A C T A U N I V E R S I T A T I S L O D Z I E N S I S

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Czesław Lipiński*, Łucja Tomaszewicz**

IMPORTANT COEFFICIENTS IN THE INPUT-OUTPUT MODELS FOR POLAND

1. Criteria for the Evaluation of Importance of Input-Output Coefficients

Consider the model:

$$y_t = (I - A_t)X_t, \tag{1}$$

where: $y_t = [y_{it}]$ is a vector of final output, $X_t = [X_{it}]$ is a vector of total output, $A_t = [a_{ijt}]$ is a matrix of input-output coefficients ($i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, n$).

A practical application of such a model in solving numerous problems connected with the formation of proper economic macro-proportions and economic equilibrium, i.e. among others with

- the determination of demand for the output of particular branches,
- the distribution of output among intermediate and final users,
- the utilization of production capacities of the branch,

is connected first of all with the analysis of the behaviour of input-output coefficients in the investigated period.

A simplifying assumption about the stability of these coefficients in the input-output systems frequently undertaken, cannot be applied without limitations due to at least the following reasons:

1) change of the structure of possible inputs in branches

*Senior Assistant, Inst. of Econometrics and Statistics, University of Łódź.

**Lecturer, Inst. of Econometrics and Statistics, University of Łódź.

induced by changes of relative prices (causing substitution among inputs), by availability of inputs, production scale etc.,

2) change of assortment structure of production resulting mainly from determined market tendencies,

3) technological innovations and introduction of new products,

4) possible differences in the methods of constructing input-output balances concerning mainly a different price basic and changes in sectoral classification of economic subjects and produced goods and services.

However, it is not always possible, and sometimes even unnecesary, to consider all the elements of matrix A_t. For practical reasons it is enough to concentrate on the important coefficients only, e.g. by consulting experts on expected changes or by constructing appropriate models of changes etc.

Other coefficients which are less important can be determined using the well-known methods: biproportional RAS-type methods (B a c h a r a c h 1970), Input-Output Relationships... or mathematical programming methods (cf. L e c o m b e r 1971).

We have assumed three basic criteria of coefficients importance:

1. The large values of input-output coefficients a_{ijt} or related values. In this case even small inaccuracy in determining the values of these coefficients can influence, to a great extent, the correctness of model solution (1).

2. Significant changeability of coefficient sequence $\{a_{ijt}\}$ in time.

3. "Strong" connections between coefficient a_{ijt} and the whole economic system. The change of one such element can cause significant changes in the processes of production and distribution in the whole economic system.

It is worth noting that each of these criteria when considered separately, will order the importance of these coefficients in a different way. It is relatively easy to point out "large", fairly stable in time values of the coefficients, most frequently linking the raw material sectors with manufacturing ones (e.g. agriculture with food industry, mining with fuel and power industry and metallurgy) which are not always strongly connected with other economic sectors. It is also easy to point out relati-

vely small although not stable in time coefficients or these strongly connected with other branches (e.g. connections of transportation with other productive branches). With relation to this it seems that the statement that only joint consideration of the above criteria will allow to evaluate properly the importance of the considered coefficient, is true.

However, it should be stressed that the testing of its importance must be performed in the context of model (1) and on the basis of a determined input-output balance or their sequence. If model (1) was a part of a general model constructed not only to obtain consistent production plans, then it probably would not be necessary to use all of these three criteria for evaluating the coefficients importance. On the other hand, it cannot be excluded that quite different measures might prove useful, e.g. in the optimization model the measure will be the degree of sensitivity of the optimal solution to the change of one of them[1].

Taking into account the above formulated criteria for the evaluation of importance of input-output coefficients, the methods for their determination can be divided into:

1) direct ones, in which the basis of evaluation are the values of particular coefficients or their sequences, and - according to the purpose of the study, connected with the application of the input-output relations - their related values such as, for instance, the value of input-output flow related to the sum of production demand or, the so-called distribution structure coefficient. The group of direct measures includes also the measures based on the investigation of coefficient changeability;

2) indirect ones - in which the basis for evaluation of importance is the measure of the influence of an identical (in per cent) change of values of each of particular coefficients on

a) the volume of final output of the branch, under the assumption that the volume of gross output by branches is unchanged,

b) the value of gross output of branches with unchanged final output.

