ACTA UNIVERSITATIS LODZIENSIS
FOLIA PHILOSOPHICA 8, 1991

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MEANING AND THE HERITAGE OF FORMALISM — THE RELEVANCE OF MATHEMATICAL PHILOSOPHY TO THE MODERN PHILOSOPHY OF NATURE

If meaning analysis is a predominant task of philosophy then mathematics as the most distinct science should be of a main interest for philosophers. Can we ever know what we are speaking about, if not in mathematics? Nowadays the influence of mathematical philosophy on general philosophy seems rather poor. This in my view has two main reasons:

– there has been a strong influence of mathematical philosophy on philosophy of science, but the traces are wiped out; unfortunately, as we will see, this influence to great extent has come in form of misconceptions of insights, the full corn of which can only be grasped in connection with the mathematical sources;

– the progress of understanding of the nature of mathematical concepts which has been the result of the so called „Grundlagenstreit", is to considerable extend hidden from philosophers eyes, because it is transformed in regular mathematical disciplines as for example model theory, present in philosophical discussion only by some difficult papers of Kreisel, Putnam et al.; mathematical model theory has become a formal tool to solve problems of philosophical semantics, but the progress it brings for epistemology and general methodology has to be spelled out.

In their book *The Mathematical Experience* Philip Davis and Reuben Hersh have given a polemical sketch of the formalist philosophy of mathematics and its influence on general philosophy. For the formalist, they tell us, „mathematics, from arithmetic on up, is just a game of logical deduction [...] For example, in plane geometry we have the undefined terms «point» and «line» and the axiom [...] Trough any two distinct points passes exactly one straight line". The formalist points out that the logical impact of this statement does not depend on any mental picture we may associate with it. Only tradition prevents us from using other words than point and line – „Through any two distinct bleeps passes exactly one neep"[1]. All talk of pure mathematics is in this sense talk about undefined terms, until it is supplied with an interpretation.

In the same fashion, logical positivism has pronounced formalization as a goal for all sciences. Physical theories should be viewed as formal calculi, related to experimental data by „interpretation rules". This mode of talk about physical theories has become outdated, but the problem of the meaning of theoretical terms has remained a burden for philosophy of science which can only be lefted by a convincing non-formalist picture of physical theories. To get an idea of this task we have to look at the sources of the formalist thesis and the fate of its opponent, the logicist program.

FREGÉS EARLY IDEA OF A LOGICAL REDUCTION OF MATHEMATICS

Reading the correspondance between Gottlob Frege and David Hilbert one can feel strange with Frege's insistance on the meaning and truth value of mathematical axioms. Again and again he critizised Hilbert for taking the name „axiom" for something for a collection of words with no definite reference: „Wenn ich ihr Axiom «Wenn A, B, C Punkte einer Geraden sind, und B zwischen A und C liegt, so liegt B auch zwischen C und A» als solches aufstellte, so setzte ich dabei die Bedeutung der Ausdrücke «etwas ist Punkt einer Geraden» und «B liegt zwischen A und C» als vollständig und unzweideutig bekannt voraus. Dann kann das Axiom nicht dazu dienen, etwa das Wort «zwischen» genauer zu erklären, und es ist selbstverständlich unmöglich, diesem Wort nachträglich noch eine Bedeutung zu geben (die von der früheren abweicht). (Andernfalls) scheint kaum etwas anderes übrig zu bleiben, als die Annahme, das Wort «zwischen» habe überhaupt noch keine Bedeutung. Dann aber kann der (mit ihm gebildete Satz) kein Axiom sein in meinem Sinne des Wortes"[2]. „Axioms" in his sense of the word are „Sätze, die wahr sind, die aber nicht bewiesen werden, weil ihre Erkenntnis aus einer von der logischen ganz verschiedenen Erkenntnisquelle fließt, die man Raumanschauung nennen kann".

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[1] Ph. Davis, R. Hersh, *The Mathematical Experience*, Boston 1981.

