of fixed exchange rate regime and the introduction of managed float in the Czech Republic in May 1997 can be considered as a structural change. More recent example of a structural change is the introduction of euro in the European Monetary Union (EMU) member countries and the loss of national central banks monetary autonomy in these countries since January 1999. A typical example of an analysis dealing with short time series is a comparison of costs and benefits associated with the EMU integration. Testing of hypotheses on the significance of costs and benefits connected with the entry into the EMU is based on insufficient number of observations (since the year 1999) or on historical experiences from different (non-European) regions. To illustrate this fact various studies dealing with the impact of the ECB's single monetary policy on economic growth and inflation in the EMU are listed in Table 1.

As can be seen from the Table 1, econometric models used in the above listed studies analysing the impact of common monetary policy of the ECB on the EMU countries are based on data which abstract from the loss of national central banks monetary autonomy. However, this structural change can be considered as highly significant and therefore the results of these analyses could be inaccurate and/or misleading. The only exception is the last study, which is also based on the current time period; nevertheless, the results of the time series regression hypothesis are insignificant. This fact can be attributed to the insufficient number of observations (i.e. 37) included in the analysis.

Lots of similar examples of insufficient observations and their impact on hypothesis testing can be found. In real macroeconomic environment short time series are often caused by structural changes in institutions and/or by impulsive shocks with long-term influence on economic growth.

Table 1

Selected empirical analysis identified ECB's single monetary policy effects

Authors	Name of the article	Publication date	Observations
Van Els P.; Locarno A.; Morgan J., Villetelle J-P.	*Monetary policy transmission in the euro area: What do aggregate and national structural models tell us?*	2001	NCB*models
Peersman G., Smets F.	*The monetary transmission mechanism in the euro area: evidence from VAR analysis*	2003	1980–1998
Mojon B., Peersman G.	*A VAR description of the effects of monetary policy in the individual countries of the euro area*	2003	1980–1998
Van Els P.; Locarno A.; Morgan J., Villetelle J-P.	*The effects of monetary policy in the euro area: evidence from structural macroeconomic models*	2003	NCB*models
Sousa J., Zaghini A.	*Monetary policy shocks in the euro area and global liquidity spillowers*	2004	1981–2001
Garnier J., Wilhelmsen B.-R.	*The natural real interest rate and the output gap in the euro area. A joint Estimation*	2005	1963:Q1–2004:Q1
Calza A., Sousa J.	*Output and inflation respondens to credit shocks. Are three threshold effects in the euro area?*	2005	1981:Q2–2002:Q3
Kapounek A., Poměnková J.	*Economic growth and single monetary policy of the European Central Bank*	2006	1997–2006

* the analysis used models of the national central banks of the EMU member states.
Source: own elaboration.

The authors of this paper have focused on hypothesis testing of cointegration. When the terms cointegration, regression or correlation are used in this paper, they refer to statistical causality, because the real causality cannot be proved by time series analysis.

The character of macroeconomic time series is often responsible for biased detection of regression even if the sample size is large. Typically, macroeconomic variables trend together and are nonstationary. Therefore it is necessary to adjust these time series for nonstationarity before the regression analysis is carried out. To avoid several critical mistakes, it is crucial to choose an appropriate method to adjust for nonstationarity. R. Hušek (1999) adds: "If a difference stationary process is detrended by including a time-trend variable as an independent variable in the regression or a trend stationary process is differenced spurious autocorrelation arises."

Consider a typical regression equation:

$$Y_t = a_0 + a_1 X_t + \varepsilon_t,\ t = 1, 2, ..., T. \quad (1)$$

The classical regression model is derived under the assumption of stationarity of the time series Y_t, X_t, and normally distributed random error term with a mean value of zero and constant variance. In case that the time series are nonstationary, spurious regression can occur. Spurious regression tends to overstate the t-scores and the reliability of the model. However, obtained results often lack economical sense or/and could be misleading.

To test for nonstationarity the Dickey-Fuller test, or the augmented Dickey-Fuller test, was designed (Hušek 1999). In case of nonstationary time series transformation to stationary time series (integrated of corresponding order) is undergone. To assess the relation among time series of the same order their linear combination must be found. To avoid misinterpreting spurious regression results, Engle and Granger came up with the so called Engel-Granger cointegration test which helps to confirm the existence of a long run relationship among time series.

The aim of this paper is to show on a real case study how the small sample size can affect detection of real regression and suggest a solution to this problem using a Monte Carlo simulation. For this purpose two time series which are by the economic theory supposed to display real causality have been chosen. Namely interbanking interest rates and selected retail banking interest rates in the Czech Republic. At first the analysis is carried out on a large sample size ($n = 73$), afterwards is the number of observations reduced to an extent to which tests start to detect spurious regression (non-cointegration) instead of cointegration. At this moment own critical t-values for this small sample are derived using a Monte Carlo simulation.

