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HOW QUANTUM PRISONER'S DILEMMA CAN SUPPORT NEGOTIATIONS[2]

Summary

Decision-making by the two negotiating parties is simulated by a prisoner's dilemma game. The game is formulated in a quantum manner, where players strategies are unitary transformations of qubits built over the basis of opposite decision options. Quantum strategies are correlated through the mechanism of quantum entanglement and the result of the game is obtained by the collapse of the resulting transformed state. The range of strategies allowed for quantum players is richer than in case of a classical game and therefore the result of the game can be better optimized. On the other hand, the quantum game is save against eavesdropping and the players can be assured that this type of quantum arbitration is fair. We show that quantum prisoner's dilemma has more favorable Nash equilibria than its classical analog and they are close to the Pareto optimal solutions. Some economical examples of utilizing quantum game Nash equilibria are proposed.

Key words: game theory; quantum game; prisoner's dilemma; Nash equilibrium; Pareto optimal solutions.

1. Negotiations as a game

Many decisions made by negotiating parties rely on the strategic interaction between them. By this we mean that negotiating parties can choose between different strategies, typically conflict or cooperation between them. They both agree that the mutual cooperation is the most desirable behavior but their choices are made simultaneously without knowing the other party's decision. It yields the temptation to refuse cooperation (defection). This kind of interactions is often described by the classical game theory.

One of the best known games of that type in the Prisoner's Dilemma [PD] game. It was first described by Flood and Dresher [Flood, Dresher, 1952] and popularized by Albert Tucker, whose two-prisoner story was a basis for the current name of the game. The popularity of PD comes from its universal game scheme, which describes a negotiation dilemma very common in everyday life. A typical scenario contains an assumption that two players, Alice and Bob, independently of each other make a choice between cooperation (*C*) and defection (*D*). The choice of the two players is a basis for a payoff matrix which is shown in Table 1.

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TABLE 1.

The payoff matrix for the Prisoner's Dilemma

		Bob	
		C	D
Alice	C	(r, r)	(s, t)
	D	(t, s)	(p, p)

The strategy (C) means cooperation and strategy (D) - defection. The first number of each pair represents Alice's payoff, while the second number is Bob's payoff. The payoff r corresponds to 'reward' for cooperation, t is related to 'temptation' of betrayal, s is 'sucker's payoff' and p stands for 'punishment' for mutual refusal of cooperation. These numbers fulfill an inequality: $t > r > p > s$ and $r > \frac{s+t}{2}$.

Source: [Rapoport, Chammah, 1970].

Analyzing the payoff matrix we can notice that independently of the opponent's move, the dominant strategy of each of the players is the defection strategy. The pair of strategies (D, D) is here the Nash equilibrium [NE]. Paradoxically, the equilibrium corresponds to mutual punishment (p, p), which is highly remote from the Pareto optimal result. Mutual cooperation (C, C) is the best move to get an optimal result. However, a game like this requires mutual trust of the players, as every change of (C, C) strategy brings each person a reward - a temptation of betrayal t, which for the second player means a punishment in form of a sucker's payoff s. If the PD is played by two parties without mutual trust, the most frequent outcome is mutual punishment for lack of cooperation. Mutual defection as the only NE of the PD is the natural reservation point for both negotiators. The goal of the present paper is to show, that within the quantum framework, mutual defection can be replaced by another NE, that is much more favorable for negotiators and therefore can be regarded as an alternative reservation point.

A natural extension of the game to a multiplayer PD is possible, in which the dilemma is generally the same as in the two player game. From a practical point of view an important extension of the game is its iterated version [Hamilton, Axelrod, 1981] in which the players' strategies depend on previous games they played. These extensions will not be considered in the present paper.

