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OPTIMUM CHEMICAL BALANCE WEIGHING DESIGN WITH CORRELATED ERRORS BASED ON BIPARTITE AND TERNARY DESIGNS

Abstract

The paper is studying the estimation problem of individual weights of objects using the chemical balance weighing design under the restriction on the number times in which each object is weighed. It is assumed that the errors have the same variances and they are equal correlated. The necessary and sufficient conditions under which the lower bound of variance of each of estimated weights is attained are given. For construction of the design matrix of the optimum chemical balance weighing design we use the incidence matrices of the balanced bipartite weighing designs and the ternary balanced block designs.

Key words: balanced bipartite weighing design, chemical balance weighing design, ternary balanced block design.
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1. Introduction

The statistical problem is to estimate the vector $\mathbf{w}$ when the experiment is taken according to the model

$$\mathbf{y} = \mathbf{X}\mathbf{w} + \mathbf{e} \qquad (1)$$

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where $\mathbf{y}$ is an $n \times 1$ random vector of observations, $\mathbf{w}$ is an $p \times 1$ column vector of unknown measurements of objects and $\mathbf{e}$ is an $n \times 1$ random vector of errors. If $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}(-1,\ 0,\ 1)$, where $\mathbf{\Phi}_{n\times p,m}(-1,\ 0,\ 1)$ is the class of the $n \times p$ matrices with elements equal to –1, 0 or 1, m is the maximum number of elements equal to –1 and 1 in each column of the matrix $\mathbf{X}$, then the model (1) is called the model of the chemical balance weighing design. We assume that there are not systematic errors, i.e. $\mathrm{E}(\mathbf{e}) = \mathbf{0}_n$, and that the errors are equal correlated with the same variances, i.e. $\mathrm{E}(\mathbf{ee}') = \sigma^2\mathbf{G}$, where

$$\mathbf{G} = g\left((1-\rho)\mathbf{I}_n + \rho\,\mathbf{1}_n\mathbf{1}_n'\right),\ g > 0,\ \frac{-1}{n-1} < \rho < 1 \qquad (2)$$

$\mathbf{G}$ in (2) is the $n \times n$ positive definite matrix of known elements, $\mathbf{0}_n$ is the $n \times 1$ column vector of zeros, $\mathbf{I}_n$ is the $n \times n$ identity matrix.

For a given matrix $\mathbf{G}$ the optimality problem is concerned with efficient estimation in some sense by a proper choice of the design matrix $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}(-1,\ 0,\ 1)$. The design is optimal if allows for estimation of all individual weights with the least possible variance for each one of them. The model is the standard Gauss-Markoff model and it is well known that if $\mathbf{X}$ is of full column rank and $\mathbf{G}$ is the positive definite matrix then $\hat{\mathbf{w}} = \left(\mathbf{X}'\mathbf{G}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{G}^{-1}\mathbf{y}$ and $\mathrm{Var}(\hat{\mathbf{w}}) = \sigma^2(\mathbf{X}'\mathbf{G}^{-1}\mathbf{X})^{-1}$.

The problems concern on determining of unknown measurements of objects in the model of the optimum chemical balance weighing design for $x_{ij} = -1$ or 1 were considered in Banerjee (1975), Raghavarao (1971), Shah and Sinha (1989). For the same matrix $\mathbf{X}$ and correlated errors in Ceranka and Katulska (1998) the necessary and sufficient conditions under which the lower bound of variances of estimators was attained are given. In Ceranka and Graczyk (2003) is considered the problem of existing of the optimum design for $x_{ij} = -1$, 0 or 1 and for equal correlated errors. In this case the optimality conditions are depended on the sign of parameter ρ. Cause of this from Ceranka and Graczyk (2003) we have

Theorem 1.1. Let $0 \le \rho < 1$. Any nonsingular chemical balance weighing design with the design matrix $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}(-1,\ 0,\ 1)$ and with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where the matrix $\mathbf{G}$ is of the form (2), is optimal if and only if

(i) $\mathbf{X}'\mathbf{X} = m\,\mathbf{I}_p$ and

(ii) $\mathbf{X}'\mathbf{1}_n = \mathbf{0}_p$.

Theorem 1.2. Let $\frac{-1}{n-1} < \rho < 0$. Any nonsingular chemical balance weighing design with the design matrix $\mathbf{X} \in \mathbf{\Phi}_{n \times p, m}(-1, 0, 1)$ and with the variance–covariance matrix of errors $\sigma^2 \mathbf{G}$, where the matrix $\mathbf{G}$ is of the form (2), is optimal if and only if

(i) $\mathbf{X}'\mathbf{X} = m\, \mathbf{I}_p - \frac{\rho\,(m-2u)^2}{1+\rho\,(n-1)}\left(\mathbf{I}_p - \mathbf{1}_p\mathbf{1}_p'\right)$,

(ii) $u_1 = u_2 = ... = u_p = u$ and

(iii) $\mathbf{X}'\mathbf{1}_n = \mathbf{z}_p$,

where $\mathbf{z}_p$ is $p \times 1$ vector, for which the j-th element is equal $(m-2u)$ or $-(m-2u)$, $j = 1, 2, ..., p$.

2. Balanced block designs

In this section we present the definition of the balanced bipartite weighing design and the ternary balanced block design. Based of their incidence matrices in the section 3 we form the design matrix of the optimum chemical balance weighing design.

A balanced bipartite weighing design there is an arrangement of v treatments in b blocks such that each weighing containing k distinct treatments is divided into 2 subblocks containing k_1 and k_2 treatments, respectively, where $k = k_1 + k_2$. Each treatment appears in r blocks. Every pair of treatments from different subblocks appears together in λ_1 blocks and every pair of treatments from the same subblock appears together in λ_2 blocks. The integers v, b, r, k_1, k_2, λ_1, λ_2 are called the parameters of the balanced bipartite block design. If $k_1 \neq k_2$ then each object exists in r_1 blocks in the first subblock and in r_2 blocks in the second subblock, $r_1 + r_2 = r$, where $r_1 = \frac{\lambda_1(v-1)}{2k_2}$, $r_2 = \frac{\lambda_1(v-1)}{2k_1}$.

A ternary balanced block design is defined as the design consisting of b blocks, each of size k, chosen from a set of objects of size v, in such a way that each treatment occurs r times altogether and 0, 1 or 2 times in each block, (2 appears at least one) and each of the distinct pairs appears λ times. Any ternary balanced block design is regular, that means, each treatment occurs once in ρ_1 blocks and twice in ρ_2 blocks, where ρ_1 and ρ_2 are constant for the design.

3. The design matrix

In this section we will present new method of construction of the design matrix $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}(-1, 0, 1)$ of the optimum chemical balance weighing design. It is based on the incidence matrices of the balanced bipartite weighing design and ternary balanced block design.

Let $\mathbf{N}_1^*$ be the incidence matrix of the balanced bipartite weighing design with parameters v, b_1, r_1, k_{11}, k_{21}, λ_{11}, λ_{21}. Using this matrix we form the matrix $\mathbf{N}_1$ by replacing k_{11} elements equal to +1 of each column which correspond to the elements belonging to the first subblock by −1. Let $\mathbf{N}_2$ be the incidence matrix of ternary balanced block design with the parameters v, b_2, r_2, k_2, λ_2, ρ_{12}, ρ_{22}. Hence the design matrix of the chemical balance weighing design is given as

$$\mathbf{X} = \begin{bmatrix} \mathbf{N}_1' \\ \mathbf{N}_2' - \mathbf{1}_{b_2}\mathbf{1}_v' \end{bmatrix} \tag{3}$$

Lemma 3.1. Chemical balance weighing design with the matrix $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}(-1, 0, 1)$ given by (3) is nonsingular if and only if

(i) $k_{11} \neq k_{21}$ or

(ii) $v \neq k_2$.

Proof. Since $\mathbf{G}$ is the positive definite matrix then $\mathbf{X}'\mathbf{G}^{-1}\mathbf{X}$ is nonsingular if and only if $\mathbf{X}'\mathbf{X}$ is nonsingular. Hence

$$\mathbf{X}'\mathbf{X} = [r_1 - \lambda_{21} + \lambda_{11} + r_2 + 2\rho_{22} - \lambda_2]\mathbf{I}_v + (\lambda_{21} - \lambda_{11} + b_2 - 2r_2 + \lambda_2)\mathbf{1}_v\mathbf{1}_v' \tag{4}$$

and $det(\mathbf{X}'\mathbf{X}) = [r_1 - \lambda_{21} + \lambda_{11} + r_2 + 2\rho_{22} - \lambda_2]^{v-1}\left[\frac{r_2}{k_2}(v-k_2)^2 + \frac{(v-1)\lambda_{11}}{2k_{11}k_{21}}(k_{11}-k_{21})^2\right]$.

Thus we get the thesis.