This paper refers to the impact of a small sample size on cointegration test. If the number of observations is insufficient, results of empirical analysis could be misleading, and furthermore, in contrast with the economic theory. We show how to solve this problem using Monte Carlo simulation and obtain more accurate results.

2. THEORETICAL BACKGROUND

Stationarity is defined as a stationary stochastic process (i.e. weak stationarity) if its mean and variance are constant over time and its covariance is time-independent (H. Arlt 1999).

Definition 1. Assume l-dimensional stochastic process $\{X_t\} = \{(X_{1t}, X_{2t}, ..., X_{lt})'\}$. This process is said to be weakly stationary, if:

i) $E(X_t) = \mu < \infty$ for all t, μ is r-dimensional mean vector

ii) $E[(X_t-\mu)(X_t-\mu)'] = \Sigma < \infty$ for all t, where Σ is the covariance matrix of the type $(l \times l)$, the diagonal elements of this covariance matrix are the variances and the off-diagonal elements are the covariances of the random variable of the stochastic processes

iii) $E[(X_t - \mu)(X_{t-k} - \mu)'] = \Gamma_k$ for all t a $k = 0, 1, \ldots$, Γ_k is the autocovariance matrix function of the type (l × l), on the main diagonal there are the autocovariances of random variables in time t and with lag k (covariances of the individual random values in time t and $t + k$), outside the main diagonal there are covariances of the random variable, one of them in time t and the second in time $t - k$.

To test for the hypothesis of time series stationarity the Dickey-Fuller test, or the augmented Dickey-Fuller test, based on the hypothesis of the existence of a unit root is used (Hušek 1999). In the case when the residuals ε_t are autocorrelated of the order p, *i.e.* $AR(p)$, $p>1$, the augmented Dickey-Fuller test is applied.

For simplicity, let us now consider a first-order autoregressive process of the form

$$Y_t = a_1 Y_{t-1} + \varepsilon_t \tag{2}$$

where ε_t is white noise. The Dickey-Fuller (DF) test used for testing of the presence of a unit root can be applied in following three different regression equations, where $\delta = (a_1 - 1)$:

$$\begin{aligned} \Delta Y_t &= \delta Y_{t-1} + \varepsilon_t \\ \Delta Y_t &= a_0 + \delta Y_{t-1} + \varepsilon_t \\ \Delta Y_t &= a_0 + a_2 t + \delta Y_{t-1} + \varepsilon_t \end{aligned} \tag{3}$$

In the case when the residuals ε_t are autocorrelated of the order p, i.e. $AR(p)$, $p > 1$, the augmented Dickey-Fuller test is used in the following forms.

$$\begin{aligned} \Delta Y_t &= \delta Y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta Y_{t-i+1} + \varepsilon_t \\ \Delta Y_t &= a_0 + \delta Y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta Y_{t-i+1} + \varepsilon_t \\ \Delta Y_t &= a_0 + a_2 t + \delta Y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta Y_{t-i+1} + \varepsilon_t \end{aligned} \tag{3}$$

Decision about the existence of unit roots in time series is based on the following hypothesis

H_0: $\delta = 0$, for non-stationarity, if $t_\delta > \tau$,

H_1: $\delta < 0$, for stationarity, if $t_\delta < \tau$,
where τ is the critical value (Seddighi, Lawler, Katos 2000, Hušek 1999).

Let us focus on the statistical properties of two integrated time series. Firstly, we define two key terms: integration process and cointegration process.

Definition 2. A process is said to be integrated of order d, i.e. $Y_t \sim I(d)$, if a difference of order d of the time series $\{Y_t\}$ is stationary.

Definition 3. Two time series are denoted as cointegrated of order d, c, i.e. $\{X_t\},\{Y_t\} \sim CI(d,c)$, if both time series $\{X_t\}$ a $\{Y_t\}$ are of the same order $I(d)$ and there exists a linear combination of these two time series $aX_t + bY_t \sim I(d-c)$, where $c > 0$.

Let us start with the typical regression equation in (1). Assume that both variables are integrated of the same order. It is obvious (Arlt 1999), that if there are processes of different order there must be at least one process of a higher order to allow for partial cointegration.

According to H. R. Seddighi, K. A. Lawler and A. Katos (2000) are both variables Y_t and X_t integrated of order 1 cointegrated of order (1,1) if there is a linear combination

$$\varepsilon_t = Y_t - a_0 - a_1 X_t, t = 1, 2, ..., T. \tag{4}$$

which is stationary, i.e. I(0). The equilibrium error ε_t is derived under the assumption of a nonzero parameter a_1. If the time series Y_t and X_t are cointegrated then can be, using linear regression model (LMR), distinguished the long-run regression from the short-run regression. If the variables Y_t and X_t in model (1) are not cointegrated, i.e. the equilibrium error ε_t is nonstationary $e_t \sim I(1)$, variables are diverging in time. As a result spurious regression can occur.