2. Quantum game definition

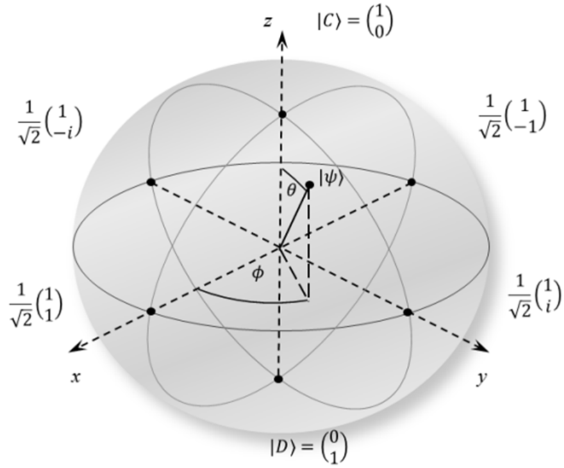
Experimental psychology shows that real human decisions in a situation of PD are often incompatible with the classical NE. Some researchers argue [Busemeyer, Wang, Townsend, 2006; Pothos, Busemeyer, 2009], that to explain human decisions making, more suitable than classical are quantum methods. As the research on quantum information processing was developed, the quantum version of PD has been formulated [Eisert, Wilkens, Lewenstein, 1999; Szopa, 2014]. In its perspective players' strategies are operators in a vector space called the Bloch sphere. This space is a collection of qubits – normalized vectors with complex coefficients spanned on a basis of two elements $\{|C\rangle, |D\rangle\}$, which, up to the phase, can be represented in the form

$$|\psi\rangle = \cos\frac{\theta}{2}|C\rangle + \mathrm{e}^{\mathrm{i}\phi}\sin\frac{\theta}{2}|D\rangle, \tag{1}$$

where $\theta \in [0, \pi]$ and $\phi \in [-\pi, \pi]$ (cf. Figure 1).

FIGURE 1.

The Bloch sphere



The Bloch sphere with marked localizations of the qubits, $|C\rangle = \binom{1}{0}$- cooperation, $|D\rangle = \binom{0}{1}$ – defection and the qubits localized in the xy plane and intersecting axes.

The qubits $|C\rangle$ and $|D\rangle$, which are quantum pure states, correspond to cooperation and defection. The following description is the same for an arbitrary 2x2 game with $|C\rangle$ and $|D\rangle$ corresponding to any other strategies. The rest of the Bloch sphere qubits are states in a superposition of two quantum states. According to the principles of quantum mechanics, unless we take a measurement we cannot tell in which of the two states, the qubit actually is. The one thing we can say is that as a result of measurement, $|C\rangle$ can occur with probability of $\cos^2\frac{\theta}{2}$ and $|D\rangle$ with probability of $\sin^2\frac{\theta}{2}$.

The measurement is followed by a collapse of the wave function, which from state changes to $|C\rangle$ or $|D\rangle$. As an example, all qubits located on the Bloch sphere equator (the circle in the *x-y* plane of Figure 1.) represent quantum states, which after the measurement collapse into state $|C\rangle$ or $|D\rangle$ with probability $\frac{1}{2}$. (The Schrödinger's cat example shows analogous situation in which states $|C\rangle$ i $|D\rangle$ correspond to the cat being dead or alive.)

In quantum game theory, neither qubits $|C\rangle$ nor $|D\rangle$ corresponds to players' strategies. Here the strategies are unitary operators $\widehat{U}_A$ – for Alice and $\widehat{U}_B$ – for Bob. The unitary strategies work at certain entangled quantum state $|\psi_0\rangle$, which is known for both players. These transformations, in general, are Bloch sphere rotations $\widehat{U}_X \in SU(2)$, defined by unitary matrices

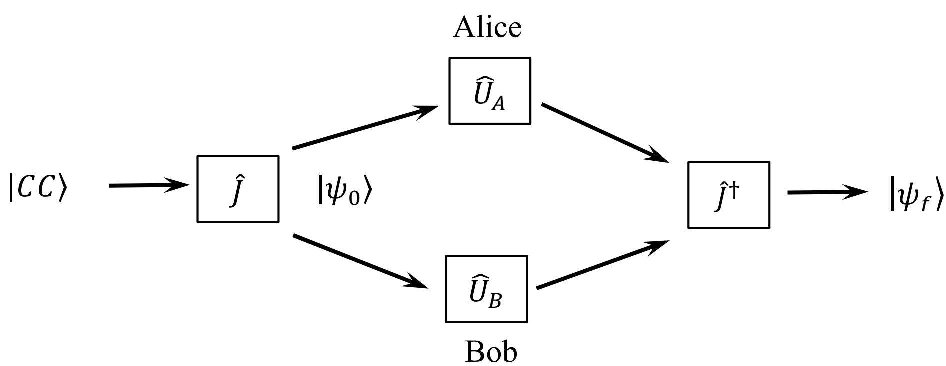
$$\widehat{U}(\theta,\phi,\alpha) = \begin{pmatrix} e^{-i\phi}\cos\frac{\theta}{2} & e^{i\alpha}\sin\frac{\theta}{2} \\ -e^{-i\alpha}\sin\frac{\theta}{2} & e^{i\phi}\cos\frac{\theta}{2} \end{pmatrix}, \tag{2}$$