Theorem 3.1. Let $0 \leq \rho < 1$. Any nonsingular chemical balance weighing design with the matrix $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}(-1, 0, 1)$ given in (3) and with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where the matrix $\mathbf{G}$ is of the form (2), is optimal for the estimation of individual unknown measurements of objects if and only if

(i) $(\lambda_{21} - \lambda_{11}) + (b_2 + \lambda_2 - 2r_2) = 0$ and

(ii) $b_2 - r_2 - \frac{\lambda_{11}(v-1)(k_{21}-k_{11})}{2k_{11}k_{21}} = 0$.

Proof. It is consequence of the equality (4) and the Theorem 1.1.

Theorem 3.2. Let $0 \le \rho < 1$. If the parameters of the balanced bipartite weighing design and the ternary balanced block design are equal to

(i) $v=5$, $b_1=5$, $r_1=5$, $k_{11}=1$, $k_{21}=4$, $\lambda_{11}=2$, $\lambda_{21}=3$ and $v=5$, $b_2=15$, $r_2=12$, $k_2=4$, $\lambda_2=8$, $\rho_{12}=8$, $\rho_{22}=2$;

(ii) $v=9$, $b_1=36$, $r_1=24$, $k_{11}=2$, $k_{21}=4$, $\lambda_{11}=8$, $\lambda_{21}=7$ and $v=9$, $b_2=24$, $r_2=16$, $k_2=6$, $\lambda_2=9$, $\rho_{12}=8$, $\rho_{22}=4$;

(iii) $v=9$, $b_1=36$, $r_1=24$, $k_{11}=3$, $k_{21}=5$, $\lambda_{11}=15$, $\lambda_{21}=13$ and $v=9$, $b_2=18$, $r_2=10$, $k_2=5$, $\lambda_2=4$, $\rho_{12}=2$, $\rho_{22}=4$;

(iv) $v=11$, $b_1=55$, $r_1=50$, $k_{11}=4$, $k_{21}=6$, $\lambda_{11}=24$, $\lambda_{21}=22$ and $v=11$, $b_2=22$, $r_2=12$, $k_2=6$, $\lambda_2=5$, $\rho_{12}=2$, $\rho_{22}=5$;

(v) $v=13$, $b_1=78$, $r_1=72$, $k_{11}=5$, $k_{21}=7$, $\lambda_{11}=35$, $\lambda_{21}=31$ and $v=13$, $b_2=26$, $r_2=14$, $k_2=7$, $\lambda_2=6$, $\rho_{12}=2$, $\rho_{22}=6$;

then $\mathbf{X}$ in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is in (2).

Theorem 3.3. Let $\frac{-1}{n-1} < \rho < 0$. Any nonsingular chemical balance weighing design with the matrix $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}(-1,\ 0,\ 1)$ given in (3), and with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where the matrix $\mathbf{G}$ is of the form (2), is optimal for the estimation of individual unknown measurements of objects if and only if

(i) $\rho = \dfrac{\lambda_{21}-\lambda_{11}+b_2+\lambda_2-2r_2}{(r_{21}-r_{11}+r_2-b_2)^2-(b_1+b_2-1)\cdot(\lambda_{21}-\lambda_{11}+b_2+\lambda_2-2r_2)}$ and

(ii) $\lambda_{21}-\lambda_{11}+b_2+\lambda_2-2r_2<0$.

Proof. It is consequence of the equality (4) and the Theorem 1.2.

Theorem 3.4. Let $\frac{-1}{n-1} < \rho < 0$. If for a given ρ the parameters of the balanced bipartite weighing design are equal to $v=2s$, $b_1=s(2s-1)$, $r_1=3(2s-1)$, $k_{11}=2$, $k_{21}=4$, $\lambda_{11}=8$, $\lambda_{21}=7$ and the parameters of the ternary balanced block design are equal to

(i) $\rho=-\left(6s^2-29s+80\right)^{-1}$, $v=2s$, $b_2=8s$, $r_2=8(s-1)$, $k_2=2(s-1)$, $\lambda_2=\rho_{12}=8(s-2)$, $\rho_{22}=4$, $s=3,4,\ldots$;

(ii) $\rho=-\left(6s^2-11s+8\right)^{-1}$, $v=b_2=2s$, $r_2=k_2=2(s-1)$, $\lambda_2=\rho_{12}=2(s-2)$, $\rho_{22}=1$, $s=3,4,\ldots$;

(iii) $\rho=-\left(s(6s+7)\right)^{-1}$, $v=2s$, $b_2=4s$, $r_2=2(2s+1)$, $k_2=2s+1$, $\lambda_2=4(s+1)$, $\rho_{12}=2(2s-1)$, $\rho_{22}=2$, $s=3,4,\ldots$;

(iv) $\rho=-\left(6s^2+19s+8\right)^{-1}$, $v=2s$, $b_2=8s$, $r_2=4(2s+1)$, $k_2=2s+1$, $\lambda_2=8(s+1)$, $\rho_{12}=4(2s-1)$, $\rho_{22}=4$, $s=3,4,\ldots$;

(v) $\rho=-3\left(10s^2+17s-20\right)^{-1}$, $v=k_2=2s$, $b_2=r_2=2(4s-3)$, $\lambda_2=8(s-1)$, $\rho_{12}=4(s-1)$, $\rho_{22}=2s-1$, $s=3,4,\ldots$;

(vi) $\rho=-3\left(10s^2+17s-2\right)^{-1}$, $v=k_2=2s$, $b_2=r_2=8s$, $\lambda_2=2(4s-1)$, $\rho_{12}=2(2s+1)$, $\rho_{22}=2s-1$, $s=3,4,\ldots$;

(vii) $\rho=-3\left(10s^2+17s+4\right)^{-1}$, $v=k_2=2s$, $b_2=r_2=2(4s+1)$, $\lambda_2=8s$, $\rho_{12}=4(s+1)$, $\rho_{22}=2s-1$, $s=3,4,\ldots$;

(viii) $\rho=-3\left(10s^2+17s+10\right)^{-1}$, $v=k_2=2s$, $b_2=r_2=4(2s+1)$, $\lambda_2=2(4s+1)$, $\rho_{12}=2(2s+3)$, $\rho_{22}=2s-1$, $s=3,4,\ldots$;

(ix) $\rho=-3\left(10s^2+5s+3u-8\right)^{-1}$, $v=k_2=2s$, $b_2=r_2=4s+u-2$, $\lambda_2=4s+u-4$, $\rho_{12}=u$, $\rho_{22}=2s-1$, $s=3,4,\ldots$, $u=1,2,\ldots$;

(x) $\rho=-\left(6s^2+4u^2-5s+4u-6su\right)^{-1}$, $v=2s$, $b_2=2su$, $r_2=2u(s-1)$, $k_2=2(s-1)$, $\lambda_2=\rho_{12}=2u(s-2)$, $\rho_{22}=u$, $s=3,4,\ldots$, $u=1,2,\ldots$;

(xi) $\rho=-\left(6s^2+9u^2-5s+6u-10su\right)^{-1}$, $v=2s$, $b_2=2su$, $r_2=u(2s-3)$, $k_2=2s-3$, $\lambda_2=2u(s-3)$, $\rho_{12}=u(2s-9)$, $\rho_{22}=3u$, $s=5,6,\ldots$, $u=1,2,\ldots$;

(xii) $\rho=-\left(6s^2+16u^2-5s+8u-14su\right)^{-1}$, $v=2s$, $b_2=2su$, $r_2=2u(s-2)$, $k_2=2(s-2)$, $\lambda_2=2u(s-4)$, $\rho_{12}=2u(s-8)$, $\rho_{22}=6u$, $s=9,10,\ldots$, $u=1,2,\ldots$;

then $\mathbf{X}$ in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is in (2).