The most frequently used tests based on testing for a unit root were designed by Engle and Granger (1987). They use the fact that in case of cointegrated variables Y_t and X_t the error term should be stationary, i.e. $e_t \sim I(0)$. Therefore they suggested testing the following hypotheses

H_0: $\delta = 0$, for non-stationarity of e_t, i.e. for non-cointegration (spurious regression), if $t_\delta > \tau$

H_1: $\delta<0$, for stationarity of e_t, i.e. cointegration (real regression), if $t_\delta < \tau$,
where τ is the appropriate critical value (Seddighi, Lawler, Katos 2000). The Engle-Granger (EG) cointegration test, or the augmented Engle-Granger (AEG) test for autocorrelated random factors or residuals, is based on the estimation of the LRM in (6) and the application of the DF τ test, or ADF τ test, is based on the following equation

$$\Delta e_t = \delta e_{t-1} + u_t \quad \Delta e_t = \delta e_{t-1} + \sum_{i=2}^{p} \beta_i \Delta e_{t-i+1} + u_t \tag{5}$$

Engle and Granger (1987), Philips and Ouliaris (1990) published their own critical values which are more appropriate for this test than the usual Dickey-Fuller τ statistics.

Methods used for detection of a unit root process are based on estimates and are therefore not precise. As can be seen in H. R. Seddighi, K. A. Lawler and A. Katos (2000) a problem connected with stationarity or cointegration tests and their inaccurate critical values can occur. A near-unit root process can have a similar shape of the autocorrelation function as that of a process containing a trend. Thus, usage of these tests and their set critical values could lead to mistakes in interpretation.

Consider a first-order autoregressive process

$$Y_t = a_1 Y_{t-1} + \varepsilon_t, \tag{6}$$

where ε_t is white noise. When testing for the null hypotheses that $a_1=1$, the Y_t sequence is generated by the non-stationary process

$$Y_t = Y_0 + \sum_{i=1}^{T} \varepsilon_i. \tag{7}$$

Therefore, if $a_1 = 1$, the variance growths rapidly as t decreases, thus it is inappropriate to use classical statistical methods for estimating and testing for significance on the coefficient a_1. The estimate obtained by using the Ordinary Least Squares (OLS) technique is biased. Hence, Dickey and Fuller designed a procedure to formally test for the presence of a unit root. This procedure is based on generating great number of random processes that enabled them to calculate the estimated value of the parameter a_1 and to derive their own critical values to test for unit roots.

As mentioned above, it is inappropriate to use the standard t-tests or F-tests to test the hypotheses about regression coefficients significance of nonstationary variables, because the distribution of the statistics is non-standard and therefore cannot be analytically evaluated. This problem can be solved on the basis of Monte Carlo simulation.

A Monte Carlo method is based on a large number of replications in the data-generating process, i.e. a random sample of size T with the parameters corresponding to the original data set is generated and this process is repeated N times (where $N \geq 10000$). Once a series has been generated, the distribution of the desired parameters and/or sample statistics can be tabulated. These empirical distributions are used as estimates of the actual distribution.

The use of the Monte Carlo method is warranted by the Law of Large Numbers. The obtained results are specific to the assumptions used to generate the simulated data and therefore cannot be applied in general (Enders 2004, p.178)

In order to generate own t-values for cointegration test; a Monte Carlo experiment can be performed as follows:

Step 1: If we use a usual set of assumption, we can draw a set of random numbers from a standard normal distribution. Of course, when needed, a Monte Carlo experiment allows us to generate data sets with other distributions. Thus, we firstly obtain a set of random numbers representing the ε_t sequence, i.e. the residuals (4)

$$\varepsilon_t = Y_t - a_0 - a_1 X_t,\ t = 1, 2, ..., T.$$

Step 2: Residuals obtained in Step 1 are used to draw a random walk process

$$\varepsilon_t = a_1 \varepsilon_{t-1} + w_t. \tag{8}$$

On the assumption of normally distributed residuals ε_t elementary statistics, i.e. $E(w_t)$ and $var(w_t)$, are computed. Then the w_t sequence of the chosen length featuring the required characteristics is randomly generated.

Step 3: The sequence ε_t described by the process (8) is generated using the sequence w_t. The initial value for ε_0 is chosen to be $\varepsilon_0 = 0$ and consequently T+50 realisations are generated. To eliminate the effect of the initial condition, i.e. $\varepsilon_0 = 0$, the first 50 realizations are discarded from further computations.

