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SELECTED PREMIUM ESTIMATION METHODS IN AUTOMOBILE LIABILITY INSURANCE

Abstract

Correct insurance premiums estimation constitutes the basis for the insurance activity. Premiums should be estimated in a way that does not let the insurance company incur financial losses and prevents the insured from paying too much or too little.

A gross premium consists of a net premium enlarged by security loading and insurance activity costs. The paper compares two premium estimation methods: the expected value method and the zero utility method. It also investigates whether premiums estimated according to the selected methods allow to design an optimal bonus-malus system. The investigation was carried out on real data from an insurance company in Łódź.

Key words: bonus-malus system, automobile liability insurance, Bayes estimators.

1. Introduction

The classification of the insured into rating groups in automobile liability insurance is carried out on the basis of *a priori* factors (observable risk factors such as, for example, car type and year of production, engine capacity, the age and sex of a driver and *a posteriori* factors (damage history of a driver). That is why premiums in automobile liability insurance are estimated in two stages. The first stage comprises calculating the basic premium on the basis of *a priori* factors and the other one involves *a posteriori* rating (L e m a i r e, 1995).

The paper will concentrate on the second stage, called a bonus-malus system. A bonus-malus system denotes individual premium estimation methods that take in to account the number of damages caused by a driver in the past. In each bonus-malus system, there must be determined: a starting class into which

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the insured without any damage history are assigned, a basic premium rates vector and the rules of transition between classes. In order to make the system work, the insurance portfolio ought to be heterogeneous, i.e. the insured should be characterised by different average damage numbers in the past (cf. Hossack *et al.*, 1999).

The annual net premium is estimated as the product of the current basic premium for a given rating class (*a priori* rating) and the coefficient that constitutes the estimated percentage rate of the premium.

The paper does not consider additional rises and reductions characteristic for individual insurers.

For property insurance, a gross premium is calculated as a sum of three components: a net premium, security loading and insurance activity costs. The paper does not consider the third of the components. Therefore, a net premium enlarged by security loading will be called a gross premium in this study.

The paper compares two most frequently used gross premium estimation methods: the expected value method and the zero utility method.

2. The investigation of portfolio homogeneity

In automobile liability insurance it is assumed that the number of damages K in a homogeneous portfolio is a random variable having the Poisson distribution with λ damage intensity parameter:

$$P(K=k)=e^{-\lambda}\frac{\lambda^{k}}{k!},\qquad (k=0,1,2,\ldots) \tag{1}$$

If the portfolio is heterogeneous, the damage intensity parameter usually has the gamma distribution with parameters α and β, whereas the number of damages, has the negative binomial distribution with parameters p and q (cf. Hossack *et al.,* 1999), i.e. the probability function distribution in the following form:

$$P(K=k)=\binom{q+k-1}{k}p^{q}(1-p)^{k},\qquad (k=0,1,2,\ldots) \tag{2}$$

where:

$$q=\alpha \quad\text{and}\quad p=\beta/(1+\beta) \tag{3}$$

The investigation was carried out on the basis of data from an insurance company in Lodz, for automobile liability insurance for the year 2000. 15 867 policies were independently drawn from the whole portfolio consisting of 31 734 policies. The drawn policies were grouped according to the age of a driver and marked: I – drivers up to the age of 25, II – drivers above the age of 25. The data is presented in Table 1.

Table 1

Number of damages structure, according to the groups of automobile liability insurance portfolio

Damage no. \ Group	I	II
0	2 907	10 221
1	592	1 843
2	66	210
3	5	18
4	0	5
Sum	3 570	12 292

Source: own research.

On the basis of the data from Table 1, we estimated: the Poisson distribution parameter, providing that:

$$\hat{\lambda} = \overline{k} \tag{4}$$

and the negative binomial distribution parameters, providing that:

$$\hat{p} = \frac{\overline{k}}{s^2} \tag{5}$$

$$\hat{q} = \frac{s^2 \hat{p}^2}{1 - \hat{p}} \tag{6}$$

where $\overline{k}$ is the sampling average value and s^2 is the sampling variance. The estimators were obtained with the use of the moment method (cf. Domański, 2001). The gamma distribution parameters were determined from formula (3).

Table 2

Parameters for claims frequency distribution

Group	Average damage number	Damage number variance	Negative binomial distribution parameters		Gamma distribution parameters	
			p	q	α	β
I	0.207	0.209	0.98	16.95	16.95	81.88
II	0.19	0.2	0.95	3.61	3.61	19

Source: own calculations on the basis of Table 1.