Theorem 3.5. Let $\frac{-1}{n-1} < \rho < 0$. If for a given ρ the parameters of the balanced bipartite weighing design are equal to $v = 2s+1$, $b_1 = s(2s+1)$, $r_1 = 3s$, $k_{11} = 1$, $k_{21} = 2$, $\lambda_{11} = 2$, $\lambda_{21} = 1$ and the parameters of the ternary balanced block design are equal to

(i) $\rho = -\left(3s^2 - 7s + 67\right)^{-1}$, $v = 2s+1$, $b_2 = 4(2s+1)$, $r_2 = 4(2s-1)$, $k_2 = 2s-1$, $\lambda_2 = \rho_{12} = 4(2s-3)$, $\rho_{22} = 4$, $s = 1, 2, \ldots$;

(ii) $\rho = -\left(3s^2 - s + 4\right)^{-1}$, $v = b_2 = 2s+1$, $r_2 = k_2 = 2s-1$, $\lambda_2 = \rho_{12} = 2s-3$, $\rho_{22} = 1$, $s = 3, 4, \ldots$, except the case $s = 5$;

(iii) $\rho = -\left(3s^2 + 9s + 7\right)^{-1}$, $v = 2s+1$, $b_2 = 2(2s+1)$, $r_2 = 4(s+1)$, $k_2 = 2(s+1)$, $\lambda_2 = 2(2s+3)$, $\rho_{12} = 4s$, $\rho_{22} = 2$, $s = 2, 3, \ldots$, except the case $s = 5$;

(iv) $\rho = -\left(3s^2 + 17s + 19\right)^{-1}$, $v = 2s+1$, $b_2 = 4(2s+1)$, $r_2 = 8(s+1)$, $k_2 = 2(s+1)$, $\lambda_2 = 4(2s+3)$, $\rho_{12} = 8s$, $\rho_{22} = 4$, $s = 1, 2, \ldots$;

(v) $\rho = -\left(3s^2 + 17s + 23\right)^{-1}$, $v = 2s+1$, $b_2 = 4(2s+1)$, $r_2 = 8(s+1)$, $k_2 = 2(s+1)$, $\lambda_2 = 4(2s+3)$, $\rho_{12} = 8s$, $\rho_{22} = 4$, $s = 2, 3, \ldots$, except the case $s = 5$;

(vi) $\rho = -3\left(7s^2 + 27s - 9\right)^{-1}$, $v = k_2 = 2s+1$, $b_2 = r_2 = 2(4s-1)$, $\lambda_2 = 4(2s-1)$, $\rho_{12} = 2(2s-1)$, $\rho_{22} = 2s$, $s = 1, 2, \ldots$;

(vii) $\rho = -3\left(7s^2 + 15s + 3\right)^{-1}$, $v = k_2 = 2s+1$, $b_2 = r_2 = 2(2s+1)$, $\lambda_2 = 4s$, $\rho_{12} = 2$, $\rho_{22} = 2s$, $s = 1, 2, \ldots$;

(viii) $\rho = -3\left(7s^2 + 27s + 9\right)^{-1}$, $v = k_2 = 2s+1$, $b_2 = r_2 = 4(2s+1)$, $\lambda_2 = 2(4s+1)$, $\rho_{12} = 4(s+1)$, $\rho_{22} = 2s$, $s = 2, 3, \ldots$, except the case $s = 5$;

(ix) $\rho = -3\left(7s^2 + 27s + 15\right)^{-1}$, $v = k_2 = 2s+1$, $b_2 = r_2 = 2(4s+3)$, $\lambda_2 = 4(2s+1)$, $\rho_{12} = 2(2s+3)$, $\rho_{22} = 2s$, $s = 1, 2, \ldots$;

(x) $\rho = -3\left(7s^2 + 27s + 21\right)^{-1}$, $v = k_2 = 2s+1$, $b_2 = r_2 = 8(s+1)$, $\lambda_2 = 2(4s+3)$, $\rho_{12} = 4(s+2)$, $\rho_{22} = 2s$, $s = 2, 3, \ldots$, except the case $s = 5$;

(xi) $\rho=-3\left(7s^2+15s+3u\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=4s+u+1$, $\lambda_2=4s+u-1$, $\rho_{12}=u+1$, $\rho_{22}=2s$, $s=5,6,\ldots$, $u=1,2,\ldots$, except the case $s=5$;

(xii) $\rho=-\left(3s^2+4u^2-2su+s+u-1\right)^{-1}$, $v=2s+1$, $b_2=u(2s+1)$, $r_2=u(2s-1)$, $k_2=2s-1$, $\lambda_2=\rho_{12}=u(2s-3)$, $\rho_{22}=u$, $s=5,6,\ldots$, $u=1,2,\ldots$, except the case $s=2$ and $u=1$;

(xiii) $\rho=-\left(3s^2+9u^2-4su+s+u-1\right)^{-1}$, $v=2s+1$, $b_2=u(2s+1)$, $r_2=2u(s-1)$, $k_2=2(s-1)$, $\lambda_2=u(2s-5)$, $\rho_{12}=2u(s-4)$, $\rho_{22}=3u$, $s=6,7,\ldots$, $u=1,2,\ldots$;

(xiv) $\rho=-\left(3s^2+16u^2-6su+s+u-1\right)^{-1}$, $v=2s+1$, $b_2=u(2s+1)$, $r_2=u(s-3)$, $k_2=2s-3$, $\lambda_2=u(2s-7)$, $\rho_{12}=u(2s-15)$, $\rho_{22}=6u$, $s=8,9,\ldots$, $u=1,2,\ldots$;

then **X** in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where **G** is in (2).

Theorem 3.6. Let $\frac{-1}{n-1}<\rho<0$. If for a given ρ the parameters of the balanced bipartite weighing design are equal to $v=2s+1$, $b_1=s(2s+1)$, $r_1=6s$, $k_{11}=2$, $k_{21}=4$, $\lambda_{11}=8$, $\lambda_{21}=7$ and the parameters of the ternary balanced block design are equal to

(i) $\rho=-\left(6s^2-23s+67\right)^{-1}$, $v=2s+1$, $b_2=4(2s+1)$, $r_2=4(2s-1)$, $k_2=2s-1$, $\lambda_2=\rho_{12}=4(2s-3)$, $\rho_{22}=4$, $s=3,4,\ldots$;

(ii) $\rho=-\left(6s^2-5s+4\right)^{-1}$, $v=b_2=2s+1$, $r_2=k_2=2s-1$, $\lambda_2=\rho_{12}=2s-3$, $\rho_{22}=1$, $s=1,2,\ldots$;

(iii) $\rho=-\left(6s^2+13s+5\right)^{-1}$, $v=2s+1$, $b_2=2(2s+1)$, $r_2=4(s+1)$, $k_2=2(s+1)$, $\lambda_2=2(2s+3)$, $\rho_{12}=4s$, $\rho_{22}=2$, $s=3,4,\ldots$;

(iv) $\rho=-\left(6s^2+25s+19\right)^{-1}$, $v=2s+1$, $b_2=4(2s+1)$, $r_2=8(s+1)$, $k_2=2(s+1)$, $\lambda_2=4(2s+3)$, $\rho_{12}=8s$, $\rho_{22}=4$, $s=3,4,\ldots$;

(v) $\rho=-3\left(10s^2+27s-9\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=2(4s-1)$, $\lambda_2=4(2s-1)$, $\rho_{12}=2(2s-1)$, $\rho_{22}=2s$, $s=3,4,\ldots$;

(vi) $\rho=-3\left(10s^2+15s+3\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=2(2s+1)$, $\lambda_2=4s$, $\rho_{12}=2$, $\rho_{22}=2s$, $s=3,4,\ldots$;

(vii) $\rho=-3\left(10s^2+27s+9\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=4(2s+1)$, $\lambda_2=2(4s+1)$, $\rho_{12}=4(s+1)$, $\rho_{22}=2s$, $s=3,4,\ldots$;

(viii) $\rho=-3\left(10s^2+27s+15\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=2(4s+3)$, $\lambda_2=4(2s+1)$, $\rho_{12}=2(2s+3)$, $\rho_{22}=2s$, $s=3,4,\ldots$;

(ix) $\rho=-3\left(10s^2+27s+21\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=8(s+1)$, $\lambda_2=2(4s+3)$, $\rho_{12}=4(s+2)$, $\rho_{22}=2s$, $s=3,4,\ldots$;

(x) $\rho=-3\left(10s^2+15s+3u\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=4s+u+1$, $\lambda_2=4s+u-1$, $\rho_{12}=u+1$, $\rho_{22}=2s$, $s=3,4,\ldots$, $u=1,2,\ldots$;

(xi) $\rho=-\left(6s^2+4u^2-6su+s+u-1\right)^{-1}$, $v=2s+1$, $b_2=u(2s+1)$, $r_2=u(2s-1)$, $k_2=2s-1$, $\lambda_2=\rho_{12}=u(2s-3)$, $\rho_{12}=u$, $s=3,4,\ldots$, $u=1,2,\ldots$;

(xii) $\rho=-\left(6s^2+9u^2-10su+s+u-1\right)^{-1}$, $v=2s+1$, $b_2=u(2s+1)$, $r_2=2u(s-1)$, $k_2=2(s-1)$, $\lambda_2=u(2s-5)$, $\rho_{12}=2u(s-4)$, $\rho_{22}=3u$, $s=5,6,\ldots$, $u=1,2,\ldots$;

(xiii) $\rho=-\left(6s^2+16u^2-14su+s+u-1\right)^{-1}$, $v=2s+1$, $b_2=u(2s+1)$, $r_2=u(2s-3)$, $k_2=2s-3$, $\lambda_2=u(2s-7)$, $\rho_{12}=u(2s-15)$, $\rho_{22}=6u$, $s=8,9,\ldots$, $u=1,2,\ldots$;

then **X** in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where **G** is in (2).