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[1] Cf. our remarks on the subject in L i p i ń s k i, T o m a s z e w i c z 1981.

2. Direct Methods for Evaluating the Coefficients Importance

The simplest measure in the group of direct measures is the coefficient value, i.e.

$$d_{ij}^{(1)} = a_{ij} = \frac{x_{ij}}{X_j}, \tag{2}$$

where x_{ij} is a value of input flow of the i-th branch to the j-th branch and X_j - the value of gross output of the j-th branch. The higher the coefficient value, the greater importance is attributed to it.

Similar results are obtained using, as a measure, the share of input originating in the i-th branch in the total material costs of the j-th user, i.e. the measure in the form:

$$d_{ij}^{(2)} = \frac{x_{ij}}{\sum_i x_{ij}} = a_{ij} \frac{X_j}{\sum_i x_{ij}}. \tag{3}$$

Both these measures supply direct information about the input importance (input-output coefficient) from the point of view of particular users. This importance is the higher, the greater the value of d_{ij}[2].

Similarly, the importance of input in the process of production distribution can be determined from the point of view of particular producers. The simplest measure of this kind is:

$$d_{ij}^{(3)} = a_{ij} \frac{X_j}{X_i}. \tag{4}$$

It is easy to note that the measure $d^{(3)}$ is the so-called distribution structure coefficient

$$h_{ij} = \frac{x_{ij}}{X_i} \tag{5}$$

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[2] It is not difficult to see that the measures $d^{(1)}$ and $d^{(2)}$ give a similar hierarchy of importance of the coefficients, if the share of value-added in gross output of the branch is not too much differentiated in particular branches.

The measure constructed in the conception as $d^{(2)}$ and based on the coefficient h_{ij} has the form:

$$d_{ij}^{(4)} = a_{ij} \frac{X_j}{\sum_j x_{ij}} . \tag{6}$$

Some generalization of these measures which enables to evaluate the significance of a given coefficient both from the point of view of the supplier and the user(of the i-th and j-th branches), would consist in assuming the following measures:

$$d_{ij}^{(5)} = \sqrt{a_{ij}\, h_{ij}} = \frac{x_{ij}}{\sqrt{X_i\, X_j}} \tag{7}$$

or

$$d_{ij}^{(6)} = \frac{x_{ij}}{\sqrt{\sum_i x_{ij} \sum_j x_{ij}}} . \tag{8}$$

Hence, we obtain the information about the importance of the coefficient only from the point of view of a given supplier and user. And still we lack the evaluation of the coefficients importance from the point of view of their influence on the behaviour of a balanced economic system as a whole[3].

Before we pass to the indirect measures, which allow to carry out such an evaluation, we shall devote some attention to the measures which we consider as direct and which are based on the changeability of the coefficients in time. If such an evaluation is made on the basis of two input-output balances describing the same economic system (in the identical branch classification)

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[3] Note that all direct methods require arbitrary determination of the value above which the given coefficient is considered as important. In a simple procedure of coefficient importance evaluation based on a direct method applied by EEC (J i l e k 1971) it was assumed, for instance, that the most important elements are these which in the decreasingly ordered rows or columns are at the beginning and constitute 90% of the sum of rows or columns.

in moments t_0 and t_1, then the most frequently used measures are relative differences

$$_{0,1}d_{ij}^{(7)} = \frac{a_{ij1} - a_{ij0}}{a_{ij0}} \tag{9}$$

or

$$_{1,0}d_{ij}^{(8)} = \frac{a_{ij0} - a_{ij1}}{a_{ij1}}. \tag{10}$$

The higher the absolute values of these measures, the more attention should be paid to respective elements, using the input-output coefficients to forecast the branch structure of production and distribution.

Another way to investigate changeability of the coefficients in time is to estimate the regression function of the form:

$$a_{ij} = f_{a_{ij}}(t), \tag{11}$$

(where t is a time variable) and to assume as a criterion of importance, the derivative of this function at some point $t = T$, generally, at the moment for which the forecast of the coefficient is evaluated.