For the purpose of assessing the homogeneity of portfolio groups from Table 1, the fitness of the claims number distribution with the Poisson distribution and the negative binomial distribution was investigated, with the help of the chi-square test (cf. Domański, 2001).

On 0.05 significance level, there is no basis for rejecting the null hypothesis that number of damages distribution in group I is the Poisson distribution ($\chi^2 = 0.5049$; $\chi^2_\alpha = 5.99$). On 0.05 significance level, we reject the null hypothesis that number of damages distribution in group I is the negative binomial distribution, in favour of the alternative hypothesis ($\chi^2 = 11.34$; $\chi^2_\alpha = 5.99$). In this connection, it can be assumed that number of damages distribution in group I is homogeneous.

On 0.05 significance level, we reject the null hypothesis that number damage of distribution in group II is the Poisson distribution, in favour of the alternative hypothesis ($\chi^2 = 27.349$; $\chi^2_\alpha = 5.99$). On 0.05 significance level, there is no basis for rejecting the null hypothesis that damage number distribution in group II is negative binomial distribution ($\chi^2 = 0.07$; $\chi^2_\alpha = 5.99$). In this connection, it can be assumed that the number of damages distribution in group II is heterogeneous.

3. Bonus-malus systems

A bonus-malus system determined with the help of the Bayesian analysis is called the optimal system (cf. [4]). In a system determined in such a way, the *a priori* premium is estimated and then an individual risk parameter is considered. In order to determine individual risk parameters, Bayes estimators are used (cf. Domański, Pruska, 2000).

The functioning of the system makes sense when the insurance portfolio is heterogeneous. In this connection, only group II from the portfolio presented in table 1 was considered in further research. Table 3 presents the bonus-malus system of the investigated insurance company. The system consists of 11 classes. Class 4 is the starting class. Table 4 illustrates the rules of transition between classes.

Let K_j be a random variable representing number of damages for year j for a given policy; $(k_1, k_2, \ldots, k_t)$ number of damages observation vector for t years for a given policy; $\lambda_{t+1}(k_1, k_2, \ldots, k_t)$ an unknown damage parameter in the year $t + 1$ for the policy described by the observation vector $(k_1, k_2, \ldots, k_t)$. Parameter λ is a realisation of a random variable Λ having the cumulative distribution function $U(\lambda)$. The unknown parameter $\lambda_{t+1}(k_1, k_2, \ldots, k_t)$ was estimated with the help of a Bayes estimator on the basis of the observation vector $(k_1, k_2, \ldots, k_t)$.

Table 3

Rises and reductions of automobile liability insurance according to classes

Class	% of the basic premium	Class	% of the basic premium
1	200	7	70
2	150	8	60
3	125	9	50
4	100	10	50
5	90	11	40
6	80		

S o u r c e: the insurance company.

Table 4

The class number to which the insured is assigned, depending on the damage number k in the preceding period and on the previous class

Class of the insured	Damage number k			
	0	1	2	3 or more
1	4	1	1	1
2	4	1	1	1
3	4	1	1	1
4	5	2	1	1
5	6	3	2	1
6	7	4	3	1
7	8	5	3	1
8	9	6	4	2
9	10	7	5	3
10	11	8	6	4
11	11	9	7	5

S o u r c e: the insurance company.

We assume that damage number distribution in this portfolio is negative binomial and described by formula (2). Damage intensity parameter Λ has *a priori* gamma distribution with parameters α and β. Therefore, the *a posteriori* distribution of the parameter is the gamma distribution with $\hat{\alpha} = \alpha + k$ and $\hat{\beta} = \beta + t$ parameters. The Bayes estimator of parameter Λ is a conditional expected value of the *a posteriori* distribution and has the following form:

$$\lambda_{t+1}(k_1,...,k_t) = E_\lambda[\Lambda|k_1,\dots,k_t] = \int_0^\infty \lambda dU(\lambda|k_1,\dots,k_t) = \frac{\hat{\alpha}}{\hat{\beta}} = \frac{\alpha+k}{\beta+t} \tag{7}$$

where $E_\lambda[\Lambda|k_1,\dots,k_t]$ is the conditional expected value of the *a posteriori* distribution of parameter Λ, $U(\lambda|k_1,\dots,k_t)$ is the conditional cumulative distribution function of random variable Λ at the observed values $(k_1, k_2, \dots, k_t)$.