Theorem 3.7. Let $\frac{-1}{n-1}<\rho<0$. If for a given ρ the parameters of the balanced bipartite weighing design are equal to $v=2s+1$, $b_1=s(2s+1)$, $r_1=7s$, $k_{11}=2$, $k_{21}=5$, $\lambda_{11}=10$, $\lambda_{21}=11$ and the parameters of the ternary balanced block design are equal to

(i) $\rho=-\left(11s^2+9s-3\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=2(4s-1)$, $\lambda_2=4(2s-1)$, $\rho_{12}=2(2s-1)$, $\rho_{22}=2s$;

(ii) $\rho=-\left(11s^2+5s+1\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=2(2s+1)$, $\lambda_2=4s$, $\rho_{12}=2$, $\rho_{22}=2s$;

(iii) $\rho=-\left(11s^2+9s+3\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=4(2s+1)$, $\lambda_2=2(4s+1)$, $\rho_{12}=4(s+1)$, $\rho_{22}=2s$;

(iv) $\rho=-\left(11s^2+9s+5\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=2(4s+3)$, $\lambda_2=4(2s+1)$, $\rho_{12}=2(2s+3)$, $\rho_{22}=2s$;

(v) $\rho=-\left(11s^2+9s+7\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=8(s+1)$, $\lambda_2=2(4s+3)$, $\rho_{12}=4(s+2)$, $\rho_{22}=2s$;

(vi) $\rho=-\left(11s^2+5s+u\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=4s+u+1$, $\lambda_2=4s+u-1$, $\rho_{12}=u+1$, $\rho_{22}=2s$, $u=1,2,\ldots$;

where $s=3,4,\ldots$, then **X** in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, **G** is in (2).

Theorem 3.8. Let $\frac{-1}{n-1}<\rho<0$. If for a given ρ the parameters of the balanced bipartite weighing design are equal to $v=2s+1$, $b_1=s(2s+1)$, $r_1=8s$, $k_{11}=3$, $k_{21}=5$, $\lambda_{11}=15$, $\lambda_{21}=13$ and the parameters of the ternary balanced block design are equal to

(i) $\rho=-\left(3\left(s^2+3s-1\right)\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=2(4s-1)$, $\lambda_2=4(2s-1)$, $\rho_{12}=2(2s-1)$, $\rho_{22}=2s$, $s=1,2,\ldots$;

(ii) $\rho=-\left(3s^2+5s+1\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=2(2s+1)$, $\lambda_2=4s$, $\rho_{12}=2$, $\rho_{22}=2s$, $s=1,2,\ldots$;

(iii) $\rho=-\left(3\left(s^2+3s+1\right)\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=4(2s+1)$, $\lambda_2=2(4s+1)$, $\rho_{12}=4(s+1)$, $\rho_{22}=2s$, $s=4,5,\ldots$;

(iv) $\rho=-\left(3s^2+9s+5\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=2(4s+3)$, $\lambda_2=4(2s+1)$, $\rho_{12}=2(2s+3)$, $\rho_{22}=2s$, $s=1,2,\ldots$;

(v) $\rho=-\left(3s^2+9s+7\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=8(s+1)$, $\lambda_2=2(4s+3)$, $\rho_{12}=4(s+2)$, $\rho_{22}=2s$, $s=4,5,\ldots$;

(vi) $\rho=-\left(4s^2-7s+35\right)^{-1}$, $v=2s+1$, $b_2=4(2s+1)$, $r_2=4(2s-1)$, $k_2=2s-1$, $\lambda_2=\rho_{12}=4(2s-3)$, $\rho_{22}=4$, $s=1,2,\ldots$;

(vii) $\rho=-\left(4s^2-s+2\right)^{-1}$, $v=b_2=2s+1$, $r_2=k_2=2s-1$, $\lambda_2=\rho_{12}=2s-3$, $\rho_{22}=1$, $s=4,5,\ldots$;

(viii) $\rho=-\left(4s^2+9s+3\right)^{-1}$, $v=2s+1$, $b_2=2(2s+1)$, $r_2=4(s+1)$, $k_2=2(s+1)$, $\lambda_2=2(2s+3)$, $\rho_{12}=4s$, $\rho_{22}=2$, $s=4,5,\ldots$;

(ix) $\rho=-\left(4s^2+17s+11\right)^{-1}$, $v=2s+1$, $b_2=4(2s+1)$, $r_2=8(s+1)$, $k_2=2(s+1)$, $\lambda_2=4(2s+3)$, $\rho_{12}=8s$, $\rho_{22}=4$, $s=1,2,\ldots$;

(x) $\rho=-\left(3s^2+5s+u\right)^{-1}$, $v=k_2=2s+1$, $b_2=r_2=4s+u+1$, $\lambda_2=4s+u-1$, $\rho_{12}=u+1$, $\rho_{22}=2s$, $s=4,5,\ldots$, $u=1,2,\ldots$;

(xi) $\rho=-\left(3s^2+2u^2-2su+s+u-1\right)^{-1}$, $v=2s+1$, $b_2=u(2s+1)$, $r_2=u(2s-1)$, $k_2=2s-1$, $\lambda_2=\rho_{12}=u(2s-3)$, $\rho_{22}=u$, $s=4,5,\ldots$, $u=1,2,\ldots$;

(xii) $\rho=-\left(4s^2+8u^2-6su+s+u-1\right)^{-1}$, $v=2s+1$, $b_2=u(2s+1)$, $r_2=u(2s-3)$, $k_2=2s-3$, $\lambda_2=u(2s-7)$, $\rho_{12}=u(2s-15)$, $\rho_{22}=6u$ $s=8,9,\ldots$, $u=1,2,\ldots$;

(xiii) $\rho=-2\left(8s^2+9u^2-8su+2s+2u-2\right)^{-1}$, $v=2s+1$, $b_2=u(2s+1)$, $r_2=2u(s-1)$, $k_2=2(s-1)$, $\lambda_2=u(2s-5)$, $\rho_{12}=2u(s-4)$, $\rho_{22}=3u$, $s=5,6,\ldots$, $u=1,2,\ldots$;

then $\mathbf{X}$ in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is in (2).

Theorem 3.9. Let $\frac{-1}{n-1}<\rho<0$. If for a given ρ the parameters of the balanced bipartite weighing design are equal to $v=4s-3$, $b_1=r_1=2(4s-3)$, $k_{11}=2(s-1)$, $k_{21}=2s-1$, $\lambda_{11}=2(2s-1)$, $\lambda_{21}=4(s-1)$ and the parameters of the ternary balanced block design are equal to

(i) $\rho=-\left(24(s-1)\right)^{-1}$, $v=k_2=4s-3$, $b_2=r_2=2(8s-9)$, $\lambda_2=4(4s-5)$, $\rho_{12}=2(4s-5)$, $\rho_{22}=4(s-1)$, $s=2,3,\ldots$;

(ii) $\rho=-\left(6(4s-3)\right)^{-1}$, $v=4s-3$, $b_2=r_2=4(4s-3)$, $k_2=4s-3$, $\lambda_2=2(8s-7)$, $\rho_{12}=4(2s-1)$, $\rho_{22}=4(s-1)$, $s=2,3,\ldots$;

(iii) $\rho=-\left(8(3s-2)\right)^{-1}$, $v=k_2=4s-3$, $b_2=r_2=2(8s-5)$, $\lambda_2=4(4s-3)$, $\rho_{12}=2(4s-1)$, $\rho_{22}=4(s-1)$, $s=2,3,\ldots$;

(iv) $\rho=-\left(2(12s-7)\right)^{-1}$, $v=k_2=4s-3$, $b_2=r_2=8(2s-1)$, $\lambda_2=2(8s-5)$, $\rho_{12}=8s$, $\rho_{22}=4(s-1)$, $s=2,3,\ldots$;

(v) $\rho=-(24s-1)^{-1}$, $v=4s-3$, $b_2=4(4s-3)$, $r_2=8(2s-1)$, $k_2=2s-1$, $\lambda_2=4(4s-1)$, $\rho_{12}=16(s-1)$, $\rho_{22}=4$, $s=2,3,\ldots$;

(vi) $\rho = -(24s-1)^{-1}$, $v = 4s-3$, $b_2 = 4(4s-3)$, $r_2 = 4(4s-5)$, $k_2 = 4s-5$, $\lambda_2 = \rho_{12} = 4(4s-7)$, $\rho_{22} = 4$, $s = 2, 3, \ldots$;

(vii) $\rho = -(4(4s-3))^{-1}$, $v = k_2 = 4s-3$, $b_2 = r_2 = 2(4s-3)$, $\lambda_2 = 8(s-1)$, $\rho_{12} = 2$, $\rho_{22} = 4(s-1)$, $s = 2, 3, \ldots$;

(viii) $\rho = -(16s-5)^{-1}$, $v = 4s-3$, $b_2 = 2(4s-3)$, $r_2 = 4(2s-1)$, $k_2 = 2(2s-1)$, $\lambda_2 = 2(4s-1)$, $\rho_{12} = 8(s-1)$, $\rho_{22} = 2$, $s = 2, 3, \ldots$;

