*Artur Mikulec**

EVALUATION OF INVESTING EFFICIENCY OF OPEN PENSION FUNDS BY MEANS OF THE METHOD OF CLUSTER ANALYSIS

Abstract

The following study aims at analyzing the activity of open pension funds so far. To evaluate their efficiency, the author uses profitability indicators of investment portfolio such as Sharpe, Treynor and Jensen Ratio as well as IR (Information Ratio), TE (Tracking Error) and M^2 (M^2-measure). The analysis was carried out by means of monthly and quarterly data.

The next stage includes rating of open pension funds from the point of view of their efficiency and conducted investment policy, analyzing, at the same time, calculated profitability ratios, rates of return and risk measures. In order to do that, the author uses such methods of cluster analysis as Tree Clustering and k-Means Clustering as well as different distance measures and Amalgamation or Linkage Rules.

Key Words: normal distribution, systematic risk, profitability of investment portfolio, Shaqpe Ratio, Jensen Ratio, M^2 – measure.

1. OPF efficiency measures

This elaboration is another attempt to analyze the OPF market from the point of view of investment results obtained by different funds (see Mikulec, 2004). The author used here longer time series of rates of return for each fund of the October 1999–July 2005 period of time, and also a wider set of efficiency measurement methods were used.

In the first stage, monthly and quarterly rates of return $R^*_{i,t}$ for each fund, risk-free rates of return RFR^*_t and the market portfolio rates of return $R^*_{M,t}$ were calculated. **Investment portfolio rate of return** calculation for each OPF was based on the value of their account unit on the first and the last day of a month (quarter). Interests of 52-week treasury bills emitted by the Minister of

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* Ph.D., student, Chair of Statistical Methods, University of Łódź.

Treasury were accounted as **risk-free assets**. The market portfolio built for the purpose of this analysis contained: **shares, bonds and interbank deposits of Polish market, as well as foreign market shares. Average engagement of all funds in each category of financial instruments for the whole period taken under consideration** was used as **weights** in calculation of mean weighted rate of return from this portfolio. Monthly rate of return for the market portfolio was calculated by the formula below where elements are rates of return as follows: WIG index, treasury bonds, 1M WIBID (Warsaw Interbank Bid Rate), DAX (blue-chip index of Deutsche Börse), FTSE–100 (100 blue-chip companies of the UK market) and DJIA (Dow Jones Industrial Average, Index of the New York Stock Exchange).

$$R_{M,t} = 0.294 R_{WIG,t} + 0.653 R_{Obl,t} + 0.043 R_{WIBID,t} + 0.009(0.33 R_{DAX,t} + 0.33 R_{FTSE,t} + 0.33 R_{DJIA,t})$$

(1)

In further calculations series of real monthly (quarterly) rates of return: for each OPF ($R_{i,t}$), risk-free assets (RFR_t) and the market portfolio ($R_{M,t}$) were used. Data was corrected by inflation in the given month (quarter) according to formula ($R^*_{i,t}$ – nominal rates of return):

$$R_{i,t} = \left(\frac{1 + R^*_{i,t}}{1 + I_t} \right) - 1$$

(2)

In the second stage, one of the main assumptions of CAPM model (Capital Asset Pricing Model) was studied. It concerned the issue whether all rates of return series (monthly and quarterly) have normal distribution (or close to normal). The obtained results showed that, with significance level $\alpha=0.01$, in none of the studied cases (either monthly or quarterly rates of return) there was any reason to reject the null hypothesis (H₀), that their distribution is normal ($p > \alpha$) (see D o m a ń s k i, P r u s k a, 2000).

In the third stage, risk measurements resulting from further proposed investment efficiency indicators were defined and calculated. Basic measurements used to evaluate funds' investment risk are standard deviation and beta coefficient. The first is the measure of a total risk and is calculated from values of rate of return in an attempt, unbiased and consistent estimator of sample variance comes from formula (see B e r n s t e i n, D a m o d a r a n, 1999):

$$\hat{\sigma}_i^2 = \frac{1}{T-1} \sum_{t=1}^{T} (R_{i,t} - \bar{R}_{i,t})^2 \implies \hat{\sigma}_i = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} (R_{i,t} - \bar{R}_{i,t})^2}$$

(3)

where $\bar{R}_{i,t}$ – is a mean return from OPF portfolio in the specified period of time. Beta coefficient states how much the rate of return from funds' "portfolio" depends on (is correlated to) the change of the rate of return from model

(market) portfolio. This coefficient is determined by a simple linear regression, in which the return from investment portfolio ($R_{i,t}$) in specified period of time is a linear function of rate of return from the market portfolio ($R_{M,t}$) (see M a y o, 1997):

$$R_{i,t} = \alpha + \beta_i R_{M,t} + \xi_t \qquad R_{i,t} = \hat{\alpha} + \hat{\beta}_i R_M \qquad (4)$$

Further stages of this analysis consist of calculation of numerous indicators, which altogether were used to build OPF rank considering their efficiency. The first measure is the Treynor Ratio, which **defines the additional rate of return from risk bonus (over RFR) per risk unit in the given portfolio** (see B r o w n, R e i l l y, 2001). Also the Treynor Ratio (sometimes called Reward-to-Variability-Ratio) relates excess return to risk; but systematic risk instead of total risk is used.

