Maintenance Optimization for Transport Vehicles in a Supply Chain

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In this paper, we are interested in determining a maintenance policy with an optimal cost to enable the company to generate significant profits, often the means of transport travelled different paths, they are characterized by the distance they covered; however each distance has an effect on the operating characteristics of means of delivery. The objective of this paper is twofold: It aims both to introduce the model of supply chain and specify distances, and codify the use of it in the proportional hazard model, later a maintenance policy was presented which takes into consideration the types of paths travelled by the means of delivery.

Keywords: maintenance policy, supply chain, proportional hazard, optimization.

1. INTRODUCTION

The economic development requires companies to be well structured. The economic development and the nature of industrial products show a strong competition between companies, the spread of competition is characterized by the establishment of foreign companies around the world and that is due to economic and logistical reasons.

In the present context the main objectives for many companies is to adapt to the market and customers' satisfaction, which usually depends on the overall performance of their supply chain. Today, the attention of enterprise is focused on the management of flows between actors of supply chain. Companies often ignore the mobile resources management, these resources can increase costs. Consequently, it is important to think about the good management of mobile resources and to take them into consideration when designing a logistics network, which provides a comprehensive visibility and control over these resources. Mismanagement of mobile resources in a supply chain lengthens the delay times of stock-outs for customers and also significantly increases the level of inventories in companies, and poor exploitation of these resources may affect the quality of the products transported. Therefore it is time for companies to think about the proper functioning of their mobile resources by applying a maintenance policy to ensure the reliability and availability of mobile resources that are the basis for global companies in the field of transport. In a context of increased competition, the main objective of the company is to be more competitive among the existing enterprises. A company must better manage its transportation with the least cost. The development of technology and their complexity have led to widely improved transport efficiency. In this context, i.e. the field of supply chain management, the research works are generally interested in minimizing costs related to the concept of logistics network to determine the optimal quantity transported and also the costs of the location of the various elements of supply chain seeking to minimize the distance and maximize profit from this work.

The research work [6] aims to study the environmental and economic challenges of logistics pooling. They believe that it is an emerging strategy to improve logistics performance. The authors noted that it aims to develop the concept of supply chain services and to reduce inventory. In the article the authors have introduced some researchers that link transport and sustainable development, which enables logistics providers and carriers to participate not only in the reduction of logistics costs, but also in the reduction of CO_2 emissions. [8] is interested in problem of optimization of vehicle tours with inventory management. In this paper, the seller is responsible for the inventory of its customers; the objective of this paper is to minimize logistics costs. The authors demonstrated that their model could reduce costs by 20% compared to what was done by the experts of the logistics domain.

[5] is interested in a recent problem of green logistics. The purpose of this article is to provide a mathematical model to minimize the overall economic costs of the supply chain i.e. direct traditional logistics costs, in closed loop and reverse logistics costs. Then they proposed another model that minimizes the effects of carbon dioxide - these costs are considered in the article as external transport costs.

Concerning the operating conditions of an industrial or logistics problems, several works are interested in the type of this problem, the researchers are always considered that these conditions are constant normal over time.

For [3], the operating conditions have an influence on the choice of maintenance policy conducted and system reliability, he defined, that these conditions are environmental or operational associated for each mission are estimated by experts naval field. According to [3], the environmental conditions depend on the environment and the place of performance of the mission; these environmental conditions have an influence on the rate of system failure. Modelling the influence for operating conditions is given by the proportional risk model.

In the works of [2] concerning the operating conditions, they determined that the variation of the production rate leads to a degradation of the system and increases the failure rate. [6] studied the impact of operating conditions on the choice of maintenance action. The aim of the author is to determine after each changing operating conditions the maintenance actions to apply.

The remainder of the paper is organized as follows: in Section II, we detailed the problem and Section III presents the maintenance strategy. In section IV we developed the optimization approach. In section V we presented a numerical example in order to apply the analytical results. Finally we concluded in section VI.

2. PROBLEM DESCRIPTION

This study deals with the case of a company composed of the transport means ensuring the delivery of merchandise between different elements of the supply chain (plants, distribution centres, customers).

In this work we assume that only one means of transport is required to achieve a set of tours during a determined period of time. The tours are characterized by types of roads as well as the time required to complete a tour. Similarly, a tour generally generates a degradation process of delivery means, which can cause failures where the maintenance cost will be high, which reduces the profit generated by the tour.