(ix) $\rho = -(12s-10)^{-1}$, $v = b_2 = 4s-3$, $r_2 = k_2 = 4s-5$, $\lambda_2 = \rho_{12} = 4s-7$, $\rho_{22} = 1$, $s = 3, 4, \ldots$;

(x) $\rho = -(16s+u-13)^{-1}$, $v = k_2 = 4s-3$, $b_2 = r_2 = 8s+u-7$, $\lambda_2 = 8s+u-9$, $\rho_{12} = u+1$, $\rho_{22} = 4(s-1)$, $s = 2, 3, \ldots$, $u = 1, 2, \ldots$;

(xi) $\rho = -(2u^2+4us-7u+8s-5)^{-1}$, $v = 4s-3$, $b_2 = u(4s-3)$, $r_2 = u(4s-5)$, $k_2 = 4s-5$, $\lambda_2 = \rho_{12} = u(4s-7)$, $\rho_{22} = u$, $s = 2, 3, \ldots$, $u = 1, 2, \ldots$, except the case when $s = 2$ and $u = 1$;

(xii) $\rho = -(8u^2+4us-11u+8s-5)^{-1}$, $v = 4s-3$, $b_2 = u(4s-3)$, $r_2 = u(4s-7)$, $k_2 = 4s-7$, $\lambda_2 = u(4s-11)$, $\rho_{12} = u(4s-19)$, $\rho_{22} = 6u$, $s = 5, 6, \ldots$, $u = 1, 2, \ldots$;

(xiii) $\rho = -2(9u^2+8us-18u+16s-10)^{-1}$, $v = 4s-3$, $b_2 = u(4s-3)$, $r_2 = 2u(2s-3)$, $k_2 = 2(2s-3)$, $\lambda_2 = u(4s-9)$, $\rho_{12} = 4u(s-3)$, $\rho_{22} = 3u$, $s = 4, 5, \ldots$, $u = 1, 2, \ldots$;

then $\mathbf{X}$ in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is in (2).

Theorem 3.10. Let $\frac{-1}{n-1} < \rho < 0$. If for a given ρ the parameters of the balanced bipartite weighing design are equal to $v = b_1 = r_1 = 4s-1$, $k_{11} = 2s-1$, $k_{21} = \lambda_{11} = 2s$, $\lambda_{21} = 2s-1$ and the parameters of the ternary balanced block design are equal to

(i) $\rho = -3(60s-17)^{-1}$, $v = 4s-1$, $b_2 = r_2 = 4(4s-1)$, $k_2 = 4s-1$, $\lambda_2 = 2(8s-3)$, $\rho_{12} = 8s$, $\rho_{22} = 2(2s-1)$, $s = 1, 2, \ldots$;

(ii) $\rho = -3(60s-11)^{-1}$, $v = 4s-1$, $b_2 = r_2 = 2(8s-1)$, $k_2 = 4s-1$, $\lambda_2 = 4(4s-1)$, $\rho_{12} = 2(4s+1)$, $\rho_{22} = 2(2s-1)$, $s = 1, 2, \ldots$;

(iii) $\rho=-3(5(12s-7))^{-1}$, $v=4s-1$, $b_2=r_2=2(8s-5)$, $k_2=4s-1$, $\lambda_2=4(4s-3)$, $\rho_{12}=2(4s-3)$, $\rho_{22}=2(2s-1)$, $s=1,2,...$;

(iv) $\rho=-3(60s-5)^{-1}$, $v=4s-1$, $b_2=r_2=16s$, $k_2=4s-1$, $\lambda_2=2(8s-1)$, $\rho_{12}=4(2s+1)$, $\rho_{22}=2(2s-1)$, $s=1,2,...$;

(v) $\rho=-(20s+19)^{-1}$, $v=4s-1$, $b_2=4(4s-1)$, $r_2=16s$, $k_2=4s$, $\lambda_2=4(4s+1)$, $\rho_{12}=8(2s-1)$, $\rho_{22}=4$, $s=1,2,...$;

(vi) $\rho=-3((36s-11))^{-1}$, $v=4s-1$, $b_2=r_2=2(4s-1)$, $k_2=4s-1$, $\lambda_2=2(2s-1)$, $\rho_{12}=2$, $\rho_{22}=2(2s-1)$, $s=1,2,...$;

(vii) $\rho=-(12s+5)^{-1}$, $v=4s-1$, $b_2=2(4s-1)$, $r_2=8s$, $k_2=4s$, $\lambda_2=2(4s+1)$, $\rho_{12}=4(2s-1)$, $\rho_{22}=2$, $s=1,2,...$;

(viii) $\rho=-(2(4s-1))^{-1}$, $v=b_2=4s-1$, $r_2=k_2=4s-3$, $\lambda_2=\rho_{12}=4s-5$, $\rho_{22}=1$, $s=2,3,...$;

(ix) $\rho=-3(36s+3u-14)^{-1}$, $v=4s-1$, $b_2=r_2=8s+u-3$, $k_2=4s-1$, $\lambda_2=8s+u-5$, $\rho_{12}=u+1$, $\rho_{22}=2(2s-1)$, $s=2,3,...$, $u=1,2,...$;

(x) $\rho=-(4u^2+4us+4s-5u-1)^{-1}$, $v=4s-1$, $b_2=u(4s-1)$, $r_2=u(4s-3)$, $k_2=4s-3$, $\lambda_2=\rho_{12}=u(4s-5)$, $\rho_{22}=u$, $s=2,3,\dots$, $u=1,2,\dots$;

(xi) $\rho=-(9u^2+4us+4s-7u-1)^{-1}$, $v=4s-1$, $b_2=u(4s-1)$, $r_2=4u(s-1)$, $k_2=4(s-1)$, $\lambda_2=u(4s-7)$, $\rho_{12}=2u(2s-5)$, $\rho_{22}=3u$, $s=3,4,...$, $u=1,2,...$;

(xii) $\rho=-(16u^2+4us+4s-9u-1)^{-1}$, $v=4s-1$, $b_2=u(4s-1)$, $r_2=u(4s-5)$, $k_2=4s-5$, $\lambda_2=u(4s-9)$, $\rho_{12}=u(4s-17)$, $\rho_{22}=6u$, $s=5,6,\dots$, $u=1,2,\dots$;

then $\mathbf{X}$ in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is in (2).

Theorem 3.11. Let $\frac{-1}{n-1}<\rho<0$. If for a given ρ the parameters of the balanced bipartite weighing design are equal to $v=4s$, $b_1=r_1=4s(4s-1)$, $k_{11}=2s-1$, $k_{21}=2s+1$, $\lambda_{11}=2(4s^2-1)$, $\lambda_{21}=2(4s^2-2s+1)$, $4s-1$ is prime power and the parameters of the ternary balanced block design are equal to

(i) $\rho=(1-s)\left(16s^3-9s+2\right)^{-1}$, $v=b_2=4s$, $r_2=k_2=2(2s-1)$, $\lambda_2=\rho_{12}=4(s-1)$, $\rho_{22}=1$, $s=2,3,\ldots$;

(ii) $\rho=(1-s)\left(16s^3+12s^2-5s+2\right)^{-1}$, $v=4s$, $b_2=16s$, $r_2=4(4s+1)$, $k_2=4s+1$, $\lambda_2=8(2s+1)$, $\rho_{12}=4(4s-1)$, $\rho_{22}=4$, $s=1,2,\ldots$;

(iii) $\rho=(1-s)\left(16s^3+12s^2-53s+26\right)^{-1}$, $v=4s$, $b_2=16s$, $r_2=8(2s-1)$, $k_2=2(2s-1)$, $\lambda_2=\rho_{12}=16(s-1)$, $\rho_{22}=4$, $s=1,2,\ldots$;

(iv) $\rho=(1-s)\left(16s^3-4s^2-5s+u^2+2u-12su+4s^2u+2\right)^{-1}$, $v=4s$, $b_2=4su$, $r_2=2u(2s-1)$, $k_2=2(2s-1)$, $\lambda_2=\rho_{12}=4u(s-1)$, $\rho_{22}=u$, $s=2,3,\ldots$, $u=1,2,\ldots$;

(v) $\rho=(1-s)\left(16s^3-4s^2-5s+4u^2+4u-20su+4s^2u+2\right)^{-1}$, $v=4s$, $b_2=4su$, $r_2=4u(s-1)$, $k_2=4(s-1)$, $\lambda_2=4u(s-2)$, $\rho_{12}=4u(s-4)$, $\rho_{22}=6u$, $s=5,6,\ldots$, $u=1,2,\ldots$;

(vi) $\rho=(1-s)\left(32s^3+40s^2-30s+3\right)^{-1}$, $v=k_2=4s$, $b_2=r_2=16s$, $\lambda_2=2(8s-1)$, $\rho_{12}=2(4s+1)$, $\rho_{22}=4s-1$, $s=2,3,\ldots$, $u=1,2,\ldots$;