$$\frac{E(R_i) - E(RFR)}{\beta_i} = E(R_M) - E(RFR) \Rightarrow T_i = \frac{\overline{R_i} - \overline{RFR}}{\hat{\beta}_i} \qquad (5)$$

where $\overline{R_i}$ and $\overline{RFR_t}$ – is a mean real rate of return from portfolio of fund "i" and risk-free investment (in year's time) respectively, and $\hat{\beta}_i$ – is an estimated systematic risk of the portfolio of fund "i", – risk due to market fluctuations (beta coefficient). The value of this coefficient for the market portfolio T_M is given by formula: $T_M = \overline{R_M} - \overline{RFR}$. A portfolio **with a higher value of T Ratio that for the market portfolio means better profitability considering the risk,** so the desired situation is $T_i > T_M$.

The second measure used for evaluation of profitability is a portfolio profitability indicator elaborated by W. F. Sharpe (Sharpe measure). It measures the **total portfolio risk** by including a standard deviation of portfolio rate of return into formula (see T a r c z y ń s k i, 2001).

$$S_i = \frac{E(R_i) - E(RFR)}{\sigma_i} \Rightarrow S_i = \frac{\overline{R_i} - \overline{RFR}}{\hat{\sigma}_i} \qquad (6)$$

This way we obtain the information about **the rate of return from the risk bonus (over risk-free rate of return) obtained per unit of total OPF risk.** Also the Sharpe Ratio (known as Reward-to-Volatility-Ratio) indicates the excess rate of return per unit of risk associated with the excess rate of return. Value of the Sharpe Ratio for the market is given by formula:

$$S_M = \frac{\overline{R_M} - \overline{RFR}}{\hat{\sigma}_M} \qquad (7)$$

where: $\overline{R}_M$ – is a mean real (annual) rate of return from the market portfolio, $\hat{\sigma}_M$ – is an estimated (annual) standard deviation of the rate of return from the market portfolio M.

From these ratios we **obtain relative**, not absolute, **values of portfolio profitability** (see B r o w n, R e i l l y, 2001). It means that, for example Sharpe Ratio for two portfolios A and B, whose profitability is higher than the market profitability, proves that the portfolio A is better than the portfolio B. However, it is hard to determine whether the difference between them is statistically significant.

Another indicator which can be used to analyze the profitability of OPF investment portfolios is **Jensen Ratio** (α-Jensen). It is determined by a simple linear regression:

$$R_{i,t} - RFR_t = \alpha_i^* + \beta_i^* \left(R_{M,t} - RFR_t \right) + U_{i,t} \tag{8}$$

Making an estimation of parameters from the above equation with OLS method we obtain:

$$R_{i,t} - RFR_t = \hat{\alpha}_i^* + \hat{\beta}_i^* \left(R_{M,t} - RFR_t \right) \tag{9}$$

The parameter under consideration is $\hat{\alpha}_i^*$, which **determines how much the portfolio rate of return reflects higher mean "rates of return" including the risk (results are better or worse from expected ones)** (see B r o w n, R e i l l y, 2001). It indicates whether a person in management foresees cycles of economic situation correctly or accurately chooses assets for portfolio.

Another proposed difference measurement is **α-Sharpe Ratio.** It is an upgraded version of Sharpe Ratio, which depends on economic situation. Using it we can separate added value of investment obtained by OPF from economic situation. Its value is positive for OPF better than market and its negative value means suffering too many costs (see W o ś, 2000).

$$\alpha - Sharpe'a = \overline{R}_i - \overline{RFR} - \frac{\hat{\sigma}_i (\overline{R}_M - \overline{RFR})}{\hat{\sigma}_M} \tag{10}$$

Tracking error of the market portfolio **TE** is a measure of variation of differences between funds' portfolio and benchmark. It is given by standard deviation of additional rate of return over benchmark $(R_i - R_M)$ (see S t e i n e r).

$$TE_i = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} (R_{i,t} - R_{M,t})^2} \tag{11}$$

The Information Ratio (also known as Appraisal Ratio) is basically a risk-adjustment of Alpha, is a quotient of additional rate of return obtained by OPF to a benchmark and benchmark tracking error. **This indicator shows how much**

the additional rate of return obtained by a fund exceeds benchmark per unit of active risk. Sharpe Ratio compares funds' results to risk-free investment, information ratio compares them to model portfolio (market portfolio) (see Steiner).

$$\alpha_i = \frac{1}{T}\sum_{t=1}^{T}\left(R_{i,t} - R_{M,t}\right) \Rightarrow IR = \frac{\alpha_i}{TE_i} \tag{12}$$

The M^2 measure is a positive linear transformation of the Sharpe Ratio and does not contain more or additional information then the Sharpe Ratio, that's why OPF rank based on it and on M^2 would be the same. For a fund with a given risk and rate of return, **M^2 equals the rate of return a fund would obtain if it had the same risk as the market portfolio** (see S t e i n e r).

$$R_{RAP,i} = \frac{\hat{\sigma}_M}{\hat{\sigma}_i}(\overline{R_i} - \overline{RFR}) + \overline{RFR} \Rightarrow R_{RAP,i} = \hat{\sigma}_M S_i + \overline{RFR} \tag{13}$$

As in the case of Sharpe Ratio, OPF rank according to M^2 depends on assumed market portfolio.