with $\widehat{U}_X = \widehat{U}(\theta_X,\phi_X,\alpha_X)$, $\theta_X \in [0,\pi]$ and$\alpha_X,\phi_X \in [-\pi,\pi]$, $X = A,B$. In particular, if the rotation is determined only by the angle θ, i.e. $\alpha = \phi = 0$, it may be written as $\widetilde{U}(\theta) = \widehat{U}(\theta,0,0) = \cos\frac{\theta}{2}\hat{C} + \sin\frac{\theta}{2}\widehat{D}$, where the identity matrix $\hat{C} \equiv \widetilde{U}(0)$ corresponds to the cooperation strategy and the matrix $\widehat{D} \equiv \widetilde{U}(\pi) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (altering the qubits $|C\rangle$ and $|D\rangle$) corresponds to the defection strategy. The strategy $\widetilde{U}(\theta)$ is equivalent to the classical mixed strategy, for which the probabilities of both pure strategies C and D are accordingly $\cos^2\frac{\theta}{2}$ and $\sin^2\frac{\theta}{2}$.

The quantum game, being a two-player game, takes place in a space of qubit pairs, one per each player. The qubits are correlated with each other by quantum entanglement (Figure 2).

FIGURE 2.

The setup of a quantum game



Such a game may be physically performed by implementing a quantum computer algorithm dependent on the strategy of players. Such algorithm was realized experimentally [Du, et al., 2002] on a two-qubit, nuclear magnetic resonance quantum computer. Details of its operation, i.e. physical implementation of the quantum algorithm are not essential for the understanding of quantum games, and in this paper will be omitted.

The initial state of the game is represented by a pair of $|CC\rangle$ qubits, the first of them representing Alice's state, the second Bob's state. The basis vectors of the space of pairs of qubits $|CC\rangle, |CD\rangle, |DC\rangle, |DD\rangle$are pure states. For computational simplicity, we define them as vectors $(1,0,0,0)$, $(0,1,0,0)$, $(0,0,1,0)$, $(0,0,0,1)$ in a four-dimensional space of states. The initial state $|CC\rangle$ is transformed by entangling operator $\hat{J} = \frac{1}{\sqrt{2}}(\hat{I} + i\hat{\sigma}_x \otimes \hat{\sigma}_x)$, where $\hat{I}$ is the 4-dim identity matrix and $\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is the Pauli matrix. As the result we get the entangled state $|\psi_0\rangle = \hat{J}\,|CC\rangle = \frac{1}{\sqrt{2}}|CC\rangle + \frac{i}{\sqrt{2}}|DD\rangle$. Then we act by a direct product of operators $\widehat{U}_A$ and $\widehat{U}_B$ representing quantum strategies of Alice and Bob. Before taking measurement on the final state we act using disentangling operator $\hat{J}^{\dagger}$. As a result, the final state $|\psi_f\rangle$ is given by

$$|\psi_f\rangle = \hat{J}^{\dagger}(\widehat{U}_A \otimes \widehat{U}_B)\,\hat{J}\,|CC\rangle \tag{3}$$

and usually it is an entangled state:

$$|\psi_f\rangle = p_{CC}\,|CC\rangle + p_{CD}\,|CD\rangle + p_{DC}\,|DC\rangle + p_{DD}\,|DD\rangle, \tag{4}$$

where $|p_{CC}|^2,\ldots, |p_{DD}|^2$are probabilities of the measurement taken on the final state being one of the four possible outcomes.