(vii) $\rho=(1-s)\left(16s^3+4s^2-5s+1\right)^{-1}$, $b_2=8s$, $r_2=2(4s+1)$, $k_2=4s+1$, $\lambda_2=4(2s+1)$, $\rho_{12}=2(4s-1)$, $\rho_{22}=2$, $s=2,3,\ldots$;

(viii) $\rho=4(1-s)\left(64s^3-16s^2-20s+9u^2+12u-64su+16s^2u+8\right)^{-1}$, $v=4s$, $b_2=4su$, $r_2=u(4s-3)$, $k_2=4s-3$, $\lambda_2=2u(2s-3)$, $\rho_{12}=u(4s-9)$, $\rho_{22}=3u$, $s=3,4,\ldots$, $u=1,2,\ldots$;

(ix) $\rho=(1-2s)\left(32s^3+40s^2-34s+5\right)^{-1}$, $v=k_2=4s$, $b_2=r_2=2(8s-1)$, $\lambda_2=4(4s-1)$, $\rho_{12}=8s$, $\rho_{22}=4s-1$, $s=1,2,\ldots$;

(x) $\rho=(1-2s)\left(32s^3+40s^2-26s+1\right)^{-1}$, $v=k_2=4s$, $b_2=r_2=2(8s+1)$, $\lambda_2=16s$, $\rho_{12}=4(2s+1)$, $\rho_{22}=4s-1$, $s=1,2,\ldots$;

(xi) $\rho=(1-2s)\left(32s^3+40s^2-42s+9\right)^{-1}$, $v=k_2=4s$, $b_2=r_2=2(8s-3)$, $k_2=4s$, $\lambda_2=8(2s-1)$, $\rho_{12}=4(2s-1)$, $\rho_{22}=4s-1$, $s=1,2,\ldots$;

(xii) $\rho=(1-2s)\left(32s^3+24s^2-26s-u+2su+5\right)^{-1}$, $v=k_2=4s$, $b_2=r_2=8s+u-2$, $\lambda_2=8s+u-4$, $\rho_{12}=u$, $\rho_{22}=4s-1$, $s=1,2,\ldots$;

(xiii) $\rho=(1-2s)\left(32s^3+40s^2-22s-1\right)^{-1}$, $v=k_2=4s$, $b_2=r_2=4(4s+1)$, $\lambda_2=2(8s+1)$, $\rho_{12}=2(4s+3)$, $\rho_{22}=4s-1$, $s=1,2,\ldots$;

then $\mathbf{X}$ in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is in (2).

Theorem 3.12. Let $\frac{-1}{n-1} < \rho < 0$. If for a given ρ the parameters of the balanced bipartite weighing design are equal to $v = 4s+1$, $b_1 = s(4s+1)$, $r_1 = 5s$, $k_{11} = 2$, $k_{21} = 3$, $\lambda_{11} = 3$, $\lambda_{21} = 2$ and the parameters of the ternary balanced block design are equal to

(i) $\rho = -\left(5s^2+s+4\right)^{-1}$, $v = b_2 = 4s+1$, $r_2 = k_2 = 4s-1$, $\lambda_2 = \rho_{12} = 4s-3$, $\rho_{22} = 1$, $s = 2, 3, \ldots;$

(ii) $\rho = -\left(5s^2+s+67\right)^{-1}$, $v = 4s+1$, $b_2 = 4(4s+1)$, $r_2 = 4(4s-1)$, $k_2 = 4s-1$, $\lambda_2 = \rho_{12} = 4(4s-3)$, $\rho_{22} = 4$, $s = 2, 3, \ldots;$

(iii) $\rho = -\left(5s^2+13s+5\right)^{-1}$, $v = 4s+1$, $b_2 = 2(4s+1)$, $r_2 = 4(2s+1)$, $k_2 = 2(2s+1)$, $\lambda_2 = 2(4s+3)$, $\rho_{12} = 8s$, $\rho_{22} = 2$, $s = 2, 3, \ldots;$

(iv) $\rho = -\left(5s^2+25s+19\right)^{-1}$, $v = 4s+1$, $b_2 = 4(4s+1)$, $r_2 = 8(2s+1)$, $k_2 = 2(2s+1)$, $\lambda_2 = 4(4s+3)$, $\rho_{12} = 16s$, $\rho_{22} = 4$, $s = 2, 3, \ldots;$

(v) $\rho = -\left(5s^2+25s+23\right)^{-1}$, $v = 4s+1$, $b_2 = 4(4s+1)$, $r_2 = 8(2s+1)$, $k_2 = 2(2s+1)$, $\lambda_2 = 4(4s+3)$, $\rho_{12} = 16s$, $\rho_{22} = 4$, $s = 2, 3, \ldots;$

(vi) $\rho = -3\left(13s^2+27s+3\right)^{-1}$, $v = k_2 = 4s+1$, $b_2 = r_2 = 2(4s+1)$, $\lambda_2 = 8s$, $\rho_{12} = 2$, $\rho_{22} = 4s$, $s = 2, 3, \ldots;$

(vii) $\rho = -3\left(13s^2+33s+15\right)^{-1}$, $v = 4s+1$, $b_2 = 3(4s+1)$, $r_2 = 12s$, $k_2 = 4s$, $\lambda_2 = 12s-5$, $\rho_{12} = \rho_{22} = 4s$, $s = 2, 3, \ldots;$

(viii) $\rho = -3\left(13s^2+51s-9\right)^{-1}$, $v = k_2 = 4s+1$, $b_2 = r_2 = 2(8s-1)$, $\lambda_2 = 4(4s-1)$, $\rho_{12} = 2(4s-1)$, $\rho_{22} = 4s$, $s = 2, 3, \ldots;$

(ix) $\rho = -3\left(13s^2+51s+9\right)^{-1}$, $v = k_2 = 4s+1$, $b_2 = r_2 = 4(4s+1)$, $\lambda_2 = 2(8s+1)$, $\rho_{12} = 4(2s+1)$, $\rho_{22} = 4s$, $s = 2, 3, \ldots;$

(x) $\rho = -3\left(13s^2+51s+15\right)^{-1}$, $v = k_2 = 4s+1$, $b_2 = r_2 = 2(8s+3)$, $\lambda_2 = 4(4s+1)$, $\rho_{12} = 2(4s+3)$, $\rho_{22} = 4s$, $s = 2, 3, \ldots;$

(xi) $\rho = -3\left(13s^2+51s+21\right)^{-1}$, $v = k_2 = 4s+1$, $b_2 = r_2 = 8(2s+1)$, $\lambda_2 = 2(8s+3)$, $\rho_{12} = 8(s+1)$, $\rho_{22} = 4s$, $s = 2, 3, \ldots;$

(xii) $\rho = -3\left(13s^2+27s+3u\right)^{-1}$, $v = k_2 = 4s+1$, $b_2 = r_2 = 8s+u+1$, $\lambda_2 = 8s+u-1$, $\rho_{12} = u+1$, $\rho_{22} = 4s$, $s = 2, 3, \ldots;$

(xiii) $\rho=-\left(5s^2+4u^2+s+u-1\right)^{-1}$, $v=4s+1$, $b_2=u(4s+1)$, $r_2=u(4s-1)$, $k_2=4s-1$, $\lambda_2=\rho_{12}=u(4s-3)$, $\rho_{22}=u$, $s=2,3,\ldots$, $u=1,2,\ldots$;

(xiv) $\rho=-\left(5s^2+9u^2-2su+s+u-1\right)^{-1}$, $v=4s+1$, $b_2=u(4s+1)$, $r_2=2u(2s-1)$, $k_2=2(2s-1)$, $\lambda_2=u(4s-5)$, $\rho_{12}=4u(2s-1)$, $\rho_{22}=3u$, $s=3,4,\ldots$, $u=1,2,\ldots$;

(xv) $\rho=-\left(5s^2+16u^2-4su+s+u-1\right)^{-1}$, $v=4s+1$, $b_2=u(4s+1)$, $r_2=u(4s-3)$, $k_2=4s-3$, $\lambda_2=u(4s-7)$, $\rho_{12}=u(4s-15)$, $\rho_{22}=6u$, $s=4,5,\ldots$, $u=1,2,\ldots$;

then $\mathbf{X}$ in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is in (2).