4. The expected value rule

The simplest rate calculation rule in automobile liability insurance is the expected value rule. According to this rule, an estimated individual net rate enlarged by security loading θ equals:

$$P_{t+1}(k_1,...,k_t) = (1+\theta)E_\lambda[\Lambda|k_1,\dots,k_t] = (1+\theta)\lambda_{t+1}(k_1,\dots,k_t) = (1+\theta)\frac{\alpha+k}{\beta+t} \tag{8}$$

In automobile liability insurance, the individual rate is equal to:

$$P_{t+1}(k_1,\dots,k_t) = (EX)\cdot(E\Lambda)\cdot b_{t+1}(k_1,\dots,k_t) \tag{9}$$

where $P_{t+1}(k_1,\dots,k_t)$ – individual net rate, (EX) – average single damage size, $(E\Lambda)$ – average damage number, $b_{t+1}(k_1,\dots,k_t)$ – estimated premium rate.

Let us assume that $(EX) = 1$ and $(E\Lambda) = \frac{\alpha}{\beta}$. Then, the equation (9) has the following form:

$$P_{t+1}(k_1,\dots,k_t) = \frac{\alpha}{\beta}\cdot b_{t+1}(k_1,\dots,k_t) \tag{10}$$

The purpose of the investigation is to estimate what percentage of the basic premium a driver who after t years reported k damages should pay. Therefore, on the basis of equations (8) and (10), the estimated premium rate in the bonus-malus system equals:

$$b_{t+1}(k_1,\dots,k_t) = \frac{(1+\theta)\frac{\alpha+k}{\beta+t}}{\frac{\alpha}{\beta}}\cdot 100\% = (1+\theta)\frac{\beta(\alpha+k)}{\alpha(\beta+t)}\cdot 100\% \tag{11}$$

Assuming that $\theta = 0$, the estimated premium rate equals:

$$b_{t+1}(k_1,\dots,k_t) = \frac{\beta(\alpha+k)}{\alpha(\beta+t)}\cdot 100\% \tag{12}$$

5. The zero utility rule

The zero utility rule is based on the assumption that the expected income utility of the insurer, when risk Z is insured for sum P, is equal to the utility of the initial income of the insurer, i.e. $v(x) = E[v(x+P-Z)]$.

Let function $v(x)$ be an exponential utility function of the following form:

$$v(x) = \frac{1}{c}\left(1 - e^{-cx}\right) \tag{13}$$

where $c > 0$ is a parameter designating aversion to risk of the insurance company. The greater the aversion, the higher the rate. In such a case, an individual net rate estimated according to the utility rule equals:

$$P_{t+1}(k_1, ..., k_t) = \frac{\alpha + k}{c}\left|\ln\left(1 - \frac{e^c - 1}{t + \beta}\right)\right| \tag{14}$$

The purpose of the investigation is to estimate what percentage of the basic premium a driver who after t years reported k damages should pay. Therefore, on the basis of equations (14) and (10), the estimated premium rate in the bonus-malus system equals:

$$b_{t+1}(k_1, ..., k_t) = \frac{\beta}{\alpha}\frac{\alpha + k}{c}\left|\ln\left(1 - \frac{e^c - 1}{t + \beta}\right)\right| \cdot 100\% \tag{15}$$

6. Applications

For group II from the insurance portfolio presented in Table 1, estimated premium rates were estimated in the bonus-malus system with the help of formula (12). Table 6 contains the results.

Table 5

Rises and reductions applied by the investigated insurance company

Year no. t \ Damage no. k	0	1	2	3	4 and more
0	100				
1	90	150	200	200	200
2	80	125	150	200	200
3	70	100	125	200	200
4	60	90	125	200	200
5	50	80	100	150	150
6	50	60	80	100	100
7	40	50	70	90	90

Source: own research.

Table 6

The estimated premium rate according to the expected value rule (E), and rises and reductions applied by the insurance company (IC)

Damage number k / Year no. t	0		1		2		3 and more	
	E	IC	E	IC	E	IC	E	IC
0	100	100						
1	95	90	121	150	148	200	148	200
2	90	80	116	125	141	150	141	200
3	86	70	110	100	134	125	134	200
4	83	60	105	90	128	125	128	200
5	79	50	101	80	123	100	123	150
6	76	50	97	60	118	80	118	100
7	73	40	93	50	114	70	113	90

Source: own calculations on the basis of Tables 1 and 5.

For group II of the insurance portfolio presented in Table 1, estimated premium rates were estimated in the bonus-malus system with the help of a formula (15). Tables 7 and 8 contain the results.

Table 7

The estimated premium rate according to the zero utility rule $c = 0.4$ (E), and rises and reductions applied by the insurance company (IC)

Damage number k / Year no. t	0		1		2		3 and more	
	E	IC	E	IC	E	IC	E	IC
0	100	100						
1	95	90	121	150	148	200	174	200
2	90	80	115	125	141	150	166	200
3	86	70	110	100	134	125	158	200
4	82	60	105	90	128	125	151	200
5	79	50	101	80	123	100	145	150
6	76	50	97	60	118	80	139	100
7	73	40	93	50	113	70	133	90

Source: own calculations on the basis of Tables 1 and 5.