2. OPF efficiency

Analysis of investment results gathered in Table 1 shows that average annual rate of return of OPF ranges from 5.60% for AIG OPF to 8.11% for ING N-N OPF. **Amongst all funds only AIG did not gain a rate of return higher than the market portfolio (5.83%).** Standard deviation which is a total risk for each OPF would vary from 7.27% for OPF Ergo Hestia to 11.64% for Bankowy OPF. β coefficient calculated from linear regression of rates of return for each OPF relating to the market portfolio turned out to be **statistically significant**. The lowest value of β coefficient was 0.946 for SAMPO OPF; the highest was 1.419 for Bankowy OPF. High values of coefficient of determination (R^2) were obtained, which means that the change of the rate of return of OPF was at least in 72.2% explained by the change of the rate of return of the market portfolio (in case of Bankowy OPF) and maximally in 85.6% for AIG OPF. **Judging from the Sharpe Ratio showed that the highest rate of return (over the rate of return from RFR assets) per unit of total risk was obtained by ING N-N OPF (0.12%), and the lowest by AIG OPF (–0.18%), – the only one which obtained below the market portfolio.** The Traynor Ratio for fully diversified portfolios shows equal results with the Shape Ratio values, but **the rate of return from the risk bonus is expressed per unit of risk in that OPF portfolio.** In the analyzed case, OPF rating according to Traynor Ratio is the same for the first 10 funds. The value of α-Jensen turned out to be statistically significant only for ING N-N OPF only at the level 0.16 and it amounted 0.2.

Table I

OPF investment results based on monthly results for October 1999 – July 2005

No.	OPF	Mean Rate of Return in % (annual)	Standard Deviation (annual)	Beta Coefficient β_i	R^2	Sharpe Ratio (annual)	Treynor Ratio (annual)	Alfa Jensen	Alfa Sharpe Ratio (annual)	TE	IR (annual)	M^2 (annual)
1	ING N-N OPF	8.1120	9.4826	**1.2319**	0.8211	**0.1173**	0.9030	**0.2084**	2.6981	1.2567	**0.5230**	7.8178
2	CU OPF	7.3788	8.2279	**1.0815**	0.8405	**0.0461**	0.3506	0.1347	1.7551	0.9697	**0.4602**	7.3211
3	OPF Polsat	7.2768	7.5209	**0.9774**	0.8217	**0.0369**	0.2836	0.1146	1.5349	0.9244	**0.4509**	7.2567
4	Generali OPF	7.2600	8.2962	**1.0770**	0.8199	**0.0314**	0.2418	0.1231	1.6477	1.0355	**0.3978**	7.2185
5	OPF PZU	6.9756	7.5791	**0.9940**	0.8368	−0.0032	−0.0241	0.0918	1.2434	0.8903	0.3704	6.9775
6	OPF DOM	6.8304	8.3627	**1.0642**	0.7879	−0.0202	−0.1590	0.0872	1.2293	1.1273	**0.2553**	6.8585
7	Bankowy OPF	6.5904	11.6491	**1.4192**	0.7221	−0.0351	−0.2883	0.1003	1.5388	1.9777	0.1105	6.7546
8	SAMPO OPF	6.3468	7.5053	**0.9460**	0.7729	−0.0870	−0.6901	0.0345	0.6023	1.0457	**0.1417**	6.3929
9	OPF Pocztylion	6.2280	8.1050	**1.0643**	0.8390	−0.0952	−0.7250	0.0383	0.5838	0.9546	0.1193	6.3356
10	OPF Skarbiec	6.0576	8.3804	**1.0464**	0.7586	−0.1124	−0.9002	0.0220	0.4594	1.2008	0.0539	6.2156
11	OPF Allianz Polska	6.0888	7.5455	**0.9465**	0.7656	−0.1207	−0.9623	0.0144	0.3510	1.0676	0.0689	6.1577
12	Credit S L&P OPF	6.0636	7.5417	**0.9914**	0.8408	−0.1241	−0.9441	0.0167	0.3252	0.8752	0.0760	6.1339
13	OPF Ergo Hestia	6.0768	7.2753	**0.9601**	0.8474	−0.1268	−0.9611	0.0143	0.2938	0.8304	0.0847	6.1149
14	Pekao OPF	5.8932	7.4759	**0.9461**	0.7791	−0.1480	−1.1694	−0.0020	0.1438	1.0274	0.0169	5.9673
15	AIG OPF	5.6088	7.8102	**1.0363**	0.8566	−0.1781	−1.3421	−0.0180	−0.0847	0.8632	−0.0753	5.7575
	Market Index R_{Mt}	5.8332	6.9750	1.0000	1.0000	−0.1672	−1.1664	0.0000	–	–	–	–
	Risk free assets RFR_t	6.9996	1.7660	–	–	–	–	–	–	–	–	–

S o u r c e: own elaboration (Funds placed by Sharpe Ratio).

