A C T A   U N I V E R S I T A T I S   L O D Z I E N S I S
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MODELLING IMPACT OF PLAN ON CAPACITY UTILIZATION

1. Introduction

In centrally planned economies the production process is, in general, subordinated to the plan. On the other hand, the plan is constructed, among others, on the basis of the course of its (past) implementation. Putting aside the problem of time lags, we may state that the magnitudes of plan QP and output Q are generated in the feed-back (see Fig. 1).

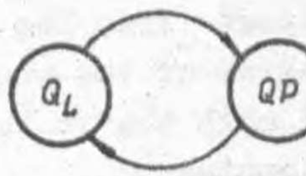

Fig. 1

In the paper we shall restrict our considerations only to one relationship in this feed-back, namely to the one in which the plan influences the output.

Of course, the analysis requires two basic assumptions:

- the plan of economic activity of the economic unit in question exists,
- the plans are obligatory (we mean by this that economic units try by all means to fulfil the plan or, possibly, to exceed it).

The analysis will be restricted to a short run. Then, the impact of a plan on the production process manifests itself in the level of output, or - more precisely - in the relation of output Q_t to (fixed) capacity output QC_t (capacity utilization coefficient). This may be formulated as

$$\frac{Q_t}{QC_t} = f(QP_t), \tag{1}$$

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where $f(QP_t)$ represents a proxy of capacity utilization coefficient based on the level of the plan (in market economies this proxy is frequently based on inventories of final goods A l l e n 1975).

Relation (1) leads to the production function

$$Q_t = QC_t f(QP_t), \tag{2}$$

showing how output Q_t is created within capacity QC_t.

The aim of the paper is to propose appropriate specifications of $f(QP_t)$ and to consider possibilities of estimation of (2).

2. The concept

Apart from the usually postulated assumptions concerning the character of the production process (G o l d b e r g e r 1972) we shall make the following assumptions with regard to the process of planning:

a) plan QP_t is expressed in the same units as output Q_t (and capacity output QC_t),

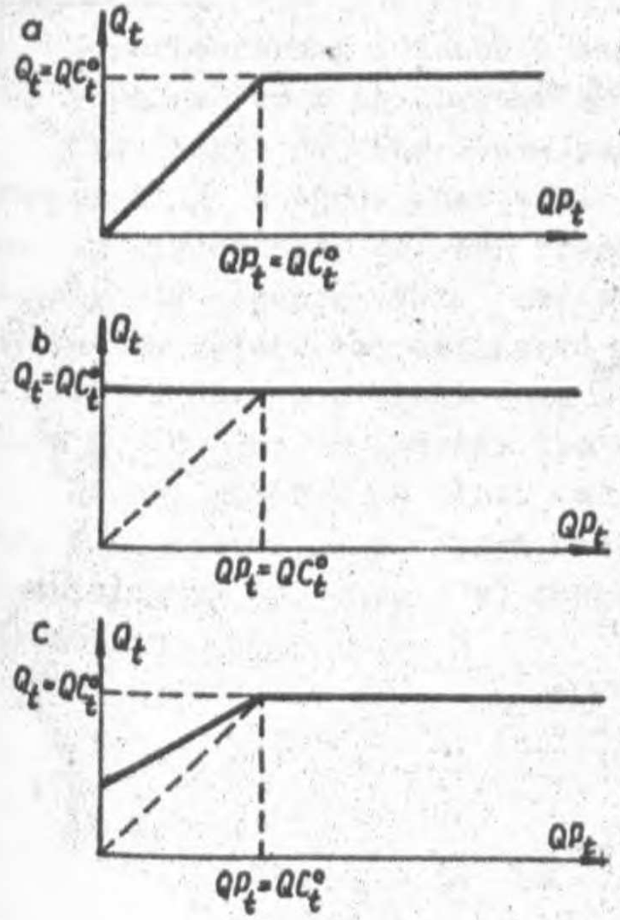
b) plan QP_t is meant as the final and obligatory version that cannot be changed during the process of its implementation,

c) the planner has no information about the current course of the production process (in particular it may be assumed that the plan for period t is completly elaborated till the end of period $t - 1$),

d) at the moment of confirmation the plan is feasible (does not exceed the expected capacity).