[2] Letter from Frege to Hilbert, 27.12.1899, relating to the „Grundlagen der Geometrie". In the same letter Frege gives an explicite exposition of „definition", declaring that Hilbert's axioms could not count as definitions in this exact sense. So Hilbert's „axioms" can neither be axioms nor definitions in Frege's view, and he complains about terminological confusion stemming from Hilbert's assigning the status of definitions" to his axioms: „scheinen mit die Grenzen zwischen Definitionen und Axiomen in bedenklicher Weise verwischt zu werden [...], scheint es mit nicht gut, dass man auch das Wort „Axiom" in schwankendem Sinne und zum Teil ähnlich wie „Definition" gebraucht".

On the same grounds Hilbert's proposal that his axioms could be seen as „implicite" definitions of the terms they consist of, is at least strongly misleading in Frege's eyes. An undefined symbol can not be replaced in the sense of „definition" by a collection of words some of which lack a definite meaning. Is this only the resistance of the exponent of a superseded research programme? Yes and no. Yes, because Frege's early idea of a logical reconstruction of mathematics aimed at a foundation of mathematics on intuitve clear logical concepts and axioms the truth values[3] of which can be seen directly or proven by steps justified only by the self-evident rules of pure logic. So the axioms should be true sentences about logical facts and the meaning of a mathematical concept, for example the concept of „two", should be analysable in definitions consisting of elementary logical concepts not itself definable by more basic concepts[4]. No, because the failure of the logicist programme to reach the aim of semantical reduction of arithmetic and analysis to the solid ground of logic plus set theory (in 1902 it became obvious with Russell's paradoxes that the ground was not as solid as Frege and Russell presupposed) does not mean, that it has failed in clarifying the nature of objects mathematical concepts speak about. Sentences with the term „1" get their meaning owing to the fact that „1" is the set of sets which are formed by propositional functions of the form „there is an element c so that 0(x) is true if

[3] Frege to Hilbert (27.12.1899): „Aus der Wahrheit der Axiome folgt, dass sie einander nicht widersprechen". Hilbert's response to this in his letter to Frege (29.12.1899): „Wenn sich die willkürlich gesetzten Axiome nicht einander widersprechen mit sämtlichen Folgen, so sind sie wahr, so existieren die durch die Axiome definierten Dinge. Das ist für mich das Criterium der Wahrheit und der Existenz. [...] Sie sagen, meine Begriffe, z.B. «Punkt», «zwischen» seien nicht eindeutig festgelegt [...] Ja, es ist doch selbstverständlich eine jede Theorie nur ein Fachwerk oder Schema von Begriffen nebst ihren notwendigen Beziehungen zueinander, und die Grundelemente können in beliebiger Weise gedacht werden. Wenn ich unter meinen Punkten irgendwelche Systeme von Dingen, z.B. das System: Liebe, Gesetz, Schornsteinfeger [...], denke und dann nur meine sämmtlichen Axiome als Beziehungen zwischen diesen Dingen annehme, so gelten meine Sätze, z.B. der Pythagoras auch von diesen Dingen. Mit anderen Worten: eine jede Theorie kann stets auf unendlich viele Systeme von Grundelementen angewandt werden. [...] Die sämmtlichen Aussagen einer Elektrizitätstheorie gelten natürlich auch von jedem anderen System von Dingen, welches man an Stelle der Begriffe Magnetismus, Electricität [...] substituiert, wenn nur die geforderten Axiome erfüllt sind. Allerdings ist zur Anwendung der Theorie auf die Welt der Erscheinungen meines Erachtens immer ein gewisses Mass von gutem Willen und Takt erforderlich: dass man für Punkte möglichst kleine Körper, für Gerade möglichst lange etwa Lichtstrahlen etc. substituiert [...] es gehört schon ein sehr grosses Mass von bösem Willen dazu, wollte man die feineren Sätze der Flächentheorie oder der Maxwell'schen Electricitätstheorie auf andere Erscheinungen anwenden, als sie gemeint sind".