Step 4: The Engle-Granger cointegration test is applied on the generated ε_t (5)

$$\Delta e_t = \delta e_{t-1} + u_t \quad \Delta e_t = \delta e_{t-1} + \sum_{i=2}^{p} \beta_i \Delta e_{t-i+1} + u_t \tag{9}$$

Step 5: After repeating Steps 1–5 10 000 times, we obtain a set of 10 000 simulated values of the parameter δ and their t-values. From the distribution of the obtained t-statistics critical values at the 5 percent and 10 percent level of significance are defined.

3. ECONOMIC ASSUMPTIONS

To objectively demonstrate the possibility of simulating own t-values using the authentic set of residuals, and therefore prove the existence of real regression even on a small sample, the authors have used the causality between the interest rates on crown time deposits of Czech residents and short-term interest rates on the Czech interbank market. This causality is based on the transmission mechanism defined by the Czech National Bank: „An increase/decrease in a monetary policy interest rate (specifically the repo rate in the Czech Republic) leads first to an increase/decrease in interest rates on the interbank market. This in turn causes banks to raise/lower their rates on credits and deposits." (ČNB 2007) This causality has been chosen deliberately. In the case of other macroeconomic indicators an agreement on the causes and effects cannot be met by various economic schools and empirical studies, whereas the causality between the chosen interest rates is obvious and therefore generally accepted.

The causality between short-term interest rates on the interbank market and retail bank interest rates is based on the relation between the operating target and the intermediate target of the central bank's monetary policy. An operating target, in the form of short-run interest rates, is directly influenced by the instruments of central bank's monetary policy and could be therefore regarded as directly controlled by the bank's governing council. Through the operating target the central bank is able to influence the intermediate target which can be defined in a form of money stock, exchange rate, long-run market interest rate or other economic indicators. The central bank uses its operating target to reach its ultimate target (Jílek 2004). In the case of the Czech National Bank the primary target is to maintain price stability and the secondary target is defined as support of the general economic policies of the Government leading to sustainable economic growth. Selection of the intermediate target is directly dependent on its long, stable and predictable linkage to the ultimate target.

Intermediate target in the form of money stock has been derived from the monetarist doctrine and the quantitative theory of money. On the ground of unstable velocity of money in economies and hardly predictable causality between the money stock growth and the ultimate target of the monetary policy in the form of price stability, the money stock targeting has become less popular among central banks in recent years. When abstracting from the exchange rate transmission mechanism, we realise, that the alternative definition of the intermediate targets is based on Keynes' concept of causality between the central bank's monetary policy and its ultimate targets. According to the Keynesian concept the only relation between monetary policy and real income are interest rates (Mach 2002).[1] On these assumptions is based the credit transmission mechanism. To his intermediate targets belong for example the total amount of loans in economy denoted in home currency, the total amount of retail bank loans in economy denoted in home currency, medium-term and long-term interest rates of loans and possible potential sources of loans for commercial banks denominated in home currency. If the intermediate target is a long-term or medium--term interest rate it is possible to speak about the interest rate transmission mechanism (Revenda 2001).

The causality of short-term interest rates on the interbank market is directly proportional to the dependency of commercial banks on the central bank's financial sources. The increase in short-term interest rates on the interbank market decreases the growth rate of client loans and further leads to the decrease in credits which commercial banks obtain from the central bank, and last but not at least, to the overall increase in interest rates in the economy. On the assumption that commercial banks are heavily dependent on the interbank market, change in short-term interest rates on the interbank market affects short-term interest rates of client loans and deposits. Client loans interest rate time series are affected/biased by the different financial standing of loan applicants and by the differences in credit channels among commercial banks[2]. Therefore, the authors have chosen for the empirical analysis only time series of deposit interest rates, which are not affected by these factors.

Increase in short-term interest rates on the interbank market leads to a slowdown in the loans growth rate or to their absolute decrease, "if repayments for loans obtained from the others bank (including the central bank) are higher than the newly granted loans. From this follows that this situation can lead – ceteris paribus - to a slowdown in the growth of newly granted clients loans, eventually to a drop in the total number of approved loans (and therefore the sum of repayments is higher than the sum of newly granted loans)" (Revenda 2001, p. 227) If the commercial banks were almost independent on the financial sources obtained from the interbank market, changes in the interest rates on this market would not affected their decisions. Thus, in case of growing short-term rates on the interbank market commercial banks would reduce their demand on interbank loans and acquire the needed financial sources in a different way, e.g. by selling foreign assets, emissions of bonds, stocks, borrowing abroad and by reducing their voluntary reserves in proportion to the granted loans.

[1] The impact of monetary policy on economic growth in long and short run and the detailed description of the Keynesians transmission mechanism are not the aim of this paper.

[2] Prutenau (2004) has identified differences in loans supply among different commercial banks in the Czech Republic on the ground of their ownership, size, liquidity, capitalisation and ratio of classified loans to total loans.