Theorem 3.13. Let $\frac{-1}{n-1}<\rho<0$. If for a given ρ the parameters of the balanced bipartite weighing design are equal to $v=4s+1$, $b_1=s(4s+1)$, $r_1=5s$, $k_{11}=1$, $k_{21}=4$, $\lambda_{11}=2$, $\lambda_{21}=3$ and the parameters of the ternary balanced block design are equal to

(i) $\rho=-\left(13s^2+9s+1\right)^{-1}$, $v=k_2=4s+1$, $b_2=r_2=2(4s+1)$, $\lambda_2=8s$, $\rho_{12}=4s$, $\rho_{22}=4s$, $s=1,2,\ldots$;

(ii) $\rho=-\left(13s^2+17s-3\right)^{-1}$, $v=k_2=4s+1$, $b_2=r_2=2(8s-1)$, $\lambda_2=4(4s-1)$, $\rho_{12}=2(4s-1)$, $\rho_{22}=4s$, $s=1,2,\ldots$;

(iii) $\rho=-\left(13s^2+17s+3\right)^{-1}$, $v=k_2=4s+1$, $b_2=r_2=4(4s+1)$, $\lambda_2=2(8s+1)$, $\rho_{12}=4(2s+1)$, $\rho_{22}=4s$, $s=1,2,\ldots$;

(iv) $\rho=-\left(13s^2+17s+5\right)^{-1}$, $v=k_2=4s+1$, $b_2=r_2=2(8s+3)$, $\lambda_2=4(4s+1)$, $\rho_{12}=2(4s+3)$, $\rho_{22}=4s$, $s=1,2,\ldots$;

(v) $\rho=-\left(13s^2+9s+u\right)^{-1}$, $v=k_2=4s+1$, $b_2=r_2=8s+u+1$, $\lambda_2=8s+u-1$, $\rho_{12}=u+1$, $\rho_{22}=4s$, $s=1,2,\ldots$, $u=1,2,\ldots$;

then $\mathbf{X}$ in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is in (2).

Theorem 3.14. Let $\frac{-1}{n-1}<\rho<0$. If for a given ρ the parameters of the balanced bipartite weighing design are equal to $v=6s$, $b_1=6s(6s-1)$,

$r_1 = 3(6s-1)$, $k_{11} = 1$, $k_{21} = 2$, $\lambda_{11} = 4$, $\lambda_{21} = 2$ and the parameters of the ternary balanced block design are equal to

(i) $\rho = -2(108s^2 - 36s + 7)^{-1}$, $v = b_2 = 6s$, $r_2 = k_2 = 2(3s-1)$, $\lambda_2 = \rho_{12} = 2(3s-2)$, $\rho_{22} = 1$, $s = 1, 2, \ldots$;

(ii) $\rho = -2(108s^2 + 72s + 7)^{-1}$, $v = 6s$, $b_2 = 24s$, $r_2 = 4(6s+1)$, $k_2 = 6s+1$, $\lambda_2 = 8(3s+1)$, $\rho_{12} = 4(6s-1)$, $\rho_{22} = 4$, $s = 1, 2, \ldots$;

(iii) $\rho = -2(108s^2 - 72s + 79)^{-1}$, $v = 6s$, $b_2 = 24s$, $r_2 = 8(3s-1)$, $k_2 = 2(3s-1)$, $\lambda_2 = \rho_{12} = 8(3s-2)$, $\rho_{22} = 4$, $s = 1, 2, \ldots$;

(iv) $\rho = -2(108s^2 - 24s + 4u^2 + 4u - 12su - 1)^{-1}$, $v = 6s$, $b_2 = 6su$, $r_2 = 2u(3s-1)$, $k_2 = 2(3s-1)$, $\lambda_2 = \rho_{12} = 2u(3s-2)$, $\rho_{22} = u$, $s, u = 1, 2, \ldots$;

(v) $\rho = -2(108s^2 - 24s + 9u^2 + 6u - 24su - 1)^{-1}$, $v = 6s$, $b_2 = 6su$, $r_2 = 3u(2s-1)$, $k_2 = 3(2s-1)$, $\lambda_2 = 6u(s-1)$, $\rho_{12} = 3u(2s-3)$, $\rho_{22} = 3u$, $s = 2, 3, \ldots$, $u = 1, 2, \ldots$;

(vi) $\rho = -2(108s^2 - 24s + 16u^2 + 8u - 36su - 1)^{-1}$, $v = 6s$, $b_2 = 6su$, $r_2 = 2u(3s-2)$, $k_2 = 2(3s-2)$, $\lambda_2 = 2u(3s-4)$, $\rho_{12} = 2u(3s-8)$, $\rho_{22} = 6u$, $s = 2, 3, \ldots$, $u = 1, 2, \ldots$;

(vii) $\rho = -2(108s^2 + 72s + 7)^{-1}$, $v = 6s$, $b_2 = 24s$, $r_2 = 4(6s+1)$, $k_2 = 6s+1$, $\lambda_2 = 8(3s+1)$, $\rho_{12} = 4(6s-1)$, $\rho_{22} = 4$, $s = 1, 2, \ldots$;

(viii) $\rho = -4(180s^2 + 60s - 11)^{-1}$, $v = k_2 = 6s$, $b_2 = r_2 = 2(12s-1)$, $\lambda_2 = 4(6s-1)$, $\rho_{12} = 12s$, $\rho_{22} = 6s-1$, $s = 1, 2, \ldots$;

(ix) $\rho = -4(180s^2 + 60s + 5)^{-1}$, $v = k_2 = 6s$, $b_2 = r_2 = 2(12s+1)$, $\lambda_2 = 24s$, $\rho_{12} = 4(3s+1)$, $\rho_{22} = 6s-1$, $s = 1, 2, \ldots$;

(x) $\rho = -4(3(60s^2 + 20s - 9))^{-1}$, $v = k_2 = 6s$, $b_2 = r_2 = 6(4s-1)$, $\lambda_2 = 8(3s-1)$, $\rho_{12} = 4(3s-1)$,, $\rho_{22} = 6s-1$, $s = 1, 2, \ldots$;

(xi) $\rho = -4(180s^2 + 12s + 4u - 11)^{-1}$, $v = k_2 = 6s$, $b_2 = r_2 = 12s + u - 2$, $\lambda_2 = 12s + u - 4$, $\rho_{12} = u$, $\rho_{22} = 6s-1$, $s = 1, 2, \ldots$, $u = 1, 2, \ldots$;

(xii) $\rho = -4(3(60s^2 + 20s - 1))^{-1}$, $v = k_2 = 6s$, $b_2 = r_2 = 24s$, $\lambda_2 = 2(12s-1)$, $\rho_{12} = 2(3s+1)$, $\rho_{22} = 6s-1$, $s = 1, 2, \ldots$;

(xiii) $\rho = -4(180s^2 + 60s + 13)^{-1}$, $v = k_2 = 6s$, $b_2 = r_2 = 4(6s+1)$, $\lambda_2 = 2(12s+1)$, $\rho_{12} = 6(2s+1)$, $\rho_{22} = 6s-1$, $s = 1, 2, \ldots$;

then $\mathbf{X}$ in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is in (2).

Theorem 3.15. Let $\frac{-1}{n-1} < \rho < 0$. If for a given ρ the parameters of the balanced bipartite weighing design are equal to $v = 6s+1$, $b_1 = ls(6s+1)$, $r_1 = 4ls$, $k_{11} = 1$, $k_{21} = 3$, $\lambda_{11} = \lambda_{21} = l$ and the parameters of the ternary balanced block design are equal to

(i) $\rho = -\left(2l^2s^2 + 6ls^2 + ls + 12s + 1\right)^{-1}$, $v = k_2 = 6s+1$, $b_2 = r_2 = 2(6s+1)$, $\lambda_2 = 12s$, $\rho_{12} = 2$ $\rho_{22} = 6s$ $s = 1, 2, \ldots$;

(ii) $\rho = -\left(2l^2s^2 + 6ls^2 + ls + 24s + 5\right)^{-1}$, $v = k_2 = 6s+1$, $b_2 = r_2 = 6(4s+1)$, $\lambda_2 = 4(6s+1)$, $\rho_{12} = 6(2s+1)$, $\rho_{22} = 6s$, $s = 1, 2, \ldots$;

(iii) $\rho = -\left(2l^2s^2 + 6ls^2 + ls + 24s - 3\right)^{-1}$, $v = k_2 = 6s+1$, $b_2 = r_2 = 2(12s-1)$, $\lambda_2 = 4(6s-1)$, $\rho_{12} = 2(6s-1)$, $\rho_{22} = 6s$, $s = 1, 2, \ldots$;

(iv) $\rho = -\left(2l^2s^2 + 6ls^2 + ls + 12s + u\right)^{-1}$, $v = k_2 = 6s+1$, $b_2 = r_2 = 12s + u + 1$, $\lambda_2 = 12s + u - 1$, $\rho_{12} = u + 1$, $\rho_{22} = 6s$, $s = 1, 2, \ldots$;

(v) $\rho = -\left(2l^2s^2 + 6ls^2 + ls + 24s + 3\right)^{-1}$, $v = k_2 = 6s+1$, $b_2 = r_2 = 4(6s+1)$, $\lambda_2 = 2(12s-1)$, $\rho_{12} = 4(3s+1)$,, $\rho_{22} = 6s$, $s = 1, 2, \ldots$;

(vi) $\rho = -\left(2l^2s^2 + 6ls^2 + ls + 24s + 7\right)^{-1}$, $v = k_2 = 6s+1$, $b_2 = r_2 = 8(3s+1)$, $\lambda_2 = 6(4s+1)$, $\rho_{12} = 4(3s+2)$, $\rho_{22} = 6s$, $s = 1, 2, \ldots$;

where μ, $l = 1, 2, \ldots$; then $\mathbf{X}$ in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is in (2).