Table 2

OPF investment results based on quarterly results for October 1999 – September 2005

No.	OPF	Mean Rate of Return in % (*annual*)	Standard Deviation (*annual*)	Beta Coefficient β_i	R^2	Sharpe Ratio (*annual*)	Alfa Jensen	Alfa Sharpe Ratio (*annual*)	TE	IR (*annual*)	M^2 (*annual*)
1	ING N-N OPF	8.6540	8.5364	0.4143	0.0950	0.2051	0.5844	3.2778	4.5620	0.3164	8.2055
2	CU OPF	7.9724	7.8974	0.4409	0.1257	0.1354	0.4213	2.4819	4.1850	0.2635	7.7629
3	OPF Polsat	7.7244	6.7464	0.4163	0.1535	0.1218	0.3512	2.0281	3.6925	0.2650	7.6761
4	Generali OPF	7.7488	7.2922	0.2752	0.0574	0.1160	0.3278	2.1501	4.3130	0.2297	7.6395
5	OPF PZU	7.5488	6.5622	0.3491	0.1141	0.0984	0.2824	1.8195	3.7958	0.2347	7.5279
6	OPF DOM	7.3648	8.1608	0.3007	0.0548	0.5660	0.2495	1.9214	4.6439	0.1720	7.2623
7	Bankowy OPF	7.1344	10.9350	0.5320	0.0955	0.0212	0.2395	2.1871	5.5244	0.1237	7.0373
8	SAMPO OPF	6.9276	6.9826	0.3913	0.1267	0.0036	0.1390	1.2735	3.8736	0.1498	6.9254
9	OPF Pocztylion	6.8260	7.8944	0.3256	0.0686	−0.0097	0.1258	1.3349	4.4640	0.1186	6.8410
10	OPF Skarbiec	6.6780	7.4112	0.4619	0.1566	−0.0303	0.1187	1.1005	3.8897	0.1171	6.7102
11	OPF Ergo Hestia	6.6344	6.6200	0.3073	0.0869	−0.0405	0.0599	0.9154	3.9352	0.1102	6.6453
12	Credit S L&P OPF	6.6536	6.1330	0.3261	0.1140	−0.0406	0.0679	0.8476	3.6703	0.1208	6.6448
13	OPF Allianz Polska	6.4896	6.5364	0.3463	0.1132	−0.0632	0.0294	0.7557	3.7921	0.0952	6.5914
14	Pekao OPF	6.3064	7.0976	0.2727	0.0595	−0.0840	−0.0256	0.6729	4.2337	0.0637	6.3692
15	AIG OPF	6.2168	7.3016	0.2899	0.0636	−0.0940	−0.0552	0.6197	4.2811	0.0525	6.3062
	Market Index R_{Mt}	5.7672	6.3502	1.0000	1.0000	−0.1788	0.0000	–	–	–	–
	Risk free assets RFR_t	6.9028	2.4854	–	–	–	–	–	–	–	–

S o u r c e: own elaboration (Funds placed by Sharpe Ratio).

This means that only this fund **obtained in average a rate of return higher by 0.2% than from the market portfolio with a given risk β for the portfolio. Comparing OPF according to α-Sharpe coefficient we can also see a high investment efficiency of ING N-N OPF, which obtained the highest annual rate of return over the market portfolio equal to 2.67%.**

Tracking error TE of benchmark shows which fund had the best perception of the strategy of the market portfolio – variations of OPF rates of return from the benchmark would vary from 0.83% for OPF Ergo Hestia to 1.98% for OPF Bankowy. Judging from IR indicator we can draw the same conclusion, that the highest rate of return over the market portfolio per unit of relative risk was obtained by ING N-N OPF 0.52%, and the lowest by SAMPO OPF 0.14%. The rest of IR values were statistically insignificant. OPF rank according to M^2 measure is the same as for Sharpe Ratio. However, it gives a different view of funds' efficiency. **Thanks to bringing OPF rates of return down to the market portfolio risk level it is possible to compare the funds.**

The second analysis, this time for quarterly results, gives us a very limited chance to evaluate funds' investment efficiency. Average annual rate of return for all funds varied from 6.22% for AIG OPF to 8.56% for ING N-N OPF. **All OPF obtained rate of return higher than the market portfolio (5.83%).** Their standard deviation, that is total risk, would vary from 6.13% for Credit S L&P OPF to 10.93% for Bankowy OPF. β coefficient calculated from liner regression of the rate of return for each OPF relating to the market portfolio turned out to be **statistically insignificant.** At the same time, **very low values of coefficient of determination R^2** were obtained. Comparison of OPF according to Sharpe Ratio has shown that the highest rate of return (over the rate of return from RFR assets) was obtained by ING N-N OPF (0.20%), and the lowest by AIG OPF (–0.094%), in that case all funds obtained results better than the market portfolio (–0.18%). **Because of the fact that β coefficients were statistically insignificant, no Treynor Ratio was calculated.** Values of α-Jensen's were also **statistically insignificant.** α-Sharpe Indicator confirmed once again the high investment efficiency of ING N-N OPF, which obtained the highest annual rate of return (including total risk) over the market portfolio 3.28%.

3. OPF classification methods – cluster analysis

Cluster analysis includes several different algorithms of classification. The general research task for many disciplines is to group data and to organize it into reasonable structures. With classification, the higher aggregation level, the lower the similarity between objects in different classes. Cluster analysis methods

prove to be a valuable tool whenever we need to classify a "heap" of information into reasonable groups (see StatSoft, Inc., 1997).