If commercial banks do not utilize any of the above mentioned alternative financial sources and are significantly dependent on acquiring financial sources on the interbank market, they can solve the problem of lacking financial sources by lowering their demand on interbank loans and by increasing deposit interest rates, and therefore obtaining more clients deposits. However, the inflow of bank deposits would not occur automatically, because savings of economic subjects depend mainly on their income. Nevertheless, it can be taken as granted that increase in interbank interest rates will lead to general increase in interest rates in economy. "Few days after the change in short-term interbank interest rates commercial banks change their client interest rates, mainly by new deposits, loans and bond emissions. The interest rate spread, i.e. the difference in yields between bank's actives (mainly granted loans) and liabilities (mainly time deposits), maintains more or less the same. The credit spread can evolve in time according to competitive changes on the market; however it does not change according to changes in short-time interest rates." (Jílek 2004, p. 468).

Similarly, the decrease in short-term interest rates on the interbank market enables commercial banks to obtain their credits on this market, whereas the alternative financial sources are not so widely used. Therefore can be expected a decrease in deposit interest rates.

The following empirical analysis is based on the assumption that there exists causality between the central bank's operating target in the form of short-term interest rates on the interbank market and its intermediate target defined as commercial banks' interest rates. As follows, Czech commercial banks are significantly dependent on financial sources obtained form the interbank market and use interest rate increase as an incentive for clients to make time deposits for them more appealing. Similarly an increase in short-term interest rates on the interbank market forces commercial banks to cut their costs and to lower the deposit interest rates. In other words, the authors expect, that the hypothesis about the real regression of short-term deposit interest rates and short-term interbank interest rates will be proved in the following empirical part.

4. EMPIRICAL ANALYSIS

Short-term interest rates (SIR) on the Czech interbank market used in this empirical analysis are monthly averages of the Prague Interbank Offered Rate (PRIBOR). The second analysed time series consists of monthly average interest rates on crown deposits (DIR) collected by commercial banks from Czech residents. The authors have chosen for their analysis the time period 2001/I – 2007/I, which allows them to abstract from the effects of the monetary crisis in

1997 and structural changes connected with transformation process of the Czech economy.

To test for the long-run relationship between SIR and DIR the ADF test was applied, namely the ADF1, ADF2 and ADF3 tests. Afterwards, cointegration was tested by the EG test of the same integration order.

In order to decide about the proper number of lags the Akaikes's information criterion and Schwartz criterion, which are built in the used software, were employed. If choosing the appropriate lag length two problems might arise. Inclusion of too few lags into the model causes that the residuals do not behave like white-noise process. On the other hand inclusion of too many lags reduces the power of the test to reject the null hypothesis of a unit root since the increased number of lags necessitates the estimation of additional parameters and a loss of degrees of freedom. From these reasons and with respect to the character of Czech data, the economic theory recommends using no more then six lags. Therefore a lag length of $p = 1, ..., 6$ was considered in our empirical analysis.

As could be seen from Table 2 short-term interest rates (SIR) on the inter-bank market and interest rates on time deposits (DIR) are stationary and cointegrated in the time period 2001/I–2007/I, i.e. between these time series exists real causality. However, it is necessary to remark that in the case of the ADF2 test (for a regression equation containing a constant) were the time series stationary only at the 10 percent level of significance.

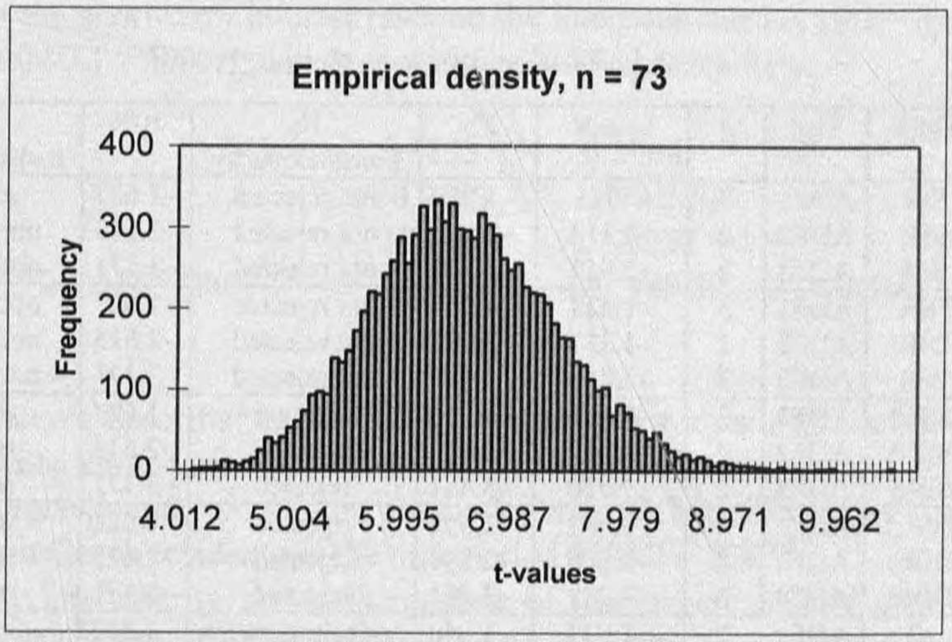
For illustration a Monte Carlo simulation for the authentic data set of 73 observation and has been performed using 10 000 replications. Obtained t-values distribution is plotted in Graph 1.