Table 8

The estimated premium rate according to the zero utility rule $c = 1.65$ (E), and rises and reductions applied by the insurance company (IC)

Damage number k / Year no. t	0		1		2		3 and more	
	E	IC	E	IC	E	IC	E	IC
0	100	100						
1	94	90	121	150	147	200	173	200
2	89	80	114	125	139	150	164	200
3	85	70	108	100	132	125	155	200
4	81	60	103	90	125	125	148	200
5	77	50	98	80	120	100	141	150
6	74	50	94	60	114	80	135	100
7	71	40	90	50	110	70	129	90

Source: own calculations on the basis of Tables 1 and 5.

Table 9

Estimation error

Premium calculation method / Error	Expected value rule	Zero utility rule			
		$c = 0.4$	$c = 1.65$	$c = 2.45$	$c = 2.85$
The sum of error absolute values	810	807	777	683	704

Source: own calculations.

Table 10 presents the differences between rises and reductions applied by the investigated insurance company, and estimated premium rates for a premium calculated with the help of the zero utility rule for higher c parameters.

Table 10

The difference between rises and reductions applied by the investigated insurance company (IC), and estimated premium rates (E). (evr– for a rate calculated according to the expected value rule, c – a parameter applied in calculating the premium with the zero utility rule)

t \ k	0			1			2			3 and more		
	evr	$c = 0.4$	$c = 1.65$	evr	$c = 0.4$	$c = 1.65$	evr	$c = 0.4$	$c = 1.65$	evr	$c = 0.4$	$c = 1.65$
0	0	0	0									
1	–5	–5	–4	29	29	29	52	52	53	26	26	27
2	–10	–10	9	10	10	11	10	9	11	35	34	36
3	–16	–16	–15	–10	–10	8	–9	–9	–7	42	42	45
4	–22	–22	–21	–15	–15	–13	–3	–3	0	49	49	52
5	–29	–29	–27	–21	–21	–18	–22	–23	–20	6	5	9
6	–25	–26	–24	–36	–17	–34	–37	–38	–34	–38	–39	–35
7	–32	–43	–31	–43	–43	–40	–43	–43	–40	–43	–43	–39

S o u r c e: own calculations on the basis of Tables 6, 7, 8.

Table 11

The difference between rises and reductions applied by the investigated insurance company (IC), and estimated premium rates (E) (c – a parameter applied in calculating the premium with the zero utility rule)

t \ k	0		1		2		3 and more	
	$c = 2.45$	$c = 2.85$	$c = 2.45$	$c = 2.85$	$c = 2.45$	$c = 2.85$	$c = 2.45$	$c = 2.85$
0	0	0						
1	–3	4	22	23	–32	–14	–25	–4
2	–6	3	32	40	44	45	–30	–8
3	–11	1	15	27	56	66	17	17
4	–16	–3	–3	12	16	31	30	42
5	–22	–8	–7	9	0	18	42	60
6	–18	–4	–11	6	7	27	52	73
7	–24	–10	–26	–9	–11	10	61	84

Source: own calculations.

7. Conclusions

In some classes, the estimated basic premium rates differ considerably from coefficients of the insurance company. It means that the system does not assess drivers in a correct way. The positive differences sign in Tables 9 and 10 means a raised insurance premium, whereas the negative sign a lowered insurance premium. Definitely, the smallest differences occur when the premium is estimated with the zero utility method, with parameter $c = 2.45$ i.e. at about 250% safety coefficient applied by the insurer. It appears from the study that the investigated insurance company presupposes a very high safety coefficient.

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Wybrane metody szacowania składek w ubezpieczeniach komunikacyjnych OC

Podstawą działalności ubezpieczeniowej jest prawidłowe szacowanie składek ubezpieczeniowych. Składki powinny być tak oszacowane, aby towarzystwo nie ponosiło strat finansowych, natomiast ubezpieczony nie płacił za dużo lub za mało.

Składka brutto to składka netto powiększona o dodatek bezpieczeństwa oraz koszty działalności ubezpieczeniowej. W pracy porównano dwie metody szacowania składek: metodę wartości oczekiwanej oraz metodę zerowej użyteczności. Zbadano również, czy oszacowane według wybranych metod składki pozwalają na budowę optymalnego systemu bonus-malus. Badanie przeprowadzono na danych rzeczywistych, pochodzących z łódzkiego towarzystwa ubezpieczeniowego.