[4] Impossibility of an explicite definition does not mean that the notion can not have a definite meaning. For the notion „point" Frege says that we have no definition (and that some vaque „definitions" by circum – descriptions like „without extension" or other „contents" of imagination must be rejected) but „point" does have a definite meaning (given by the standard model of geometry).

and only if x = c". So the attributive use of „1" means the assignment of a concept (or a propositional function) to a certain set of concepts (or propositional functions) defined by logical means. Seen from the perspective of model theory the characterization of number terms thus given is in no way unique. In terms of logic only the class of propositional functions (or in Frege's terminology the concept of numerical identity) can be defined without arbitraryness. So, as Russell himself remarks, this definition has an absolutely fatal defect. It does not show that only one objects satisfies the definition" we get a whole class of properties instead a single property representing in «natural way» the concept «1»"[5].

The way out is to take the extensional point of view, i.e. to define as number of a class the classes of the same kind (i.e. the class of all classes belonging to the propositional function above). The point is stressed already in Frege's „Grundlagen der Arithmetik", where he describes the step from the concept of „numerical identity" to the concept ot „number" as a special case of the method of abstraction[6]. In the same way we ascend from the concept of „parallel straight lines" to the concept „direction" by the definition „direction of a = direction of b: <=> a parallel b". By this „definition" of „direction" no single object is determined as in the case of „number", the definition only demands that whatever shall be replaced for „direction" must obey the relation of equivalence given by „parallel". So we are free to choose as „direction" all kinds of objects obeying this demand, for example the angle the members of a equivalence-class of parallels built to a given straight line, the equivalence-classes $<\ >$ itself, or some arbitrary product of equivalence-classes $<\ >^n = <\ > \times \overset{n}{\ldots} \times <\ >$. So Frege himself uses in 1884 the same kind of indefinite, „implicite" definition which he later critizises in Hilbert's work. But Frege is not inconsequent in doing this because he never emphasizes that this sort of definition d e t e r m i n e s[7] the objects of arithmetics. What it determines are structures or „relational structures", i.e. collections of set-theoreticals relations operating on a set of individuals, realizing the rules formulated in a set of axioms. So the structures are abstract set-theoretical models of axiomatic formalisms. Interpreting the relations and the domain of individuals changes the abstract structure into a concrete structure, the concrete models of mathematical discourse. In this perspective the logicist programme has not

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[5] B. Russell, *Foundation of Mathematics* (1903); after: O. Becker, *Grundlagen der Mathematik*, Frelburg 1964, p. 323.

[6] G. Frege, *Die Grundlagen der Arithmetik*, Kap. 62–68, Breslau 1884; Ch. Thiel, *Gottlob Frege – Die Abstraktion*, [in:] *Studien zu Frege*, ed. M. Schirn, Stuttgart 1976.

[7] „Wenn man sagen wollte: q ist eine Richtung, wenn es durch die oben ausgesprochene (implizite!) Definition eingeführt ist, so würde man die Weise, wie der Gegenstand q eingeführt ist, als dessen Eigenschaft behandeln, was sie nicht ist". Frege, *Die Grundlagen der Arithmetik*, Kap. 67.

been successful in detecting the logical definition of „number", because there is no unique logical object, and no object at all that can be singled out as „number". Instead it has shown that numbers can be reconstructed in a relational structure which is a standard set theoretical model of logic calculus, by interpreting the relation of numerical identity in the „most natural way" as the set it defines and the individuals of the abstract model as the concepts of ordinary language.

This is far away from the original logicist dream to detect the objects people have looked for thousend of years, when they asked „What are the numbers?" It is far away from foundation of mathematics on self-evident axioms. But it shows that mathematical concepts used by the mathematician and by ordinary people do have meaning, as they can be shown to be concrete models of some calculus, neither the calculus nor the choosen model being unique.