Hence,

$$d_{ij} = \frac{d f_{a_{ij}}(t)}{dt} \quad (t = T). \tag{12}$$

This way requires, however, a sufficiently long series a_{ijt} on the basis of which the class of functions $f_{a_{ij}}$ can be chosen and their parameters estimated. Since in this case we use, in fact, not a derivative but its estimate, obtained on the basis of the sample, we should also take into account mean errors of the obtained results. Hence, it seems possible to consider not one but three criteria:

1) the already mentioned evaluation of the derivative at the moment $t = T$,

2) lower bound of confidence interval assuming that the coefficient is important if, at high probability, it changes significantly in time, and

3) upper bound of this interval, assuming that the coefficient is important if it is possible (e.g. 5% probability) that it changes significantly.

The choice of these criteria will certainly depend on additional information concerning the coefficient, which results from the application of other measures of importance.

3. Indirect Methods for the Evaluation of Coefficient Importance

Indirect methods of evaluating the importance of input-output coefficients generally denote the determination, through the change of the value of a given coefficient, of the change in fixed relations between the output of a given branch and the input for this output. A change of one coefficient being a primary impulse, can lead, in many cases, to significant changes in the structure of production and distribution through many secondary impulses.

Let us consider two extreme cases which can result from a change of some coefficient a_{ij}:

1. At unchanged level of gross output in particular branches.
2. At unchanged level of final production.

In the first case exactly one element of vector y is changed in fact. This is the element y_i. The change is

$$y_i = -\frac{p}{100} a_{ij} x_j, \tag{13}$$

where: p - per cent of the change of coefficient a_{ij}.

This value could constitute some measure of importance of the coefficient being changed. However, this evaluation of importance is equally one-sided as the above presented direct measures.

It has been included into the group of indirect measures only because we consider here directly not the coefficients, but indirectly - their importance, through changes in the final production.

In the other case, due to the necessity of maintaining the relation

$$X^* = \left(I - A^*\right)^{-1} y, \tag{14}$$

where: A^* - matrix of input-output coefficients with element a_{ij} replaced by a^*_{ij}, X^* - a respectively changed vector of gross output, the substitution of a_{ij} by some element a^*_{ij} can induce a change in the gross output level of each branch, since then there are changes in all elements of the matrix of full material - input coefficients $(I - A)^{-1}$.

The most frequently used measure of sensitivity of gross output to a change of one of the input-output coefficients is the so-called coefficient of tolerable limits.

$$d^{(9)}_{ij} = \frac{1}{a_{ij}\left(0.01\, b_{ji} + \max\limits_{k} \left\{ b_{ki} \frac{X_j}{X_k} \right\}\right)}, \tag{15}$$

where b_{ji}, b_{ki} are the elements of matrix $(I - A)^{-1}$.

The values assumed by this measure are interpreted as a per cent change of the value of coefficient a_{ij}, which causes a change in the output level of the i-th branch by 1%.It is evident that the lower the value of the coefficient of torelable limits, the more important is a given element of the matrix A for the economic system as a whole. Even slight changes of its values can cause significant changes in the level of gross output of particular branches.

4. An Attempt at Joint Evaluation of the Importance of Coefficients a_{ij} on the Basis of Various Criteria

In order to compare simultaneously the importance of input-output coefficients with respect to various criteria,the following procedure for ranking of the importance measures can be proposed.

Particular elements a_{ij} are ordered within each criterion in decreasing importance order by giving them the ranks

$$R^{(k)}_1 = 1,\ R^{(k)}_2 = 2,\ \ldots,\ R^{(k)}_m = m,$$

where: $k = 1, 2, \ldots, k$ - successive criteria, $m = n \times m$ - the number of elements of matrix A.