The first method is **Tree Clustering**. This algorithm is designed to join objects by amalgamation or linkage rules into ever-larger classes (clusters) using some specified similarity or distance measure. A typical result of this method is a hierarchical tree. In this analysis Single Linkage Method, Complete Linkage Method and Ward Method were used, as a distance measure the Euclidean's metric was used.

Single Linkage Method (nearest neighbor) defines the distance between two clusters as the distance between two nearest objects (nearest neighbours) belonging to different clusters. **Complete Linkage Method (furthest neighbour)** is much more similar to the former one, but the distance between two clusters is determined by the largest distance between two arbitrary objects belonging to different clusters (furthest neighbours).

Ward Method differs from the two mentioned above, as it uses variation analysis to estimate the distance between clusters. It tends to minimize the sum of squares of arbitrary two clusters, which can be formed in next step.

Tree Clustering method uses distance measures between objects to form clusters. The most direct ways of determining distance in multidimensional space is calculating **Euclidean distance**. It is a geometrical distance in multi-dimensional space.

K-Means Clustering is the second cluster analysis method used here. This method differs much from Tree Clustering methods. Let us suppose that we formed a hypothesis about the number of clusters of our cases (variables). This algorithm allows us to create a given number of clusters that would be as different as it is possible. It starts from "k" random clusters and then moves objects between clusters in order to minimize the differentiability inside these clusters and maximize the differentiability between clusters.

4. OPF classification

Three OPF classifications were made, two concerning funds' **efficiency** and one concerning funds' **investment policy**. A different set of variables was used in the analysis.

For the first classification (C1) concerning efficiency three measures were used, which do not show any significant correlation – **the Sharpe Ratio, the β coefficient and the coefficient of determination R^2**. Variable standardization was made by subtracting from each variable its mean value and dividing by variable's standard deviation.

Correlation coefficients for first classification

Variable	Correlation		
	Sharpe Ratio	Beta Coefficient β_i	R^2
Sharpe Ratio	1.00	0.44	0.08
Beta Coefficient β_i	0.44	1.00	−0.36
R^2	0.08	−0.36	1.00

S o u r c e: own calculations in STATISTICA 6.0 PL.

Next, Tree Clustering with Single Linkage, Complete Linkage and Ward Method were used. **To verify object belonging to clusters Z. Hellwig Method was used.** It states that two subclasses of a class are regarded as significantly different if the smallest distance between two points belonging to two different subclasses is larger than a some critical W_k. Critical value is given by formula $W_k = \bar{x} + 2\sigma$. In order to calculate it we need to find a minimal value for every row of distance matrix and next calculate mean "$\bar{x}$" and standard deviation "σ" of these values.
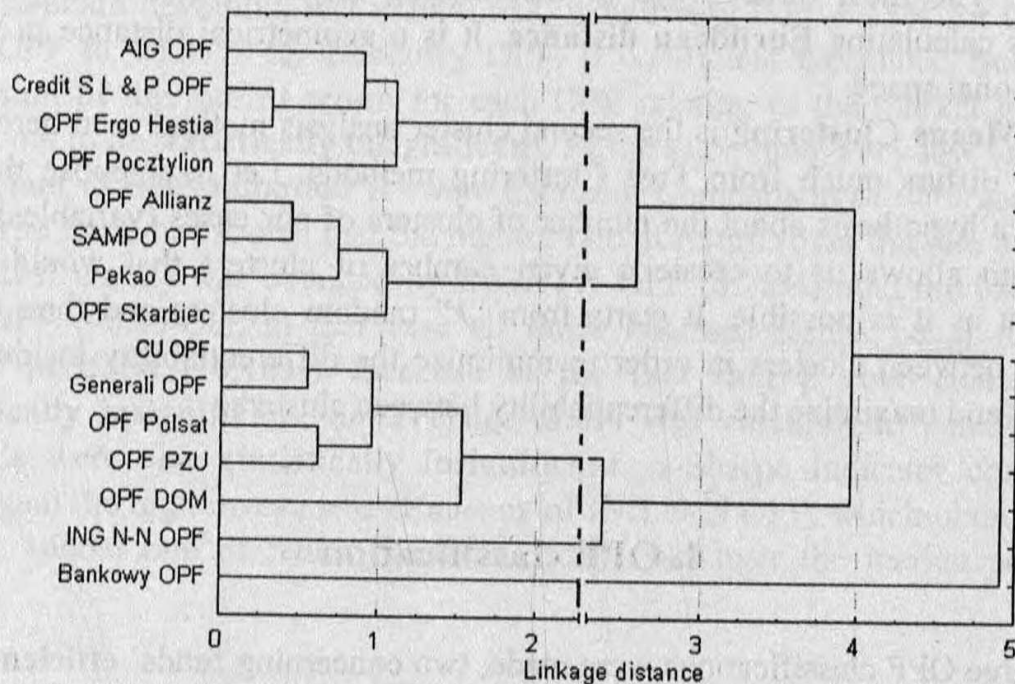

Chart 1. Classification (C1). Tree Diagram with Complete Linkage Method and Euclidean distance

S o u r c e: own calculation in STATISTICA 6.0 PL ($W_k = 2.282$).