Some calculus may seem more appropriate to us, some models more „natural" than others; the meaning of the concept „two" will be unique for us in so far as we all can recognize our use of the term „two" as application of a certain model. Logic has lost its privileged status as ontological and semantical fundament of mathematics (already reflected by Russell in his remarks about the unsharp distinction between arithmetics and logic)[8] and Bernays is completely right in 1930[9] to diminish, the logicist pretension to a standpoint over and above the program of epistemological clarification by formalization of mathematical reasoning: the logicists only show that the

[8] B. Russell, *Einführung in die mathematische Philosophie*, München 1923, p. 198: „Indem wir die Überlegungen der Arithmetik verallgemeinern, folgen wir nur einer Vorschrift, die in der ganzen Mathematik anerkannt ist. Durch diese Verallgemeinerung haben wir tatsächlich eine Reihe von neuen deduktiven Systemen geschaffen, in denen die traditionelle Arithmetik aufgelöst und erweitert ist. Ob eines dieser neuen deduktiven Systeme, z.B. die Auswahltheorie, zur Logik oder zur Arithmetik gehören soll, ist vollkommen willkürlich und lässt sich nicht zwingend entscheiden". Russell gesteht hier zu, dass die Ausweitung des formalen Standpunkts in der Mathematik essentielle Unterscheidungen mathematischer Disziplinen, die sich auf vorstellungsmässige Inhalte ihrer Begriffe stützen, gegenstandslos geworden sind.

[9] P. Bernays, *Die Philosophie der Mathematik und die Hilbertsche Beweistheorie*, „Blätter für Deutsche Philosophie" 1930/31, Bd. 4, H. 3/4, p. 326–367: „In Hinsicht auf das Formale stellt aber... die mathematische Betrachtung gegenüber der *begrifflich* logischen den Standpunkt der höheren Abstraktion dar. Wir können also für die mathematischen Erkenntnisse durch ihre Einordnung in die Logik gar keine höhere Allgemeinheit gewinnen, sondern vielmehr umgekehrt nur eine Spezialisierung durch logische Interpretation, d.h. eine Art von logischer Einkleidung [...] Ein typisches Beispiel einer solchen logischen Einkleidung bildet die Methode, nach der die natürlichen Zahlen von Frege und in Anschluss an ihn, [...] von Russell definiert verden [...] erweist sich der Anzahlbegriff als ein elementarer *Strukturbegriff*. Der Anschein, als ob dieser Begriff aus den Elementen der Logik gewonnen würde, entsteht bei der betrachteten logischen Anzahldefinition nur dadurch, dass der Begriff mit logischen Elementen, [...] verkoppelt wird, welche an sich für den Anzahlbegriff unwesentlich sind. Wir haben also hier in der Tat eine logische Einkleidung eines formalen Begriffes vor".

formal calculus of logic may be an appropriate tool to characterize arithmetic notions. Frege gives a logical equipment of arithmetic. The mathematical formalism of pure logic gets a specialization by Frege's non formal interpretation. From the model theoretical standpoint the logicist programme has led to nothing but a new model for arithmetics, therefore only following the guidelines of formalist axiomatic method.

THE LEGACY OF THE LATE FREGE'S CRITICS ON THE FORMALIST PHILOSOPHY OF MATHEMATICS

But this is not the true end of the story, it is too one-sided. As Kambartel[10] has pointed out, Frege saw that Hilbert's axioms could not be treated as predicators of the first level, generating true or false statements, when individual constants are given. Instead they are predicators of second level, refering to Predicates $P_1, ..., P_n$ so that the predicator comes true of the predicates if and only if by replacing the predicator variables in the axioms by the predicates $P_1, ..., P_n$ the axioms change in true statements. The Hilbertian axioms are therefore propositional functions and this is another way of saying that they are relational structures. Frege's insisting that Hilbert's axioms are not axioms in the usual sense in refering not to concrete objects but to abstract structures[11] is at least to this respect no narrow minded adherence to an old fashioned concept of mathematical truth but a sensible advice that – speaking metaphorically – the hilbertian axioms have nothing to do with meaning only because they are not the sort of things that can have meaning. The meaning of mathematical concepts can – on logical grounds – not be found on the level of formalisms, but only on the level of models.