In the quantum game the expected value of Alice's payoff $\$_A$ is the weighted mean of four classical values r, s, t and p from the payoff matrix (Table 1)

$$\$_A = r\,|p_{CC}|^2 + s\,|p_{CD}|^2 + t\,|p_{DC}|^2 + p\,|p_{DD}|^2, \tag{5}$$

where weights correspond to quantum probabilities of corresponding pure states, which up to the phase are equal to [Chen, Hogg, 2006]:

$$\begin{aligned}
p_{CC} &= \cos\frac{\theta_A}{2}\cos\frac{\theta_B}{2}\cos(\phi_A+\phi_B) - \sin\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\sin(\alpha_A+\alpha_B),\\
p_{CD} &= \sin\frac{\theta_A}{2}\cos\frac{\theta_B}{2}\cos(\alpha_A-\phi_B) - \cos\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\sin(\phi_A-\alpha_B),\\
p_{DC} &= \sin\frac{\theta_A}{2}\cos\frac{\theta_B}{2}\sin(\alpha_A-\phi_B) + \cos\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\cos(\phi_A-\alpha_B),\\
p_{DD} &= \cos\frac{\theta_A}{2}\cos\frac{\theta_B}{2}\sin(\phi_A+\phi_B) + \sin\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\cos(\alpha_A+\alpha_B).
\end{aligned}$$

Bob's expected payoff is given by formula after the $s \leftrightarrow t$ replacement. Let us notice that the quantum situation can be simulated by a classical experiment, where the coefficients in front of r, s, t and p in can be replaced by 0 or 1 by using a classical device, which draws 1 with probabilities $|p_{CC}|^2,\ldots, |p_{DD}|^2$respectively and 0 otherwise. The result shall be the same as an average result after the quantum game is played multiple times. When taking into account a quantum device entangling qubits and performing particular unitary transformations , the game result is given by a measurement taken on the final state. The final state, obtained as a result of the collapse of the wave function, shall give, with proper probabilities, one of four possible states. Using entangled quantum strategies by the players gives an opportunity to have mutual interaction between them, which has no counterpart in classical games. The main advantage of the quantum game is that the result of the wave function collapse is hidden by Nature. Even if one of the players eavesdrops the other player strategy, he can't be sure about the result of the game, which is given by (3) until the measurement of the final state $|\psi_f\rangle$.

3. Quantum PD in classical limit

The quantum game becomes a classical game when strategies do not include complex phase factors i.e.: $\alpha = \phi = 0$. In fact, entangling operator $\hat{J}$ is commuting with direct product $\widetilde{U}_A \otimes \widetilde{U}_B$ of each pair of classical operators, but $\hat{J}^\dagger \hat{J} = \hat{I}$ and therefore $|\psi_f\rangle = (\widetilde{U}_A \otimes \widetilde{U}_B)\,|CC\rangle$ and consequently Alice's expected payoff is

$$\$_A(\theta_A, \theta_B) = r\ \cos^2\frac{\theta_A}{2}\cos^2\frac{\theta_B}{2} + s\cos^2\frac{\theta_A}{2}\sin^2\frac{\theta_B}{2} + t\sin^2\frac{\theta_A}{2}\cos^2\frac{\theta_B}{2} + p\ \sin^2\frac{\theta_A}{2}\sin^2\frac{\theta_B}{2}, \tag{6}$$

this gives, a result identical with the classical game, where both players choose mixed strategies (Table 2).

TABLE 2.

The payoff matrix of quantum PD in the classical limit

		Bob $C\left(\cos^2\frac{\theta_B}{2}\right)$	**Bob** $D\left(\sin^2\frac{\theta_B}{2}\right)$
Alice	$C\left(\cos^2\frac{\theta_A}{2}\right)$	(r, r)	(s, t)
Alice	$D\left(\sin^2\frac{\theta_A}{2}\right)$	(t, s)	(p, p)

In the classical limit $\alpha_A = \phi_A = \alpha_B = \phi_B = 0$. The payoffs are reproduced by assuming that players use mixed strategies determined by θ_A and θ_B. Their payoffs are to be multiplied by the probability of particular strategy choice (in the parenthesis).