Under the above assumptions it is reasonable to assume additionally that when plan exceeds capacity $QP_t \geqslant QC_t$ then full capacities are utilized $Q_t = QC_t$. This will express the principle that in the "quest" for unrealistic plan the economic unit utilizes its full capacity. Notice, that the requirement of the plan's reality insures the above described mechanism against the simple strategy of the planning unit to extort full capacity utilization from the economic unit by fixing plans beyond the reach of the latter.

The most simple specifications for relation(2) when $QP_t < QC_t$ would be:



$Q_t = QP_t$

/fulfilling the plan but revealing no part of excess capacity over plan/

$Q_t = QC_t$

/utilizing full capacity, no matter what the plan's level is/

$Q_t = QP_t + \gamma /QC_t - QP_t/$

$0 \leqslant \gamma \leqslant 1$,

/fulfilling the plan and revealing fixed part of excess capacity over plan /.

Fig. 2

The last specification (linear convex combination of a and b) is already convenient and estimable in the form

$$Q_t = \begin{cases} QC_t & \text{for } QP_t \geqslant Q_t, \\ \gamma QC_t + (1-\gamma) QP_t & \text{for } QP_t < Q_t, \end{cases} \tag{3}$$

$0 \leqslant \gamma \leqslant 1$,

but only for linear specifications of QC_t.

For more frequently used specifications of QC_t, namely that of Cobb-Douglas form

$$QC_t = \alpha_0 X_{1t}^{\alpha_1} X_{2t}^{\alpha_2} \dots X_{kt}^{\alpha_k} e^{\varepsilon_t}, \tag{4}$$

where QC_t - capacity output, X_{it} $(i = 1, 2, \ldots, k)$ - possessed resources of production factors, α_i $(i = 0, 1, \ldots, k)$ - unknown structural parameters, ε_t - disturbance term, e - base of natural logarythms,
it is convenient to assume instead of c):

$$Q_t = QP_t\left(\frac{QC_t}{QP_t}\right)^{\delta}, \tag{5}$$
$$0 \leqslant \delta \leqslant 1,$$

which may be also written in the form

$$Q_t = QC_t^{\delta} QP_t^{1-\delta}, \tag{6}$$
$$0 \leqslant \delta \leqslant 1,$$

Thus instead of (3) we obtain

$$Q_t = \begin{cases} QC_t & \text{for } QP_t \geqslant Q_t, \\ QC_t^{\delta} QP_t^{1-\delta} & \text{for } QP_t < QC_t, \end{cases} \tag{7}$$

with the following picture

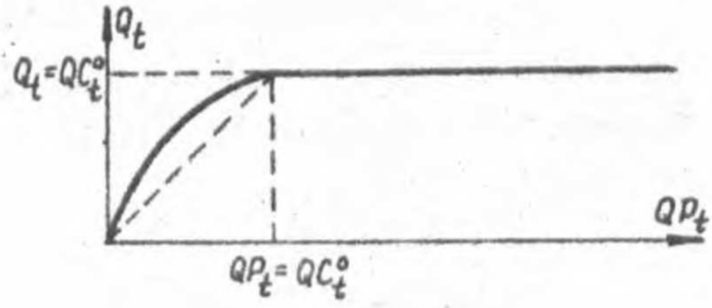


Fig. 3

Substituting (4) into (7) we obtain

$$Q_t = \begin{cases} \alpha_o X_{1t}^{\alpha_1} \ldots X_{kt}^{\alpha_k} e^{\varepsilon_t} & \text{for } QP_t > Q_t, \\ \alpha_o^{\delta} X_{1t}^{\delta\alpha_1} \ldots X_{kt}^{\delta\alpha_k} QP_t^{1-\delta} e^{\delta\varepsilon_t} & \text{for } QP_t < Q_t, \end{cases} \tag{8}$$
$$0 \leqslant \delta \leqslant 1.$$

Notice, that:

- disturbance terms in(8) are heteroscedastic (if $D^2(\varepsilon_t) = \sigma^2$ then $D^2(\delta\varepsilon_t) = \delta^2\sigma^2$).
- parameters α_i in both segments are related,
- parameter δ is limited to the interval $<0, 1>$.