Table 2

Cointegration analysis results for $n = 73$

n	Data	Test	p	t-value	5%	H_0 nonstacionarity	10%	H_0 nonstacionarity
73	SIR	ADF1	3	–4.166	–1.95	rejected	–1.628	rejected
73	SIR	ADF2	2	–2.637	–2.973	not rejected	–2.615	not rejected
73	SIR	ADF3	3	–0.602	–3.461	not rejected	–3.171	not rejected
73	DIR	ADF1	3	–4.335	–1.95	rejected	–1.627	rejected
73	DIR	ADF2	2	–3.283	–2.973	rejected	–2.615	rejected
73	DIR	ADF3	1	–0.452	–3.461	not rejected	–3.171	not rejected
72	SIRd	ADF2	2	–11.699	–2.973	rejected	–2.615	rejected
72	SIRd	ADF3	2	–12.318	–3.461	rejected	–3.171	rejected
72	DIRd	ADF2	2	–9.055	–2.973	rejected	–2.615	rejected
72	DIRd	ADF3	2	–9.98	–3.461	rejected	–3.171	rejected
73	e2	ADF1	2	–3.52	–1.95	cointegration	–1.627	cointegration

Source: own calculations.



Graph 1. Distribution of t-values obtained by a Monte Carlo simulation for the authentic data set containing 73 observations

Source: own calculation.

In the next part of the empirical analysis the sample size was repeatedly and systematically decreased. For each sample size the tests of stationarity and cointegration were applied and in all cases real regression using the EG test was approved.

The shortest time period, i.e. the smallest sample size, analysed was the period 2004/XI – 2007/I. Obtained results are displayed in Table 3. As can be seen in the case of the ADF1 test (using a regression equation without a constant and a time trend) applied on the SIR time series, SIR was nonstationarity only at the 5 percent level of significance. Whereas the EG cointegration test indicated at the 5 percent level of significance non-cointegration, i.e. spurious regression, but at the 10 percent level of significance cointegration, i.e. real regression.

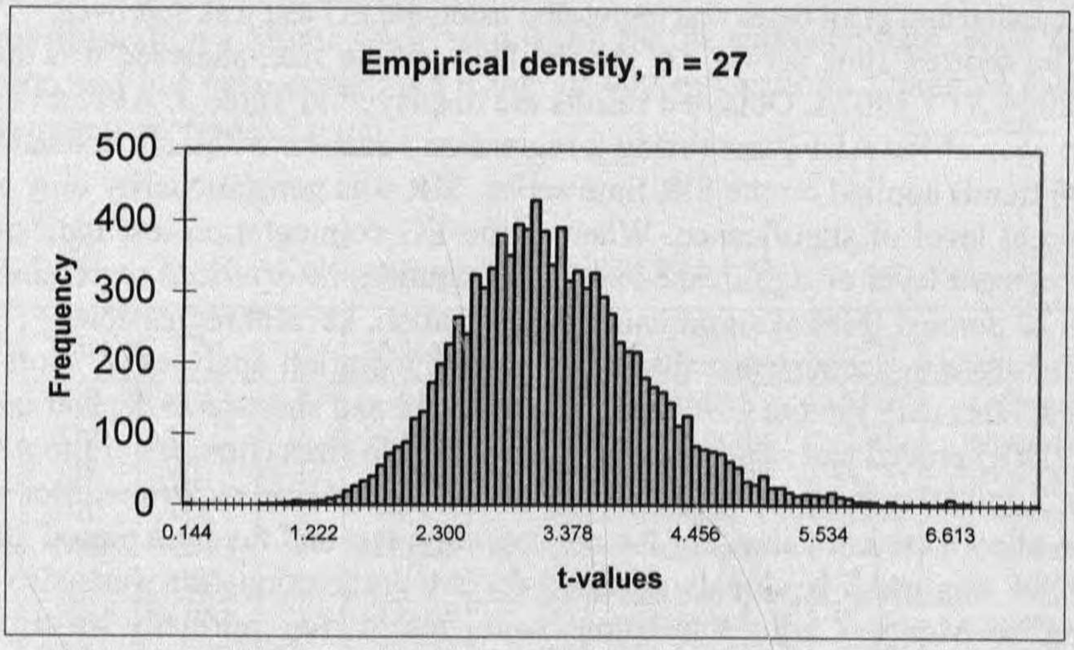
Taking into account that the results of cointegration analyses of short-term interest rates (SIR) in the Czech interbank market and short-term deposit interest rates (DIR) proved real regression for various sample sizes (from $n = 73$ to $n = 27$) at the 5 and 10 percent level of significance, a Monte Carlo experiment for simulation of own critical values for the cointegration test and the time period 2004/*X* – 2007/I was used. Residuals obtained for a cointegration test were also used during a Monte Carlo simulation were tested for normally distribution (Shaphiro-Wilk test, Hebák 2004, normally distribution test based on skewness) which was verified. Critical values computed on the base of a Monte Carlo experiment (10 000 replications) are displayed in the last row of Table 3 and confirm cointegration at the 5 percent level of significance. Graph 2 shows empirical distribution for t-values obtained by a Monte Carlo experiment for a data set of $n = 27$ values and 10 000 replications.