For example if Alice chooses cooperation $\theta_A = 0$ and Bob defection $\theta_B = \pi$, the game result is (s, t), so Bob wins. The only difference between the classical PD and the classical limit of quantum PD is that in the former the player choosing mixed strategy has to take draw by his or her own and eventually choose the option C or D. Between drawing and choosing an option there is a moment in time when the player can change his or her mind (as well as probability distribution) or the opponent can eavesdrop information regarding the planned movement (and respond to it adequately). In quantum PD a player decides only which strategy to use. The strategy is determined by an angle θ and all the rest is done by the quantum computer. There is no opportunity to eavesdrop quantum information by the opponent, at least in case without the quantum noise. Any such attempt would lead to a collapse of the wave function and ultimately end of the game.

Analyzing (6) it can be easily shown that the NE for the classical PD is the pair of mutual defections (D, D), corresponding to $\theta_A = \theta_B = \pi$. If one of players uses a classical strategy, e.g. $\widetilde{U}(\theta)$ and the other quantum strategy $\widehat{U}\left(\theta + \pi, 0, -\frac{\pi}{2}\right)$ then the game result shall be (s, t) in favor of the quantum player. This shows the advantage of quantum strategy over classical strategy - independently of the strategy used by the classical player, the quantum player always finds the best answer, which gives him the maximum payoff t and leaving the classical player with the sucker's payoff s.

4. Nash equilibria of quantum PD

The quantum strategies give in fact, a great variety of outcomes, normally not achievable by conventional strategies. Let us assume that Alice chooses arbitrary quantum strategy $\hat{A} = \widehat{U}(\theta_A, \phi_A, \alpha_A)$. Bob may answer with strategy $\hat{B} = \widehat{U}(\theta_B, \phi_B, \alpha_B) = \widehat{U}\left(\theta_A + \pi, \alpha_A, \phi_A - \frac{\pi}{2}\right)$. Note that regardless of Alice's choice Bob's move gives coefficients $p_{CC} = p_{DC} = p_{DD} = 0$ and $|p_{CD}| = 1$. Transformation used by Bob „cancels" any of Alice's moves and drives to a situation when Alice's final strategy written in $|\psi_f\rangle$ becomes 'cooperation' whereas Bob plays 'defection'. The game results in Bob's maximum reward $(\$_A, \$_B) = (s, t)$. However, the quantum game is symmetrical, so that Alice can respond to Bob's strategy $\hat{B}$ with $\widehat{A'} = \widehat{U}(\theta'_A, \phi'_A, \alpha'_A) = \widehat{U}\left(\theta_A, \phi_A - \frac{\pi}{2}, \alpha_A - \frac{\pi}{2}\right)$ strategy. Here the only non-zero factor of $|\psi_f\rangle$ is $|p_{DC}| = 1$, which means that now Alice plays 'defection' while Bob plays 'cooperation' and the payoff gets reversed $(\$_A, \$_B) = (t, s)$. The best answer for Alice's strategy $\widehat{A'}$ is Bob's$\widehat{B'} = \widehat{U}(\theta'_B, \phi'_B, \alpha'_B) = \widehat{U}\left(\theta_A + \pi, \alpha_A - \frac{\pi}{2}, \phi_A - \pi\right)$, because the game's result is again $(\$_A, \$_B) = (s, t)$. Eventually, the best answer to $\widehat{B'}$ is Alice's initial strategy, which gives her a winning position again$(\$_A, \$_B) = (t, s)$.