The last of the above restrictions would require nonlinear programming methods what would considerably complicate the estimation procedure. So, we propose to act as it is usually done, namely to ignore this restriction in estimation and then to accept or reject the estimated model on its basis.

In order to present some further problems concerning estimation of (8) let us rewrite the relation in a more compact way.

First, without a lost of generality, assume that relation $QP_t \geqslant Q_t$ occurs in n_1 first observations, and $QP_t < Q_t$ - in the remaining n_2 $(n_1 + n_2 = n)$ observations. Now, let us logarythm(8) to obtain

$$\ln Q_t = \begin{cases} \ln\alpha_o + \alpha_1 \ln X_{1t} + \dots + \alpha_k \ln X_{kt} + \varepsilon_t \\ \delta\ln\alpha_o + \delta\alpha_1 \ln X_{1t} + \dots + \delta\alpha_k \ln X_{kt} + \\ \qquad + (1-\delta)\ln QP_t + \delta\varepsilon_t \end{cases} \tag{9}$$

with the appropriate conditions concerning the relation between QP_t and Q_t.

Let us introduce the following symbolics

$$\mathbf{X} = \begin{bmatrix} 1 \ \ln X_{11} \ \ln X_{21} \dots \ln X_{k1} \\ 1 \ \ln X_{12} \ \ln X_{22} \dots \ln X_{k2} \\ \\ \dots\dots\dots\dots\dots\dots \\ 1 \ \ln X_{1n} \ \ln X_{2n} \dots \ln X_{kn} \end{bmatrix} \quad \mathbf{q} = \begin{bmatrix} \ln Q_1 \\ \ln Q_2 \\ \\ \dots\dots \\ \ln Q_n \end{bmatrix} \quad \mathbf{p} = \begin{bmatrix} \ln QP_1 \\ \ln QP_2 \\ \\ \dots\dots \\ \ln QP_n \end{bmatrix}$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \ln\alpha_o \\ \alpha_1 \\ \dots \\ \alpha_k \end{bmatrix}$$

$$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{n_1} \\ \delta\varepsilon_{n_1+1} \\ \vdots \\ \delta\varepsilon_n \end{bmatrix}$$

where n is the sample size.

The following partition

$$X = \begin{bmatrix} X_1 \\ \hline X_2 \end{bmatrix} \quad q = \begin{bmatrix} q_1 \\ \hline q_2 \end{bmatrix} \quad p = \begin{bmatrix} p_1 \\ \hline p_2 \end{bmatrix} \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \hline \varepsilon_2 \end{bmatrix}$$

allows to rewrite (9) as

$$q = \mathcal{X}\gamma + \varepsilon \qquad (10)$$

with k + 1 nonlinear restrictions

$$\gamma_2 - \gamma_1 + \gamma_3\gamma_1 = 0$$

where:

$$\mathcal{X} = \left[\begin{array}{c|c|c} X_1 & 0 & 0 \\ \hline 0 & X_2 & p_2 \end{array}\right] \quad \gamma = \begin{bmatrix} \gamma_1 \\ \hline \gamma_2 \\ \hline \gamma_3 \end{bmatrix} \quad \begin{array}{l} \gamma_1 = \alpha \\ \gamma_2 = \delta\alpha \\ \gamma_3 = 1 - \delta \end{array}$$

Random vector in (9) is heteroscedastic with the covariance matrix

$$D^2(\varepsilon) = \sigma^2 \left[\begin{array}{c|c} I_{n_1} & 0 \\ \hline 0 & \delta^2 I_{n_2} \end{array}\right]$$

Thus, estimation of (9) requires generalized methods (for heteroscedasticity) with nonlinear restrictions.