Tree Clustering with Complete Linkage and with Ward Method gave 5 identical clusters, and k-Means Clustering confirmed that result. The obtained clusters look as follows:

1 Cluster)   AIG OPF, Credit S L&P OPF, OPF Ergo Hestia, OPF Pocztylion,

2 Cluster)   OPF Allianz, Pekao OPF, SAMPO OPF, OPF Skarbiec,

3 Cluster)   CU OPF, OPF DOM, Generali OPF, OPF Polsat, OPF PZU,

4 Cluster)   ING N-N OPF,

5 Cluster)   Bankowy OPF.

The above analysis was made using monthly data. No cluster analysis was made for quarterly data as β coefficients were statistically insignificant and coefficients of determination R^2 were poor.

The second classification (C2) concerning OPF efficiency was made using **monthly rates of return during period under investigation (70 samples).** After checking correlation between rates of return for separate funds, some cases where high (significant) correlation was observed were removed, limiting to 25 samples. The same Tree Clustering methods were used (distance measures and linkage rules) and object belonging to clusters was also confirmed by Hellwig Method. Ward Method gave us 7 following clusters.

1 Cluster)   AIG OPF, OPF Ergo Hestia, OPF Polsat,

2 Cluster)   CU OPF, Pekao OPF,

3 Cluster)   OPF Pocztylion,

4 Cluster)   OPF DOM, Generali OPF,

5 Cluster)   OPF PZU, SAMPO OPF,

6 Cluster)   OPF Allianz, Credit SL& P OPF, ING N-N OPF, OPF Skarbiec,

7 Cluster)   Bankowy OPF.

K-Means Clustering confirmed these results – initial clusters centers were chosen so to maximize clusters distance. The same classification was made for quarterly rates of return. Out of 24 quarterly rates of return for each OPF 12, which showed no statistically significant correlation, were chosen. Complete Linkage and Ward Method gave 4 and 5 clusters respectively, but no starting condition for *k*-means method was able to give the same results.

For the last classification (C3), concerning investment policy, the following 3 variables were chosen: **the average monthly OPF rate of return** (expressed per year), its **standard deviation** (total risk measure) and **mean monthly participation of bonds and treasury bills in OPF portfolio** in the period under investigation. Further, correlation of variables was analyzed and the assumed variables were standardized. Two different results were obtained.
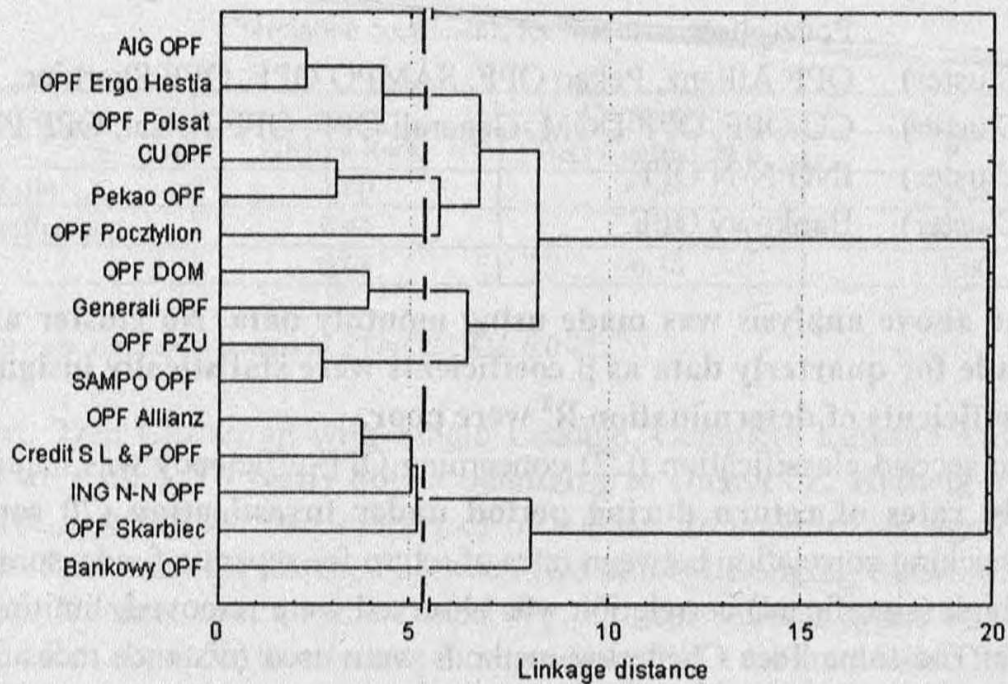
Chart 2. Classification (C1). Tree Diagram with Ward Method and Euclidean distance

S o u r c e: own calculation in STATISTICA 6.0 PL ($W_k = 5.355$).

T a b l e 4

Correlation coefficients for the third classification

Variable	Correlation		
	Standard Deviation	OPF rate of return	T-bonds and T-bills
Standard Deviation	1.00	0.44	0.08
OPF rate of return	0.44	1.00	−0.36
T-bonds and T-bills	0.08	−0.36	1.00

S o u r c e: own calculations in STATISTICA 6.0 PL.