Let us now assume that Alice chooses her meta-strategy which mixes two of her quantum strategies$\cos^2\frac{\gamma_A}{2}\hat{A} + \sin^2\frac{\gamma_A}{2}\widehat{A'}, \gamma_A \in [0, \pi]$. We call it meta-strategy because it is a classical mixture of two quantum strategies. At the same time, Bob takes a similar step by playing his meta-strategy $\cos^2\frac{\gamma_B}{2}\hat{B} + \sin^2\frac{\gamma_B}{2}\widehat{B'}, \gamma_B \in [0, \pi]$. Alice's expected payoff is equal to:

$$\$_A = s\left(\cos^2\frac{\gamma_A}{2}\cos^2\frac{\gamma_B}{2} + \sin^2\frac{\gamma_A}{2}\sin^2\frac{\gamma_B}{2}\right) + t\left(\cos^2\frac{\gamma_A}{2}\sin^2\frac{\gamma_B}{2} + \sin^2\frac{\gamma_A}{2}\cos^2\frac{\gamma_B}{2}\right). \tag{7}$$

Note that in this game the sum of Alice's and Bob's payoffs is fixed and equals $\$_A + \$_B = s + t$. The game has only one NE for $\gamma_A = \gamma_B = \frac{\pi}{2}$ - at that point the players' payoffs are $\$_A = \$_B = \frac{s+t}{2}$ and none of them can increase his or her payoff by unilateral change of strategy (Flitney & Abbott, 2002). In fact, to get an equal reward $\frac{s+t}{2}$ per each player, it is enough that one of them uses strategy $\gamma = \frac{\pi}{2}$.

The game restricted to Alice's $\hat{A}, \widehat{A'}$ and Bob's $\hat{B}, \widehat{B'}$ strategies is in fact a constant sum game. The quantum PD has infinitely many NEs, each determined by three initial parameters of Alice's strategy $\hat{A} = \hat{U}(\theta_A, \phi_A, \alpha_A)$. Therefore we have shown that an appropriate choice of mixed quantum strategies can provide both players a result only slightly worse than mutual cooperation (remember that $r > \frac{s+t}{2}$ by PD definition). Taking into account that in the classical PD the payoff of the only NE is (p, p), the quantum game gives players a much better equilibrium, not achievable in the classical game.

Note that, in particular, if we set $(\theta_A, \phi_A, \alpha_A) = (0, 0, 0)$, then

$$\hat{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \hat{I}, \hat{B} = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} = -i\hat{\sigma}_x \tag{8}$$

and

$$\widehat{A'} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = i\hat{\sigma}_z, \widehat{B'} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i\hat{\sigma}_y. \tag{9}$$

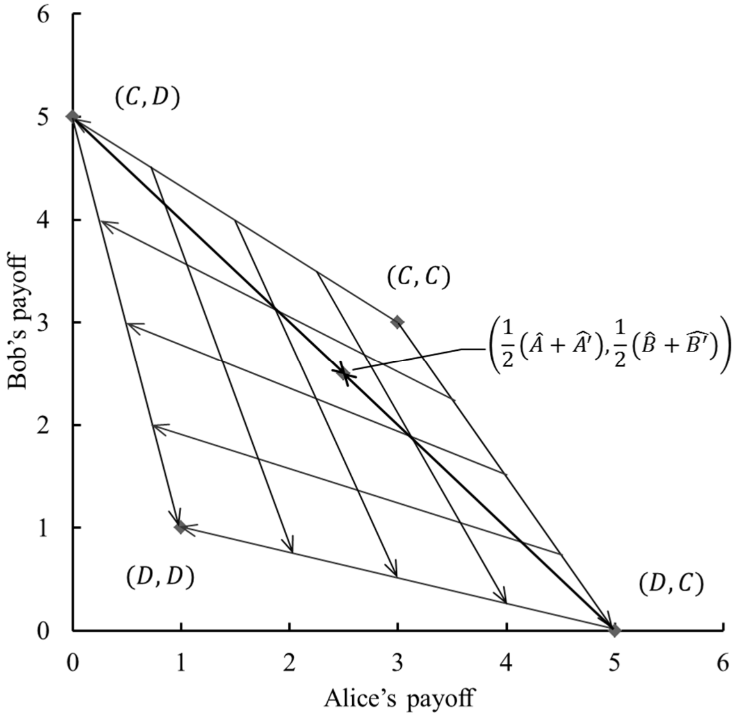
Strategies $\widehat{A}, \widehat{A'}, \hat{B}, \widehat{B'}$, lead, up to a constant, to four matrices (an identity matrix plus three Pauli matrices) from the group of unitary matrices $SU(2)$. The Pauli matrices $\hat{\sigma}_x, \hat{\sigma}_y$ and $\hat{\sigma}_z$ are generators of rotations by the angle π about the x, y and z axes respectively (green, blue and red circles at Figure 1).