We assume that a single vehicle index v ensures the delivery of products from the factory k to distribution centre, then from distribution centre to the customer, and finally back to the factory from the customer. This loop builds a tour to the delivery means, then a tour to the factory begins and returns finally to the same plant.

Our goal is to determine a jointly maintenance policy and an ordered set of tours for the delivery means before performing tours, which reduces the cost of corrective maintenance and the average number of breakdowns.

The specification of this problem lies, by the fact, the influence of the distance travelled by delivery means in the number of failures and preventive actions to maintenance.

The problem is illustrated in figure 1.

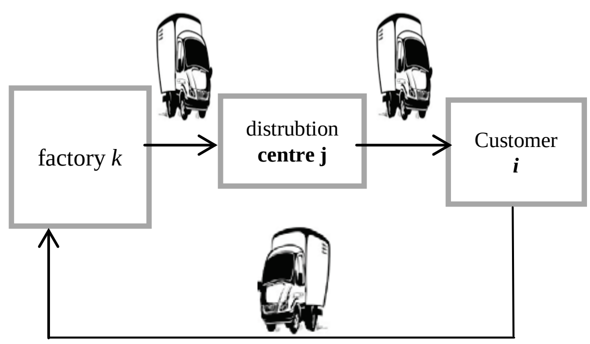

Fig. 1. Problem Description.

2.1. NOTATIONS

✓ the subscript:

k∈K: index of manufactories k= {1,2,…, K}
j∈J: index of the distribution centres j= {1,2,…, J}
i∈I: index of Customers i= {1,2,…I}
v∈V: index of vehicles v= {1,2,…, V}
Y: tours number
q: index of the tours q= {1, 2,3,…, Y}

✓ the model parameters:
r_j: demand of distribution centre j
r_i: demand of customer i
c_{jiv}: cost of transport from the distribution centre j to the customer i by the vehicle v
c_{kjv}: cost of transport from the manufactory k to distribution centre j by the vehicle v
u_{kjv}: cost of displacement from manufactory k to distribution centre j by displacement unity for vehicle v.
u_{jikv}: cost of displacement from distribution centre to customer and from customer to manufactory by displacement unity for vehicle v
$capa_j$: capacity of distribution centre
$capa_k$: capacity of manufactory k
d_{kjv}: distance between manufactory k and the distribution centre j travelled by the vehicle v.
d_{jikv}: distance between distribution centre j and customer i and the customer i to manufactory k travelled by the vehicle v.
M_{cv}: cost of corrective maintenance for vehicle v
M_{pv}: cost of preventive maintenance for vehicle v
mu: monetary unit

✓ the decision variables:
x_{jiv}: transported quantity from distribution centre j to customer i by the vehicle v
x_{kjv}: transported quantity from manufactory k to distribution centre j by the vehicle v
y_{jikv} = 1 if the distance djik travelled by the vehicle v, 0 if no
Nv: number of the preventive maintenance intervals for the vehicle v.

2.2. TOTAL FUNCTION COST

The idea is to minimize the cost of the transport between different elements of the logistic network, and the road cost, and, as a result, the cost of maintenance actions.

The total costs are as follows:

$$\min(z) = \sum_{v\in V}[(\sum_{j\in J}\sum_{i\in I}(c_{jiv}x_{jiv}) + \sum_{k\in K}\sum_{j\in J}\left(c_{kjv}x_{kjv}\right) + \sum_{j\in J}\sum_{i\in I}\sum_{k\in K}(u_{jikv}d_{jikv}\times y_{jikv}) + \sum_{j\in J}\sum_{i\in I}(u_{jiv}d_{jiv}) + M_{cv}\phi_{cv} + M_{pv}(N_v - 1)]$$

Subject to:

$$\sum_{j\in J}\mathbf{x_{jiv}} \geq \mathbf{r_i} \ \forall\ \mathbf{i}\in\mathbf{I} \tag{1}$$

$$\sum_{k\in K}\mathbf{x_{kjv}} \geq \mathbf{r_j}\ \forall\ \mathbf{j}\in\mathbf{J} \tag{2}$$

$$\sum_{i\in I}\mathbf{x_{jiv}} \leq \mathbf{capa_j}\ \forall\ \mathbf{j}\in\mathbf{J} \tag{3}$$

$$\sum_{j\in J}\mathbf{x_{kjv}} \leq \mathbf{capa_k}\ \forall\ \mathbf{k}\in\mathbf{K} \tag{4}$$

$$\mathbf{x_{jiv}} \geq 0\ \forall\ \mathbf{j}\in\mathbf{J};\ \mathbf{x_{kjv}} \geq 0\ \forall\ \mathbf{k}\in\mathbf{K} \tag{5}$$

$$\mathbf{y_{jikv}} \in \{0,1\} \tag{6}$$

Constraints (1) and (2) require the satisfaction of demands. Constraints (3) and (4) define the capacity bounds of the distribution canters and the manufactories. Constraint (5) denotes that the product flows are not negative. Constraint (6) is a binary constraint equal to 1 if the distance d_{jik}, travelled by the vehicle v.