Table 3

Cointegration analysis results for $n = 27$

n	Data	Test	p	t-value	5%	H_0 nonstacionarity	10%	H_0 nonstacionarity
27	SIR	ADF1	3	–1783	–1.950	not rejected	–1.627	rejected
27	SIR	ADF2	2	–2.118	–2.973	not rejected	–2.615	not rejected
27	SIR	ADF3	2	–2.952	–3.461	not rejected	–3.171	not rejected
27	DIR	ADF1	3	–1.429	–1.950	not rejected	–1.627	not rejected
27	DIR	ADF2	1	–1.337	–2.973	not rejected	–2.615	not rejected
27	DIR	ADF3	3	–1.880	–3.461	not rejected	–3.171	not rejected
26	SIRd	ADF1	2	–7.560	–1.950	rejected	–1.627	rejected
26	SIRd	ADF2	2	–7.389	–2.973	rejected	–2.615	rejected
26	SIRd	ADF3	2	–7.920	–3.461	rejected	–3.171	rejected
26	DIRd	ADF2	4	–3.362	–1.950	rejected	–1.627	rejected
26	DIRd	ADF2	4	–3.200	–2.973	rejected	–2.615	rejected
26	DIRd	ADF3	3	–3.091	–3.461	rejected	–3.171	rejected
27	e2	ADF1	2	–1.941	–1.950	noncointegration	–1.627	cointegration
				MC simulation	2.044	cointegration	2.285	cointegration

S o u r c e: own calculations.



Graph 2. Empirical distribution of residuals obtained by a Monte Carlo simulation for a data set of $n = 27$ values

S o u r c e: own calculations.

The following estimated regression, that was confirmed by the F-test of overall significance, displays real regression of short-term deposit interest rates

(DIR) on the short-term interest rates on the interbank market (SIR) for the time period 2004/XI – 2007/I, which contains only 27 observations.

	DIR	=	0.8153	+	0.2084	SIR
			(0.062)		(0.0296)	
			13.144		7.022	
2004/X – 2007/I			$n = 27$		$F = 49.32$	$R^2 = 0.664$

The above described methodology was used for an analysis of real regression between short-term interest rates (SIR) on the Czech interbank market and monthly average interest rates on crown deposits (DIR) collected by commercial banks from Czech residents in the time period 2001/I – 2007/I.

Firstly, stationarity and cointegration of the largest sample size (n = 73) available in our analysis were tested. Obtained results have approved our expectations arising from the economic theory, i.e. the existence of real regression between SIR and DIR (see Table 2).

Secondly, a reduction of the sample size was undergone until the preformed stationary and cointegration test did not start giving divergent results which are in contrast with the economic theory. More concretely, the EG cointegration test indicated at the 5 percent level of significance non-cointegration, i.e. spurious regression, but at the 10 percent level of significance cointegration, i.e. real regression.

Seeing that the critical values for the EG test were robust a Monte Carlo simulation on a reduced data set was performed. However, several problems about Monte Carlo simulations have arisen. Performed simulations are specific to the assumptions used to generate the simulated data, and therefore an entirely new Monte Carlo simulation needs to be performed when there is a change in the underlying data set. This fact can be considered both as an advantage and a disadvantage. The disadvantage is that you need to repeat the whole experiment if the initial conditions of the experiment change. On the other it is advantageous if you can create your own critical values for a concrete data set. Another drawback of this method is, that for precise results you need to know the exact distribution of the data set, which could be difficult in case of small samples. Therefore it is essential to focus on distribution testing of the underlying data set. On the top of it you can obtain different own critical values according to the software used and mainly small samples could be sensitive to the t-values used.

5. CONCLUSION

As sketched in the introduction to this paper on an example of the ECB's monetary policy impact on economic growth, insufficient number of observations is not only a problem in the area of hypothesis testing and macroeconomic time series analysis, but it is widely spread across different disciplines. The main focus of this paper is therefore on testing hypothesis of two macroeconomic time series causality through cointegration.