So Hilbert's knocking through the gordian knot was a little bit to strong. His fruitful detection is that axioms build by formal reasoning do not have

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[10] F. Kambartel, *Frege und die axiomatische Methode*, [in:] *Studien zu Frege...*, p. 215, 218.

[11] Cf. Frege an Hilbert, 6.1.1900: „Die Merkmale, die sie in ihren Axiomen angeben, sind wohl sämtlich höherer als erster Stufe; d.h. sie antworten nicht auf die Frage «welche Eigenschaften muss ein Gegenstand haben, um ein Punkt (eine Gerade, Ebene u.s.w.) zu sein?», sondern sie enthalten z.B. Beziehungen zweite Stufe, etwa des Begriffes Punkt zum Begriff Gerade. Es scheint mir, dass sie eigentlich Begriffe zweiter Stufe definieren wollen, aber diese von denen erster Stufe nicht deutlich unterscheiden". Hilbert's response to Frege's remarks relating to notions of different levels (and the remark that relations between notions could only be erected, when the notions have formerly been fixed in a sharp way – not in the same move with the definition of the notions), came in a letter from 22.09.1900: „Meine Meinung ist eben die, dass ein Begriff nurdurch seine Beziehungen zu anderen Begriffen logisch festgelegt werden kann. Diese Beziehungen, in bestimmten Aussagen formuliert, nenne ich Axiome und komme so dazu, dass die Axiome die Definitionen der Begriffe sind".

truth value by themselves and that the concepts they consist of must not thought to have definite referents. But it is a failure to conclude that mathematical concepts do have no meaning, because it must not be true that the only way to detect the semantical content of mathematical concepts is the method of formalization up to some point of „real fundaments", as it has been the common intuition in the beginning of the logicist and the formalist programme. The model theoretical reformulation of the logicist programme has shown that fundamentalist programmes are vain. Frege's criticisms on Hilbert have got an late legitimation though the appearence of the Löwenheim-Sklolem-theorems. These theorems show that even seemingly simple general notions of arithmetics as „finite" have their meaning only relative to some choosen model. There are „finite" models of infinite sets, because it is possible in some non-standard-model of physical time, to count an infinite set in a finite time[12]. In the same way „countable" and „uncountable" are model relative notions. So we have no chance to detect „the" meaninig of a mathematical term by formalization; we only generate a proliferation of non-standard models. So the really strange epistemological lesson from this is that there are no absolute meanings of concepts we could ascent to outgoing from our ordinary unprecise knowledge of meanings. It is, as if someone has made a joke of platonism: knowledge is represented in the „imperfect" realizations, the shadows are the ideas.

Before Hilbert generalized his revolutionary epistemological interpretation of the „axiomatic method" developed for geometry, he followed with respect to arithmetics more traditional paths of thinking about axiomatization, represented still even 1918 in his paper *Axiomatisches Denken*. Here Hilbert describes the aim of axiomatization as to bring „Ordnung" and „Orientierung"[13] in the conceptual building. To lay a deeper axiomatic ground for some mathematical discipline allows one to check the consistency of the relative to the ground: „Da aber die Prüfung der Widerspruchslosigkeit eine unabweisbare Aufgabe ist, so scheint es nötig, die Logik selbst zu axiomatisieren und nachzuweisen, daß Zahlentheorie sowie Mengelehre nur Teile der Logik sind"[14]. That is, the axiomatization of logic following up by

[12] Cf. H. Putnam, The Thesis that Mathematics is Logic, [in:] id., *Mathematics, Matter and Method*, London 1975, p. 24.

[13] D. Hilbert, *Axiomatisches Denken*, „Mathematische Annalen" 1918, Bd. 78, p. 405–415; reprinted in: id., *Gesammelte Abhandlungen*, Bd. 3, Berlin 1970, p. 148.