Theorem 3.16. Let $\frac{-1}{n-1} < \rho < 0$. If for a given ρ the parameters of the balanced bipartite weighing design are equal to $v = 8s+5$, $b_1 = (2s+1)(8s+5)$, $r_1 = 9(2s+1)$, $k_{11} = 4$, $k_{21} = 5$, $\lambda_{11} = 10$, $\lambda_{21} = 8$ and the parameters of the ternary balanced block design are equal to

(i) $\rho = -\left(17s^2 + 40s + 20\right)^{-1}$, $v = 8s+5$, $b_2 = 3(8s+5)$, $r_2 = 12(2s+1)$, $k_2 = 4(2s+1)$, $\lambda_2 = 24s + 7$, $\rho_{12} = \rho_{22} = 4(2s+1)$, $s = 1, 2, \ldots$;

(ii) $\rho=-2\left(36s^2+48s+19\right)^{-1}$, $v=b_2=8s+5$, $r_2=k_2=8s+3$, $\lambda_2=\rho_{12}=8s+1$, $\rho_{22}=1$, $s=1,2,\ldots$;

(iii) $\rho=-2\left(36s^2+72s+97\right)^{-1}$, $v=8s+5$, $b_2=4(8s+5)$, $r_2=4(8s+3)$, $k_2=8s+3$, $\lambda_2=\rho_{12}=4(8s+1)$, $\rho_{22}=4$, $s=1,2,\ldots$;

(iv) $\rho=-2\left(36s^2+80s+37\right)^{-1}$, $v=8s+5$, $b_2=2(8s+5)$, $r_2=4(4s+3)$, $k_2=2(4s+3)$, $\lambda_2=2(8s+7)$, $\rho_{12}=8(2s+1)$, $\rho_{22}=2$, $s=1,2,\ldots$;

(v) $\rho=-2\left(36s^2+120s+73\right)^{-1}$, $v=8s+5$, $b_2=4(8s+5)$, $r_2=8(4s+3)$, $k_2=2(4s+3)$, $\lambda_2=4(8s+7)$, $\rho_{12}=16(2s+1)$, $\rho_{22}=4$ $s=1,2,\ldots$;

(vi) $\rho=-4\left(68s^2+140s+57\right)^{-1}$, $v=k_2=8s+5$, $b_2=r_2=2(8s+5)$, $\lambda_2=8(2s+1)$, $\rho_{12}=2$, $\rho_{22}=4(2s+1)$, $s=1,2,\ldots$;

(vii) $\rho=-4\left(68s^2+204s+73\right)^{-1}$, $v=k_2=8s+5$, $b_2=r_2=2(16s+7)$, $\lambda_2=4(8s+3)$, $\rho_{12}=2(8s+3)$, $\rho_{22}=4(2s+1)$, $s=1,2,\ldots$;

(viii) $\rho=-4\left(68s^2+204s+105\right)^{-1}$, $v=k_2=8s+5$, $b_2=r_2=2(16s+11)$, $\lambda_2=4(8s+5)$, $\rho_{12}=2(8s+7)$, $\rho_{22}=4(2s+1)$, $s=1,2,\ldots$;

(ix) $\rho=-4\left(68s^2+204s+113\right)^{-1}$, $v=k_2=8s+5$, $b_2=r_2=8(4s+3)$, $\lambda_2=16(s+11)$, $\rho_{12}=2(16s+1)$, $\rho_{22}=4(2s+1)$, $s=1,2,\ldots$;

(x) $\rho=-4\left(68s^2+140s+4u+53\right)^{-1}$, $v=k_2=8s+5$, $b_2=r_2=16s+u+9$, $\lambda_2=16s+u+7$, $\rho_{12}=u+1$, $\rho_{22}=4(2s+1)$ $s=1,2,\ldots$, $u=1,2,\ldots$;

(xi) $\rho=-2\left(36s^2+40s+4u^2+8us+6u+9\right)^{-1}$, $v=8s+5$, $b_2=u(8s+5)$, $r_2=u(8s+3)$, $k_2=8s+3$, $\lambda_2=\rho_{12}=u(8s+1)$, $\rho_{22}=u$, $s=1,2,\ldots$, $u=1,2,\ldots$;

(xii) $\rho=-2\left(36s^2+40s+9u^2+4us+4u+9\right)^{-1}$, $v=8s+5$, $b_2=u(8s+5)$, $r_2=2u(4s+1)$, $k_2=2(4s+1)$, $\lambda_2=u(8s-1)$, $\rho_{12}=4u(2s-1)$, $\rho_{22}=3u$, $s=1,2,\ldots$, $u=1,2,\ldots$;

(xiii) $\rho=-2\left(36s^2+40s+16u^2+2u+9\right)^{-1}$, $v=8s+5$, $b_2=u(8s+5)$, $r_2=u(8s+1)$, $k_2=8s+1$, $\lambda_2=u(8s-3)$, $\rho_{12}=u(8s-11)$, $\rho_{22}=6u$, $s=1,2,\ldots$, $u=1,2,\ldots$;

(xiv) $\rho=-4\left(68s^2+204s+97\right)^{-1}$, $v=k_2=8s+5$, $b_2=r_2=4(8s+5)$, $\lambda_2=2(16s+9)$, $\rho_{12}=4(4s+3)$, $\rho_{22}=4(2s+1)$, $s=1,2,\ldots$;

then **X** in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, where **G** is in (2).

Theorem 3.17. Let $\frac{-1}{n-1} < \rho < 0$. If for a given ρ the parameters of the balanced bipartite weighing design are equal to $v = 10s+1$, $b_1 = s(10s+1)$, $r_1 = 6s$, $k_{11} = 1$, $k_{21} = 5$, $\lambda_{11} = 1$, $\lambda_{21} = 2$ and the parameters of the ternary balanced block design are equal to

(i) $\rho = -\left(26s^2 + 21s + 1\right)^{-1}$, $v = k_2 = 10s+1$, $b_2 = r_2 = 2(10s+1)$, $\lambda_2 = 20s$, $\rho_{12} = 2$, $\rho_{22} = 10s$;

(ii) $\rho = -\left(26s^2 + 41s - 3\right)^{-1}$, $v = k_2 = 10s+1$, $b_2 = r_2 = 2(20s-1)$, $\lambda_2 = 4(10s-1)$, $\rho_{12} = 2(10s-1)$, $\rho_{22} = 10s$;

(iii) $\rho = -\left(26s^2 + 41s + 3\right)^{-1}$, $v = k_2 = 10s+1$, $b_2 = r_2 = 4(10s+1)$, $\lambda_2 = 2(20s+1)$, $\rho_{12} = 4(5s+1)$, $\rho_{22} = 10s$;

(iv) $\rho = -\left(26s^2 + 41s + 5\right)^{-1}$, $v = k_2 = 10s+1$, $b_2 = r_2 = 2(20s+3)$, $\lambda_2 = 4(10s+1)$, $\rho_{12} = 2(10s+3)$, $\rho_{22} = 10s$;

(v) $\rho = -\left(26s^2 + 41s + 7\right)^{-1}$, $v = k_2 = 10s+1$, $b_2 = r_2 = 8(5s+1)$, $\lambda_2 = 2(20s+3)$, $\rho_{12} = 4(5s+2)$, $\rho_{22} = 10s$;

(vi) $\rho = -\left(26s^2 + 21s + u\right)^{-1}$, $v = k_2 = 10s+1$, $b_2 = r_2 = 20s+u+1$, $\lambda_2 = 20s+u-1$, $\rho_{12} = u+1$, $\rho_{22} = 10s$, $u = 1, 2, \ldots$;

where $s = 1, 2, \ldots$, then **X** in the form (3) is the design matrix of the optimum chemical balance weighing design with the variance–covariance matrix of errors $\sigma^2\mathbf{G}$, **G** is in (2).

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Optymalny chemiczny układ wagowy o skorelowanych błędach oparty na układach dwudzielnych i trójkowych

W artykule rozważa się zagadnienie estymacji nieznanych miar poszczególnych obiektów w chemicznym układzie wagowym przy ograniczeniu liczby pomiarów poszczególnych obiektów. Zakłada się, że błędy mają jednakowe wariancje i są równo skorelowane. Podane zostały warunki konieczne i dostateczne, przy spełnieniu których wariancja estymatorów osiąga dolne ograniczenie. Do konstrukcji macierzy optymalnego układu wykorzystuje się macierze incydencji dwudzielnych układów bloków oraz trójkowych zrównoważonych układów bloków.