5. Is it possible to use PD quantum equilibria?

Figure 3 shows PD utility diagram (for $t = 5, r = 3, p = 1$ and $s = 0$) with four classical strategies $(C, D), (C, C), (D, C)$ and (D, D). A series of red lines corresponds to pairs of classical strategies $(\theta_{A,} \theta_B)$ with constant θ_A while blue lines correspond to $(\theta_{A,} \theta_B)$ with constant θ_B. The arrows point the players' preferences and the only classical NE is at the point (D, D). Black line linking points (C, D) and (D, C), refers to a game of constant sum $\$_A + \$_B = s + t = 5$ and comprises all payoffs of Alice's meta-strategies $\cos^2 \frac{\gamma_A}{2} \hat{A} + \sin^2 \frac{\gamma_A}{2} \widehat{A'}$ and Bob's meta-strategies $\cos^2 \frac{\gamma_B}{2} \hat{B} + \sin^2 \frac{\gamma_B}{2} \widehat{B'}$.

FIGURE 3.

Prisoner's Dilemma utility diagram



PD utility diagram for $t=5, r=3, p=1$ and $s=0$. Regardless of Alice's mixed strategies (red lines), the best strategy for Bob (arrows) is D. Likewise in case of Bob's mixed strategies (blue lines), the best strategy for Alice is D. The quantum PD has NE for the pair of meta-strategies $\left(\frac{1}{2}(\hat{A}+\widehat{A'}),\frac{1}{2}(\hat{B}+\widehat{B'})\right)$ which is far more favorable than NE for classical PD strategy (D,D).

As shown in the previous section, the only NE here corresponds to the pair of meta-strategies$\left(\frac{1}{2}(\widehat{\mathrm{A}}+\widehat{\mathrm{A}'}),\frac{1}{2}(\widehat{\mathrm{B}}+\widehat{\mathrm{B}'})\right)$. Due to characteristic entanglement of classical strategies (C,D) and (D,C), quantum equilibrium gives the players payoff $\frac{s+t}{2}=2.5$, which is more favorable than the classical equilibrium and not achievable using classical strategies.

Bringing PD to a constant sum game is only possible in a quantum manner. An analysis of this solution shows that its essence is in specific correlation of the 'cooperation' - 'defection' solutions, in such a way that players are unable to foresee whether their strategies lead to one or other option. When Alice chooses strategy $\hat{A}$ or A' she does not know whether Bob responds with $\hat{B}$ or $\widehat{B'}$ strategy, so she does not know whether her move is 'cooperation' or 'defection'. The same applies to Bob's strategies. With that kind of strategy selection the quantum PD is then led to the matching pennies - zero sum game. This game has no pure strategy NE and the only NE is in mixed strategies - each player chooses any option with equal probability. If one of the players uses this strategy, then his payoff is at lest $\frac{s+t}{2}$ therefore it is his quantum reservation point which is much better than p, which can only be guaranteed be the classical NE strategy.

Naturally, here appears a question whether the quantum version of PD can be used to resolve negotiations, which may have a nature of the prisoner's dilemma. Examples from different areas indicate that quantum strategies are better than classical in cases such as solving stock and market game issues [Piotrowski, Sładkowski, 2002], as well as auction and competition issues [Piotrowski, Sładkowski, 2008], gambling [Goldenberg, Vaidman, Wiesner, 1999] or artificial intelligence [Miakisz, Piotrowski, Sładkowski, 2006]. As shown in this paper, solving a dilemma using quantum strategies could lead to better outcomes than using classical solutions. Since classical PD inevitably leads negotiators to the only rational solution, which is mutual refusal of cooperation, which leads to punishment for lack of cooperation. Quantum PD has NE yielding much better payoff, which is an average of 'temptation to betrayal' and 'sucker's payoff'. Is it in fact possible to use quantum strategies in everyday life?