3. MAINTENANCE STRATEGY

In this work, we will use the periodic policy of maintenance strategy; this policy is to perform preventive maintenance actions at constant time intervals. The maintenance strategy under consideration is the well-known preventive maintenance policy with minimal repair at failure.

The analytic expression of the total maintenance is expressed as follows:

$$\mathbf{\Gamma(Y,N) = M_{cv}\phi_{cv} + M_{pv}(N-1)} \tag{7}$$

Where ϕ_{cv} the average number of failures is expressed in the following way:

$$\phi_{cv}(\mathbf{Y},\mathbf{N}) = \sum_{\mathbf{n}=1}^{\mathbf{N}}\left(\sum_{\mathbf{q}_v=\mathbf{d}(\mathbf{n})}^{\mathbf{f}(\mathbf{n})}\left(\int_{\tau_{q-1_v}}^{\tau_{q-1_v}+\sigma_{q_v}}\lambda_{\mathbf{q}_v}(\mathbf{t})\,\mathbf{dt}\right)\right) \tag{8}$$

$\lambda_{q_v}(\mathbf{t}) = g(z)_{q_v}\,\lambda_n(t)$ The failure rate associated to the tour q realized by vehicle v.

In the equation (8), the terms *d*(n) and *f*(n) that respectively represent the tours that beginning and finished the interval *n* with n= {1,2,3,…, N}. The realization of tours can be done in several intervals of preventive maintenance; therefore it is necessary to determine the duration of these tours for each maintenance interval. This duration noted σ_{q_v} is still less than or equal to the rotated length δ_{q_v}.

✓ Proportional failure rate

In this work, we use the Cox model [1] that established a semi-parametric relationship between the risk factors for failure and distribution of lifetimes.

The Cox model is expressed by

$$\lambda(t,z) = g(z)\lambda_n(t) \tag{9}$$

with:

$\lambda_n(t)$ the function of nominal failure rate

$\lambda(t,z)$ The instantaneous risk of failure at time t, under the conditions z

In equation (9) the term g (z) is the risk function. It is an exponential function:

$$g(z) = e^{\sum_i b_i z_i} \tag{10}$$

The coefficient b_i is the influence of z_i factor, as in our case the factor z_i represents the type of the path travelled by any transport means throughout the tour,

In our case there is only one factor i = 1, it has three levels of severity:

Z_{1c}: the distance is a short path.

Z_{1M}: the distance is a medium type.

Z_{1L}: the distance is a long type

In addition, the b_i vector is estimated by maximizing the likelihood of Cox. We noted that V* the partial likelihood, from [4]:

$$V^* = \prod_{i=1}^{n} V_i = \prod_{i=1}^{n} \frac{e^{b_i Z_i}}{\sum_{k \in n(t_i)} e^{b_i zk}} \tag{11}$$

We noted t_i i= 1…n, with n is the number of observed failures and z_i i=1…n the n associated constraints. n(t_i) represent the entire population at risk, that is to say, all components with a lifetime greater than t_i

[4] developed this expression and calculated the log-likelihood

$$L^* = \ln\left(V^*\right) = \sum_{i=1}^{n}\left[b_i z_i - \ln\left(\sum_{j \in n(t_i)} e^{b_j z_j}\right)\right] \tag{12}$$

In our study, the function risk is expressed as follows:

$$g(Z)_{q_v} = e^{(z_1 b_1)} \tag{13}$$

Besides the z_1 should have three levels depending on the distance d_{qv}, which d_{qv} is the distance from the tour index q.

$$Z_1 = \begin{cases} 1 & if \quad d_{q_v} \in]a,b] \\ 2 & if \quad d_{q_v} \in]b,c] \\ 3 & if \quad d_{q_v} \in]c,d[\end{cases} \tag{14}$$

✓ Functional age:

It is understood that the function of the failure rate varies depending on the distance traveled (type of path), the reliability of transport means must be continuous over time.