The maximum likelihood method applied to (9) gives the following results:

$$\sigma^2 = \frac{1}{n}(q - \mathcal{X}\gamma)^T (q - \mathcal{X}\gamma), \tag{11}$$

$$\alpha = (\mathcal{X}^T\mathcal{X})^{-1}\mathcal{X}^T_{y}, \tag{12}$$

where:

$$y = \begin{bmatrix} q_1 \\ 1/\delta \, (q_2 - (1-\delta)p_2) \end{bmatrix},$$

$$\delta = \left[(X_2\alpha - p_2)^T (X_2\alpha - p_2)\right]^{-1} (X_2\alpha - p_2)^T (q_2 - p_2). \tag{13}$$

The above interdependent system can be solved by use of iterative methods with the starting value $\hat{\alpha}^0$ being, for example, the OLS estimate of any segment of (8).

Iterative methods can also be applied directly to (8) what seems to be more natural and easier.

3. Alternative propositions

In the economic practice we frequently observe an influence of the current production process on the plan's level in the current period. This concerns especially yearly plans - their final versions are confirmed in the first months of the years. Such a procedure enables to include additionally the production results from these months into the set of information to complete the final version of the plan. Additionally the planning systems functioning in most planned economies allow for plan's corrections during its implementation. In this case the discussed influence is evident.

Thus, it would be more realistic to consider a feed-back by supplementing the proposed production function by an equation explaining the plan formation. The set of the explanatory variables of the additional equation should include a variable characterizing the course of the production process in the current period (e.g. actual output Q_t).

The restrictions concerning the way in which the plan influences the output may be relaxed by introducing a more general statement $\lim_{QP_t \to \infty} Q_t = QC_t$ allowing for not fulfilling of the plan not only when it is unrealistic but also when it is tense enough. If we are not additionally interested too much in the behaviour of the function in the neighbourhood of point $QP_t = 0$, then the following specification may be proposed

$$Q_t = QC_t e^{-\lambda/QP_t}. \tag{14}$$

The greatest advantage of the above function is its simlicity. However, there are also some disadvantages. One of them is rather negligible - appearance of the inflexion point $QP_t = \lambda/2$. The other concerns the form of the capacity utilization coefficient $Q_t/QC_t = \exp\{-\lambda/QP_t\}$ which is independent of QC_t. This can be avioded by assuming the following function

$$Q_t = QC_t e^{-\kappa QC_t/QP_t}. \tag{15}$$

However, estimation of the above function requires much more complicated procedures.

All the functions considered in this paper will have a common disadvantage in practical applications - multicollinearity of the explanatory variables set. This is caused by evident correlation of the three considered categories - output, capacity and plan. However, this cannot be treated as a fault of the presented propositions, but rather as a technical problem which should be dealt with in the process of estimation.

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WPŁYW PLANU NA WYKORZYSTANIE ZDOLNOŚCI PRODUKCYJNYCH

Zdaniem autora w gospodarkach centralnie planowanych z obligatoryjnym planem obserwujemy jego wyraźny wpływ na przebieg procesu produkcyjnego. W szczególności wpływ ten objawia się w kształtowaniu się współczynnika wykorzystania zdolności produkcyjnych.

W artykule przedstawiona została teoretyczna koncepcja włączenia planu do zbioru zmiennych objaśniających wielkości produkcji. Rozważania swe autor ograniczył głównie do funkcji produkcji typu Cobb-Douglasa. Zaproponował on dwusegmentową jej modyfikację, w której:

- jeden z segmentów opisuje wielkość produkcji w przypadku planu nierealnego, przewyższającego zdolności produkcyjne (autor zakłada, że w tym przypadku układ gospodarczy w pełni wykorzystuje swe zdolności)
- drugi z segmentów opisuje wielkość produkcji w sytuacji, gdy plan jest dopuszczalny, tzn. ustalony na poziomie niższym od zdolności (w tym przypadku tylko część rezerwy zdolności produkcyjnych ponad plan jest angażowana w procesie produkcyjnym).

Odrębna część rozważań została poświęcona propozycjom procedur estymacyjnych przedstawionego modelu oraz dalszym jego modyfikacjom.