If the values of s elements, starting with the element h + 1, are identical, we give them a medium rank.

$$R_{h+1}^{(k)} = R_{h+2}^{(k)} = \ldots = R_{h+s}^{(k)} = \frac{h + 1 + h + 2 + \ldots + h + s}{s} =$$

$$= h + \frac{s + 1}{2}. \qquad (16)$$

The sum of ranks obtained by a given element with respect to all criteria, i.e.

$$R_i = \sum_{k=1}^{K} R_i^{(k)} \quad (i = 1, 2, \ldots, m),$$

or a mean of ranks

$$\bar{R}_i = \frac{1}{K} \sum_{k=1}^{K} R_i,$$

determines then a measure of importance of a given element. The importance is the greater the lower the rank.

The decision, regarding which level of R_i or $\bar{R}_i$ still characterizes the important input-output coefficients, is arbitrary and depends especially on the "scatter" of importance measures obtained with respect to various criteria. It can be therefore different for each case of the input-output balance.

5. Empirical Results

Input-output coefficients have been analysed for two input-output balances prepared for Poland for the years 1975 and 1980. These balances have been chosen to compare the structure of economy in the period of relatively stable development (1975) and in the period when economic depression began (1980).

The balances are presented at current producers' prices in 15 aggregated industries:

1) fuel and power industry,
2) metallurgy,

Important input-output

	1	2	3	4	5	6	7
1	1 2 3 4 5 6 9	1 3 5 6 9		5	1 2 5 6		
2		1 2 3 4 5 6 9	1 2 3 4 5 6 9				
3			1 2 3 4 5 6 9		1 2		
4			3 5 6 9	1 2 3 4 5 6 9			1 2 3 4 5 6 9
5					1 2 3 4 5 6 9	1	
6			3 9			2 3 4 5 6 9	
7							1 2 3 4 5 6 9
8							
9			9				3 9
10	4 6						
11							
12						1 2 3 4 5 6 9	
13					1		
14		4	4	4			4
15			4				

3) metal and electro-engineering industry,
4) chemical industry,
5) building materials, glass and pottery industry,
6) wood and paper industry,
7) light industry,

Table 1

coefficients for the year 1975

	8	9	10	11	12	13	14	15
1						1 2 3 4 5 6 9		1 2
2			1 3 5 6 9					
3			1 2 5 6 9			1 2		
4				3 5 9				
5			1 3 4 5 6 9					
6		1 5 6 9	3 4 9					
7								
8	1 2 3 4 5 6 9	1 2 4 5 6						
9	3 9	1 2 3 5 6 9		3 4 5 6 9				1 2 3 5 6 9
10			4 5		2		6	
11	1 2 3 4 5 6 9	1 2		1 2 3 4 5 6 9				
12								
13	3		1 2 3 4 5 6 9		2	1 2 5 6	1 2 3 5 6 9	
14		4 5	4					
15	4							

8) food industry,
9) other industrial branches,
10) construction industry,
11) agriculture,
12) forestry,

Important input-output

	1	2	3	4	5	6	7
1	1 2 3 4 5 6	1 3 5 6 9					
2		1 2 3 4 5 6 9	1 2 3 4 5 6 9				
3	1 2		1 2 3 4 5 6 9		1 2		
4			3 4 9	1 2 3 4 5 6 9			2 3 5 6 9
5					1 2 3 4 5 6 9		
6			3 9			1 2 3 4 5 6 9	
7							1 2 3 4 5 6 9
8							
9							
10	4						
11							
12						1 2 3 4 5 6 9	
13							
14	5		4 6				4
15			3 4 9				

13) transportation and communication,
14) trade,
15) other material goods and services.
These numbers are also used in the tables.

Table 2

coefficients for the year 1980

	8	9	10	11	12	13	14	15
						1 2 3 4 6 9		
			3 5 9					
		1	1 2 5 6 9			1 2 5 6 9		1 2
				3 9				
			1 3 4 5 6 9					
			3 4 9					
	1 2 3 4 5 6 9	1 2 4 5 6						
		1 2 3 4 5 6 9		3 4 5 6 9				2 3 5 6 9
			4 6					
	1 2 3 4 5 6 9	1 2 5 6		1 2 3 4 5 6 9				
					1 2 3 4 5 6 9			
	3		1 2 3 4 5 6 9		1 2	1 2 3 5 6 9	1 2 3 4 5 6 9	
			4	4		4 5 6		

First of all the above presented direct measures $d^{(1)} - d^{(6)}$ and the coefficient of tolerable limits $d^{(9)}$ have been taken into account. The series of these measures ordered from the point of view of decreasing importance as well as the one obtained from

summing up of ranks, given to the input-output coefficients, have been analysed. Each of these series consists of 30 elements since it was observed that the scatter of values of different importance criteria for further elements in the ordered series increased significantly. In the ranking procedure the changeability of the coefficients in time has not been taken into account. This criterion has been considered separately.