[14] Hilbert, *Gesammelte Abhandlungen*, p. 153. But yet in 1904 in *Über die Grundlagen der Logik und der Arithmetik* Hilbert saw a fly in this ointment; cf. P. Bernays, *Hilberts Untersuchungen über die Grundlagen der Arithmetik*, [in:] Hilbert, *Gesammelte Abhandlungen*, p. 199: „Für die Axiome der Geometrie erfolgt der Nachweis der Widerspruchsfreiheit durch eine arithmetische Interpretation des geometrischen Axiomensystems". On the other hand concerning the consistency proof for arthmetics" erscheint die Berufung auf eine andere Grunddisziplin unerlaubt". Reduction to logics is no way out here, as Bernays declares: „Allein bei aufmerksamer

Russel and Whithead, is seen as a tool to bring „Ordnung and Orientierung" in the building of arithmetics and set theory, enabling one to check the consistency of them relative to the consistency of the logical axioms, the later taken for granted by the stability of the systems build on it and the inner plausiblity of these axioms. Hilbert's demands for axiomatization seems here in harmony with the logizist programme, sharing with it the idea that logic can give a secure and privileged ground for all mathematics and that at least the logical axioms are not only formal rules, but have their own content. With logic's loss of its priviledged role it became clear to Hilbert, that the transfer of his method of axiomatization to arithmetics needed a new revolutionary step, the step from the ground of logic to the ground of metamathematics. It is a step Hilbert took not before the 20tes. (There has been a break in Hilbert's studies in the foundations of arithmatics between 1904 and 1918 as Paul Bernays points out in *Hilberts Untersuchungen über die Grundlagen der Arithmetik*).

Hilbert's transition to the perspective of metamathematics is characterized by the following moves: first, logic looses its priviledged role. Because there is no sharp boundary between set theory and logic, the paradoxes of set theory infected logic too. Another argument for Hilbert is that logic uses „transfinite" arguments (tertium non datur), following here the anti-logicist convictions of the intuitionists. From this on Hilbert views logic like the intuitionists as a collection of formal operations, abstracted from self-evident practical operations in nature, the generalization of them to realms outside the human experience is not more legitimized than for other mathematical operations. So logic can be saved from the intuitionists razor only by declaring it as a collection of pure formulas. From this follows the second move: proofs of consistency can not further rest on secure logic, they cannot be „indirect", but have to be „absolute" for each mathematical discipline. The tool to accomplish this is „metamathematics", a system of finitary operation rules with a status reminding to Cartesian axioms: they are *a priori* for all operations of the mind, may they relate to sensory experiences or abstract mathematical formulas, and show themselves by being clear and distinct[15]. A mathematical system is

Betrachtung werden wir gewahr, dass bei der hergebrachten Darstellung der Gesetze der Logik gewisse arithmetische Grundbegriffe, z.B. der Begriff der Menge, zum Teil auch der Begriff der Zahl [...] bereits zur Verwendung kommen. Wir geraten so in eine Zwickmühle und zur Vermeidung von Paradoxien ist daher eine teilweise gleichzeitige Entwicklung der Gesetze der Logik und der Arithmetik erforderlich".

[15] D. Hilbert, *Die Grundlagen der Mathematik*, Abhandlungen des Hamburger Mathematischen Seminars, Bd. 4, 1928, p. 65–85: „Die Mathematik wie jede andere Wissenschaft kann nie durch Logik allein begründet werden, vielmehr ist als Vorbedingung für die Anwendung logischer Schlüsse und für die Bestättigung logischer Operartionen uns schon etwas in der Vorstellung gegeben: gewisse ausserlogische konkrete Objekte, die anschaulich als unmittelbares

consistent, if and only if from the axioms of the system under the rules of metamathematics no formula like $0 \neq 0$ can be derived. In a radical way consistency is not longer seen as in traditional axiomatics as a consequence of truth of the premisses, but as the methodological virtue to be in harmony with the metamathematical „rules of thinking".