By Tree Clustering with Complete Linkage the following 4 clusters were obtained:

1 Cluster) AIG OPF, OPF Skarbiec, OPF Allianz, SAMPO OPF, OPF Ergo Hestia, Credit S L&P OPF, Pekao OPF, OPF Pocztylion,
2 Cluster) CU OPF, Generali OPF, OPF Polsat, OPF PZU, ING N-N OPF,
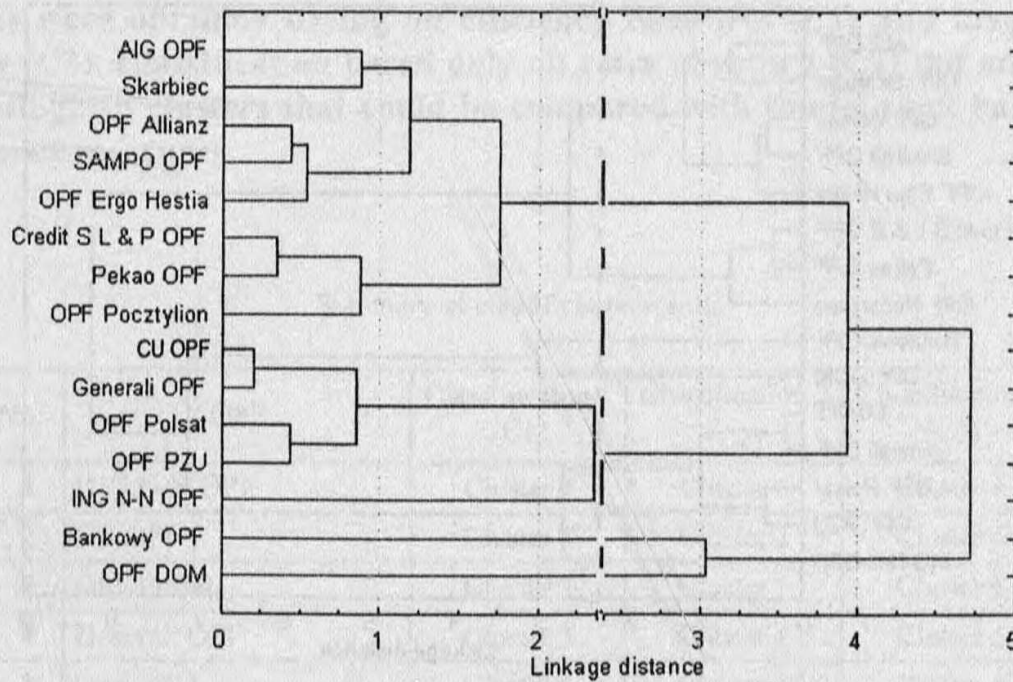3 Cluster) Bankowy OPF,
4 Cluster) OPF DOM.

Chart 3. Classification (C3). Tree Diagram with Complete Linkage Method and Euclidean distance

S o u r c e: own calculation in STATISTICA 6.0 PL ($W_k = 2.378$).

A detailed analysis of Tree Clustering and distances showed that ING N-N OPF creates a cluster with CU OPF, Generali OPF, OPF Polsat and OPF PZU on the very border of calculated critical value (2.372 compared to 2.378) and it cannot be treated as reliable.

Tree Clustering with Ward Method showed, that there are 6 clusters, that is ING N-N OPF and Credit S L&P OPF, Pekao OPF, OPF Pocztylion should be treated as separate clusters:

1 Cluster)   AIG OPF, OPF Skarbiec, OPF Allianz, SAMPO OPF, OPF Ergo Hestia,
2 Cluster)   Credit S L&P OPF, Pekao OPF, OPF Pocztylion,
3 Cluster)   Bankowy OPF,
4 Cluster)   OPF DOM,
5 Cluster)   CU OPF, Generali OPF, OPF Polsat, OPF PZU,
6 Cluster)   ING N-N OPF.

The opposite k-Means Clustering with initial condition such that "initial cluster centers were chosen to maximize cluster distances" confirmed the results from **Complete Linkage and Ward Method.**
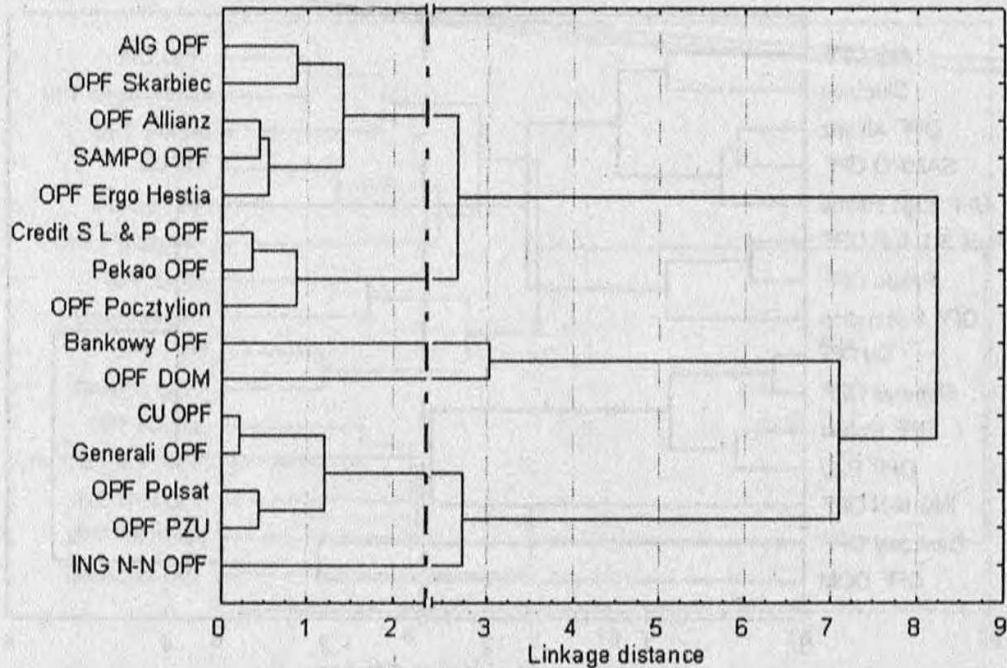
Chart 4. Classification (C1). Tree Diagram with Ward Method and Euclidean distance

S o u r c e: own calculation in STATISTICA 6.0 PL ($W_k = 2.378$).