Tables 1 and 2 present the estimation of input-output coefficients importance for the 1975 and 1980 balances. Each field corresponds to a particular coefficient. The numbers placed in these fields inform us about the measure with respect to which the given element has been estimated as the most important. The fields in thick frames correspond to the elements which are most important from the point of view of the total sum of ranks.

On the basis of the analysis of the above tables it can be concluded that the most important input-output coefficients are placed first of all on the main diagonal (it may be a result of sometimes significant aggregation of branches).The characteristic connections between sectors and branches of raw-material and manufacturing type are also assumed as important. The input-output coefficients for the following relations correspond to them:

- fuel and power industry for the metallurgy and transportation and communication,
- metallurgy for metal and electro-engineering industry,
- metal and electro-engineering industry for construction and transportation and communication,
- chemical industry for light industry,
- agriculture for food industry,
- forestry for wood and paper industry,
- transportation for construction and trade.

The importance of these coefficients is almost identical for both years from the point of view of the above mentioned measures. An additional evidence supporting this hypothesis can be the results obtained on the basis of choosing 5 subsequent most important coefficients for each measure and each balance. It appeared that in most cases these are the same coefficients. The below mentioned coefficients

$a_{2,2}$ (metallurgy - metallurgy),

$a_{7,7}$ (light industry - light industry),
$a_{11,11}$ (agriculture - agriculture),
$a_{11,8}$ (agriculture - food industry),
occurred most frequently (in 9-12 cases out of 14). Once again the hypothesis of significant importance of direct technological connections has been proved.

For the coefficients presented in Tab. 1 and 2 their rates of changes $\left(d_{ij}^{(7)}\right)$ between 1975 and 1980 have been established.

The rate of changes of more than a half of the coefficients ranged from -15% to +15%. A significant increase (over 30%) of unit inner inputs of fuel and power and inputs of fuel and power for electro-engineering industry has been observed. Unit inputs of production of other branches (fodder industry) for agriculture and electro-engineering industry production for transportation also showed significant increase. The inner inputs also increased in forestry.

A decreasing tendency of unit inputs (over 30%) was observed in inputs of construction for the light industry, in inputs of wood industry and forestry for other branches, in inputs of metallurgy for construction and in inputs of transportation for fuel and power industry. Therefore, at least half of the presented important coefficients changed significantly between the years 1975 and 1980.

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Czesław Lipiński, Łucja Tomaszewicz

WAŻNE WSPÓŁCZYNNIKI W TABLICACH PRZEPŁYWÓW MIĘDZYGAŁĘZIOWYCH DLA POLSKI

Zagadnienie poszukiwania ważnych współczynników ma istotne znaczenie dla wielu prac praktycznych związanych z analizą input-output, przede wszystkim zaś dla przewidywania zmian współczynników nakładów bezpośrednich.

W artykule przeprowadzono pewną systematyzację i analizę metod stosowanych przy wyznaczaniu ważnych współczynników. Sformułowane zostały trzy zasadnicze grupy kryteriów ważności współczynników:

1) przyjmowanie dużych wartości przez współczynniki nakładów bezpośrednich $\{a_{ijt}\}$ lub wielkości pochodne,

2 niestabilność ciągu współczynników $\{a_{ijt}\}$ w czasie,

3) "silne powiązanie" współczynników z całym układem

Wyrazem praktycznego zastosowania omawianych metod było zbadanie ważnych współczynników w bilansach input-output dla lat 1975 i 1980 w układzie 15 gałęzi.