Hilbert's sacrifice of the self-evidence of mathematical statements, often praised as the break trough of modern philosophy of science, is accomplished only at cost of reference to an „intuitive" and necessary methodological basis, similar to that of the intuitionists. This fact often is played down by the adherents of formalist ideology as a pragmatic necessity to start with some unproven and only plausible principles. But in contrast to „first principles" in a theory of mathematical physics, the metamathematical fundament can never be overthrown. It is an unfalsifiable cornerstone of clear and distinct ideas, i.e. the principles of metamathematics can be understood in a direct and unique manner, not refering to some background or model – they carry absolute meanings. So Hilbert has banished meaning from mathematical concepts only to restitute absolute meanings somewhere else. It might be a worth while task to look whether the consequent following of the structualist programme for physical theories must lead to a similiar residue of „absolute meaning".

THE TRANSITION FROM THE FORMALIST PICTURE OF MATHEMATICS TO THE FORMALIST PICTURE OF PHYSICAL THEORIES

In what sense has the formalist picture of mathematics become the prototype of the concept of physical theories in logical empirism? And why should we care about this, i.e, whre are the lessons yet to learn after the burial of logical empirism?

Rudolf Carnap[16] took his notion of „partial interpretation" of theoretical terms from Hilbert's „implicite definition" mathematical axioms give for the concepts they consist of. With „implicite" Hilbert meant something like the intern boundary conditions defined for the possible substitutes of the variables of a system of equations by the equations themselves. Frege an Hilbert, 6.1.1900 „Ihr System von Definitionen gleicht

Erlebnis vor allem Denken da sind. Soll das logische Schliessen sicher sein, so müssen sich diese Objekte vollkommen in allen Teilen überblicken lassen und ihre Aufweisung, ihre Unterscheidung, ihr Aufeinanderfolgen oder Nebeneinandergereihtsein ist mit den Objekten zugleich unmittelbar anschaulich gegeben als etwas, das sich nicht noch auf etwas anderes reduzieren lässt oder einer Reduktion bedarf" (the paper cited above gives a comprehensive survey of Hilbert's program for metamathematics).

[16] Cf. F. Kambartel, *Frege und die axiomatische Methode*, [in:] *Studien zu Frege...*, p. 222.

einem System von Gleichungen mit mehreren Unbekannten, bei dem die Auflösbarkeit und besonders die Eindeutigkeit der Bestimmung der Unbekannten zweifelhaft bleibt." The mode of determination of the individual constants is internal, not external (by interpretation) and in this sense „implicite" and it is „partial", because the determination of the constants by these internal conditions ist not unique; to be a definition, the internal conditions must be supplemented by some external rules. Because the connections between the secure basis of physics, the dates of sensory events, and the „theoretical tems" is be no means unique, Carnap was lead to the suggestion, that the partial external interpretation of the terms of a theory (the external interpretation of a part of the terms) defines the referents of the theoretical terms (which are not interpreted themselves external) by propositions, which contain theoretical as much as non-theoretical, external interpreted terms, just in the same way the mathematical systems of equations implicitely define the substitutes for the individual variables, because they contain also some external defined constants, i.e. numbers[17].

The main failure of this picture is not so much as some „realist" critics of logical empiricism mention that Carnap denies a „genuine" meaning of the theoretical terms in form of real referents, but his supposition that sensory experiences could supply such „genuine" meaning. This supposition Carnap's theory does not only share with Hilbert's conception of metamathematics but also with all conceptions of physical theories that make use of so called „interpretation rules", connecting the „formalism" of a physical theory with some basis of already understood concepts. The idea not buried with the body of logical empirism is that there is some knowledge outside the model that enables one to select the intended model of a theoretical formalism, not propositional knowledge, but knowledge about meanings, may it be knowledge of the meaning of sensory concepts or knowledge of the referents of theoretical terms. In the formalist picture this question can not be solved because in this picture the problem appears in the form of a „cut off" of the