This classification was not made for quarterly results again, as there was a strong correlation between the participation of bonds and treasury bills in OPF portfolios with the total risk of these portfolios, which seems to be reasonable taking into account the contribution of these assets in funds' investment portfolios.

5. Summary

Presented OPF investment efficiency analysis should be considered reliable. Much care was taken during data collection and elaboration (completeness and uniformity of data and methodology) and construction of the market portfolio used for comparison. D. W. French and G. V. Henderson proved that if you eliminate the random risk and shares evaluation errors, then **Sharpe and Treynor Ratios are splendid in classification of portfolios according to their real positions** (see B r o w n, R e i l l y, 2001) One disadvantage – already mentioned – of these indicators is that they measure relatively portfolios profitability and besides classification it is hard do determine how much one fund is better from another and whether the difference between them in statistically significant.

In order to solve this "problem" a cluster analysis was made. Similar results were obtained basing on efficiency measures (C1) and investment policy (C3). Classification based only on rates of return (C2) did not allow to distinguish clusters that could be compared with funds' rank basing on efficiency measures.

T a b l e 5

Summary of classification results

No.	OPF	Classification 1 (C1)	Classification 2 (C2)	Classification 3 (C3)
1	ING N-N OPF	Cluster 4	Cluster 6	Cluster 6
2	CU OPF	Cluster 3	Cluster 2	Cluster 5
3	OPF Polsat	Cluster 3	Cluster 1	Cluster 5
4	Generali OPF	Cluster 3	Cluster 4	Cluster 5
5	OPF PZU	Cluster 3	Cluster 5	Cluster 5
6	OPF DOM	Cluster 3	Cluster 4	Cluster 4
7	Bankowy OPF	Cluster 5	Cluster 7	Cluster 3
8	SAMPO OPF	**Cluster 2**	Cluster 5	*Cluster 1*
9	OPF Pocztylion	*Cluster 1*	Cluster 3	**Cluster 2**
10	OPF Skarbiec	**Cluster 2**	Cluster 6	*Cluster 1*
11	OPF Allianz Polska	**Cluster 2**	Cluster 6	*Cluster 1*
12	Credit S L&P OPF	*Cluster 1*	Cluster 6	**Cluster 2**
13	OPF Ergo Hestia	*Cluster 1*	Cluster 1	*Cluster 1*
14	Pekao OPF	**Cluster 2**	Cluster 2	**Cluster 2**
15	AIF OPF	*Cluster 1*	Cluster 1	*Cluster 1*

S o u r c e: own elaboration (OPF order as in Table 1).

The presented classification is helpful, as it is important for funds' customers who are saving for pension for a longer time to chose one of the most effective (but not necessarily the best) fund.

Table 5 shows classification results and obtained groups (OPF order as in Table 1). **In two classifications (C1 and C3) ING N-N OPF is treated as separate cluster. Taking into account its efficiency position one has to state, that it is indeed the best pension fund.** The second group; in classifications C1 and C3 is very similar, with exception of OPF DOM, which in C3 is treated as a separate cluster. Another fund – Bankowy OPF was also classified to a separate group it is a consequence of surcharges it made in the past, which had distinguished it from the rest of funds. For the rest of funds from places 8–15 no

classification was able to determine "uniform clusters". This could be caused by very small differences between funds which made it impossible to distinguish precisely the groups, so that they would comply with their position in efficiency rank. What is characteristic (Chart 1), funds belonging to cluster 1 and 2 created these clusters on the same distance (1.08 and 1.03), and then created a new class (above critical value). It seems justified to treat them all as a separate group.

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Artur Mikulec

Ocena efektywności inwestowania Otwartych Funduszy Emerytalnych metodą analizy skupień

Celem niniejszego opracowania jest analiza dotychczasowej działalności Otwartych Funduszy Emerytalnych (OFE) z punktu widzenia osiągniętych wyników inwestycyjnych. Do oceny efektywności tych funduszy wykorzystano wskaźniki rentowności portfela inwestycji: Sharpe'a, Treynora i Jensena, a także IR, TE czy M^2 (M^2-measure). Analizę przeprowadzono na danych miesięcznych i kwartalnych. W kolejnym etapie dokonano klasyfikacji OFE z punktu widzenia ich efektywności i prowadzonej polityki inwestycyjnej, analizując obliczone wskaźniki rentowności, stopy zwrotu i miary ryzyka. W tym celu wykorzystano takie metody analizy skupień, jak: aglomerację, metodę k-średnich oraz różne miary odległości i metod łączenia lub wiązania.