One of the market processes regulated by PD is a price equilibrium known as the Nash 'beautiful equilibrium'. Companies which are likely to sell its products with higher prices, *de facto* decrease their prices in order to optimize profits [Dixit, Nalebuff, 2008]. Here, a PD shows that bilateral (or multilateral) cooperation, which is to maintain the high prices, is in an almost impossible market situation because there will always be a company which wants to sell cheaper ('defection'), compensating for the lower prices by increasing the number of customers and abandoning the more expensive manufacturer (who 'cooperates') without customers. However, in some cases PD mechanism does not work properly, as in the example of price collusion. A good example is an incident known from 1950s which happened on American turbine market [Dixit, Nalebuff, 2008]. Three companies made an agreement that they would use inflated prices. But under condition that, depending on the date of the invitation to tender, one of them would win. The winner in each auction took everything (temptation to betrayal), the others were left with nothing (sucker's payoff). Randomness of announcing tendering periods ensured that all partners earned, each in their time. The most important issue in the whole affair was to correctly correlate the company that should win the tender with the moment of its publication. They had to be correlated in such a way that interested parties had no doubt and nobody else could to predict the algorithm (because price collusion is illegal). If the price-fixing partners used an algorithm based on quantum entanglement, none of external observers could prove them collusion. Nevertheless, the directors of the companies got imprisoned because they used a less sophisticated, easy to uncover correlating system – a winner of the tender was chosen on basis of the lunar calendar – each company won depending on the number of days that had elapsed since the new moon.

Another example of the possible use of quantum solution would be the situation where companies, competing on the same market, come to an agreement to limit the amount of advertisements of their products to the agreed level. Unless any of them will not increase their quota, the number of customers and profits reach certain equilibrium. However, there is always a temptation for a given company to increase the amount of adverts above the agreed level ('defection'). This will lead to the increase of the number of customers and their profits but only temporarily, until other parties do the same. Finally they will reach the same or similar customers' equilibrium but their profits will be smaller ('punishment payoff') as they all spend more money for advertisements. In the quantum game the NE assures that at the same time only one company is exceeding the quota of adverbs while the other keeps the limits, the roles of companies are changing in time in a random manner. Therefore, the quantum setup designed to coordinate the amount of advertisements can lead to more favorable results for competing companies.

6. Summary and discussion

In negotiations there is always a difference in the knowledge level of involved parties. The players often have no interest in disclosing all of their preferences because it may be used against them by others. On the other hand, in integrative negotiations the full knowledge of preferences of both parties can help in finding the optimal possible result. This contradiction is common in negotiations and can only be resolved by mutual disclosure of both parties preferences. Alternatively, they can use the third-party arbiter, who secretly collects information from both parties and then proposes optimal solution that is binding on the parties. In this paper we show that quantum entanglement can play the role of such an arbiter.

We formulate negotiation as a quantum game where the players strategies correspond to unitary transformations of the given initial state in the Hilbert space. Quantum strategies are correlated through the mechanism of quantum entanglement and the result of the game is obtained by the collapse of the resulting transformed state. The range of strategies allowed for quantum players are richer than in a classical case and therefore the result of the game can be optimized. On the other hand, the quantum game is completely save against eavesdropping and the players can be assured that this type of quantum arbitration is fair.

In quantum games an essential element of a game mechanism is entanglement. Does this phenomenon also have its counterpart in the real classical game? Can macroscopic objects that are controlled or observed only by our senses be entangled? These are open questions. Problems with decoherence of the wave function means that even at a level of well-controlled experiments, held in extreme isolation from surroundings, it is difficult to maintain two entangled qubits. Building a quantum computer whose work is based on a register with a number of entangled qubits, transformed by unitary operations of quantum gates, which is capable of solving practical problems, using quantum algorithms, or capable of simulation of quantum games, is a real challenge to modern physics. Over the last decade, these challenges have been successfully undertaken, which led to building of first quantum computers [Vandersypen, Steffen, Breyta, Yannoni, Sherwood, 2001; Haroche, 2012; van der Sar, et al., 2012]. Progress of physics and technology creates opportunities for application of quantum games to real life problems.

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