[17] In a discussion about the „foundations of mathematics" (7.09.1930), „Erkenntnis" (1931), p. 135, Rudolf Carnap tried a reconciliation of the logicist and the formalist standpoint in the foundations of mathematics. His idea was that a subsequent logical analysis of the formalist construction would yield to the meaning of the pure formal mathematical symbols. Therefore not only the axiomatics systems but also the „transformations" of mathematical formulas has to been considered, metamathematics alows (the existence of this transformation, Carnap mentiones as necessary for application of mathematics). Together with the allowed transformations the meaning of symbols should be accessible for logical analysis. This idea fails because of the underdetermination of meaning by all the logical properties of the symbols. The presentation of an abstract model (abstract structure as model of the axiomatic system) does not give the „logical meaning" of the symbols, when logical meaning is understood as in Frege's reconstruction of arithmetics as meaning qua logical (concrete) model.

nonstanstandard models, which can only be accomplished from outside by playing out some „genuine" or „direct" knowledge of meaning, independend of the models themselves.

SKETCH OF A NON-FORMALIST VIEW OF „MEANING"

The only way out is the sacrifice of the hidden essentialism of meaning which is the heritage of formalism in philosophy of science. The fate of the logicist philosophy of mathematics and the Löwenheim-Skolem-theorems show that in the „absolutist" perspective not even the knowledge what it means that the set of natural numbers is infinite, can be reconstructed as knowledge. Frege looked for a logical „absolute" definition of natural numbers and the elementary concepts of number theory, but he gets only a model of the logic calculus. Understanding of a given realm of reality comes about by representation of the realm in form of a model of a theoretical system, a model whose elements we might already be used with through applications of other theoretical systems. What we learn by understanding is that something familiar to us, for example the natural numbers, build up a model of a system, which has some also familiar and some very unfamiliar applications. The formalization is only a tool in this game, it is not the formalized system itself we are refering to when we sharpen our concepts. Because we always have an „intuitive" realm of reality and a model we associate with it, there is never in reality the problem of „interpretation of formalism"[18]. The „cut off" of unintended models does not come about by a metatheory outside the models, it simply „happens", because the familiar realm determines what model of the formalism we choose: that model.

The failure with formalism is to think we could generalize our knowledge of meaning over and above the model in which it is represented. But no formal system can represent it. So sometimes we think of „redness" as the quality of our red-experiences, supposing that there must be something universal, an intentional content, determining the set of all red things. But theoretical knowledge about „red" tells us, that colors like red are parts of a model of the theoretical color-spectrum given by electrodynamics. Thereby we understand a little better what colors really are, and no quality-knowledge is missing in it. What we look for when we speak of „Qualia" is a theoretical knowledge,

[18] The „Interpretations" of QM for example are not really interpretations of the quantem-mechanical formalism, but metatheories talking about the *relations* between models and formalisms; for example, how can we understand that this component of a piece of matter is a model of $formalism_1$ whereas the piece of matter itself satisfies $formalism_2$.

saying what „red-experiences" are independent from the factual conditions of red-experiences, from the model, and saying it in a unique way. Our critical review of the heritage of formalism has shown that this is impossible.

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ZNACZENIE A DZIEDZICTWO FORMALIZMU — DONIOSŁOŚĆ MATEMATYKI DLA WSPÓŁCZESNEJ FILOZOFII PRZYRODY

Autor uważa, że słabnie dobroczynny wpływ wywierany przez filozofię matematyki na filozofię ogólną. Część tego wpływu rozmyła się bowiem w filozofii nauki, a rozwój samej filozofii matematyki uczynił z niej dyscyplinę zbyt hermetyczną i niedostępną dla większości filozofów. Matematyczny model teorii jest formalnym narzędziem, które – wyjąwszy filozoficzną semantykę – we wciąż niezadowalającym stopniu służy rozwiązywaniu problemów epistemologii czy ogólnej metodologii. Przedmiotem dalszych rozważań autor czyni dziedzictwo – dokonanej przez późnego Fregego – krytyki formalnej filozofii matematyki, relacje pomiędzy formalnym obrazem matematyki i teorii fizycznych, a także nieformalne ujęcie „znaczenia".