Critical values commonly used in testing of stationarity and cointegration, as mentioned e.g. by Seddighi, Lawler and Katos (2000), Engle and Granger (1987), Philips and Ouliaris (1990), not always exactly correspond to the empirical distribution function of the analysed data set. As mentioned above in this paper, it is not appropriate to test for stationarity and cointegration while using biased OLS estimates, therefore, in this case classical t-test based on the assumption of classical regression model cannot be utilized. This problem can be solved by obtaining own critical values by running a Monte Carlo simulation. These critical values are influenced by the characteristics of the analysed data and are therefore sample specific. As can be shown, results of hypothesis testing using own critical values are more accurate than when using commonly tabulated "gross" critical values.

In the empirical part the problem of short time series is demonstrated on the example of causality between short-term interest rates on the interbank market and short-term deposit interest rates in the Czech Republic in the time period 2001/I–2007/I. Using standard critical values statistical causality between short-term interest rates on the interbank market and short-term deposit interest rates was found to be significant at the 5 percent and 10 percent level of significance when analysing a data set of 73 observations. When the number of observations was deliberately reduced down to 27 values, the EG test failed to detect the cointegration. Therefore the authors run a Monte Carlo experiment to derive own critical values especially for the small sample containing only 27 observations. When using these own critical values to test the hypothesis once again, different results were obtained: this time the cointegration test revealed significant statistical causality at the 10 percent level as well as at the 5 percent level of significance.

Monte Carlo method can be of use in the process of calculating own critical values for analysing short time series. In the case of small samples (i.e. less then 30 observations) this methodology can improve not only obtained estimates, but also applied tests and their results. Thus, identification of own critical values has become an essential condition for application of quantitative methods and for interpretation of gained results.

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TESTOWANIE HIPOTEZ W WARUNKACH NIEPEŁNEJ INFORMACJI – IDENTYFIKACJA WŁASNYCH WARTOŚCI KRYTYCZNYCH

Transformacja gospodarcza w Europie Centralnej i Wschodniej oraz procesy towarzyszące integracji europejskiej stanowią istotne źródła przemian strukturalnych tych gospodarek, z drugiej jednak strony powodują, żeszeregi czasowe są zbyt krótkie. Co za tym idzie, przeprowadzając

badania empiryczne dotyczące gospodarek unijnych borykamy się ze problemami związanymi ze zbyt krótkimi szeregami czasowymi.

W konsekwencji, modelowanie na podstawie szeregów czasowych i obciążoność wyników analiz ekonometrycznych przeprowadzanych dlaeuropejskich makroekonomicznych szeregów czasowych są utrudnione ze względu na długość próby. Liczba obserwacji odgrywa bowiem znaczenie nie tylko dla klasycznego modelowania szeregów czasowych i prognozowania w oparciu o nie, ale także – a może przede wszystkim – dla weryfikacji teoretycznych założeń. Stąd metody ilościowe, stosowane zazwyczaj do testowania hipotez, w przypadku niewystarczającej liczby obserwacji są często uzupełniane poprzez analizę jakościową, która jednak wnosi pewną dozę subiektywizmu.

Jednym z czynników oddziałujących na stabilność testowania jest wielkość próby. Dla dużych prób estymatory są zwykle asymptotyczne i wykazują dość dobrą stabilność. Z drugiej strony, jeśli próba jest mała (mniej niż 30 obserwacji) stabilność testów statystycznych jest niska. Co więcej, dla małych prób prawdopodobieństwo błędnego odrzucenia hipotezy zerowej, określane jako błąd I rodzaju, bywa wysokie. Innymi słowy, niedostateczna moc testu ogranicza możliwości zastosowania metod ekonometrycznych dla krótkich szeregów czasowych obejmujących dane makroekonomiczne.

Jedną z możliwości rozwiązania tego problemu jest zastosowanie symulacji Monte Carlo i ustalenie własnych wartości krytycznych. Wartości krytyczne zostają zidentyfikowane na bazie symulacji przeprowadzonych na szeregach czasowych o odpowiedniej długości. Autorzy zastosowali tę metodologię tylko dla potrzeb analizy makroekonomicznych szeregów czasowych (uwzględniając kointegrację) bez dalszych ograniczeń w ich zastosowaniu. Bazowym założeniem analizy jest odporność hipotezy o stacjonarności szeregów czasowych. Metodologia zaprezentowana i opisana w tej pracy zastosowana została do określenia związków przyczynowo-skutkowych pomiędzy międzybankową stopą procentową a wybranymi stopami oprocentowania w bankowości detalicznej w Czechach.

Słowa kluczowe: sumulacja Monte Carlo, test ADF, kointegracja, analiza szeregów czasowych, stopa procentowa.