When a tour is over and another begins, this change involves a change in the failure function, this change reproduced on the reliability function. Thereby to ensure the continuity of reliability, we will call the concept of functional age whose objective is to ensure continuity. To determine that age, it is necessary to know the reliability of the transport means to end of the previous tour and the type of path for the risk function of the next tour.

In addition, to ensure the continuity, the functional age τ_q associated of the tour q+1 must satisfy the following relationship:

$$\left[\mathbf{R}\left(\mathbf{t}_{\mathbf{q}_v}\right)\right]^{g(\mathbf{Z}_{\mathbf{q}_v})} = \left[\mathbf{R}\left(\boldsymbol{\tau}_{\mathbf{q}_v}\right)\right]^{g(\mathbf{Z}_{\mathbf{q+1}_v})} \tag{15}$$

with

$$\boldsymbol{\tau}_{\mathbf{q}_v} = \begin{cases} 0 \quad if \quad \mathbf{q}_v = 0 \\ \mathbf{R}^{-1}\left[\left(\mathbf{R}(\mathbf{t}_{\mathbf{q}_v})\right]^{\mathbf{g}(\mathbf{Z}_{\mathbf{q}_v}) / \mathbf{g}(\mathbf{Z}_{\mathbf{y}_{\mathbf{q+1}_v}})}\right. \end{cases} \tag{16}$$

with

$$\mathbf{t}_{\mathbf{q}_v} = \boldsymbol{\tau}_{\mathbf{q}-1_v} + \boldsymbol{\delta}_{\mathbf{q}_v} \tag{17}$$

$\mathbf{t}_{\mathbf{q}_v}$: age of transport means v at the end of tour q

Denoted that Δ the constant duration of preventive maintenance intervals, this duration is determined by the cumulative duration of tours and the number of intervals N.

$$\Delta = \frac{\sum_{\mathbf{q}=1}^{\mathbf{y}} \boldsymbol{\delta}_{\mathbf{q}_v}}{\mathbf{N}_\mathbf{v}} \tag{18}$$

The following figure presented an example of this maintenance policy

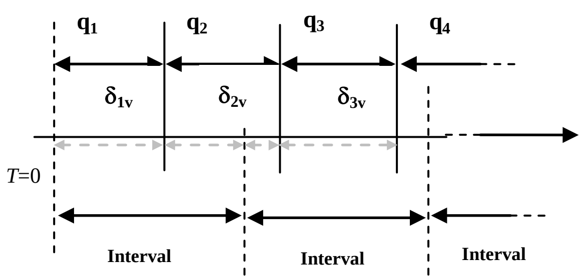

Fig. 2. Periodic policy example.

From Figure (2) it is certain that tour may belong to one or more intervals. To calculate the average number of failures of the equation (7) it is necessary to know the times of the beginning and end of each interval n where n= {1,2,…, N}, these periods are denoted respectively d(n) and f(n). Consequently, the duration of a tour q in an interval is less than the actual duration of the completion of the tour q in an interval n noted σ_{q_v} with $\sigma_{q_v} \le \delta_{q_v}$

The duration of a round is given by

$$\sigma_{q_v} = \begin{cases} \delta_{q_v} & if\ d(n) \neq q_v \neq f(n) \\ \sum_{q=1}^{d(n)} \delta_{q_v} - (n-1).\Delta & if\ d(n) = q_v \neq f(n) \\ n.\Delta - \sum_{q=1}^{f(n)-1} \delta_{q_v} & if\ d(n) \neq q_v = f(n) \\ \Delta & if\ d(n) = q_v = f(n) \end{cases} \tag{19}$$

Other terms in equation (8) remain to be defined. These terms correspond to tours q_v at the bounds of maintenance intervals d (n) and f (n)

$$d(n) = \begin{cases} 1 & if \quad n = 1 \\ q_v & solution\ of \ \begin{cases} \sum_{l=1}^{q} \delta_{q_{l_v}} > (n-1).\Delta \\ \sum_{l=1}^{q-1} \delta_{q_{l_v}} \le (n-1).\Delta \end{cases} \end{cases} \tag{20}$$

$$f(n) = \begin{cases} 1 & if \quad n = 1 \\ q_v & solution\ if \ \begin{cases} \sum_{l=1}^{q} \delta_{l_v} \ge n.\Delta \\ \sum_{l=1}^{q-1} \delta_{l_v} < n.\Delta \end{cases} \end{cases} \tag{21}$$

4. OPTIMSATION

4.1. SHORTEST PATH

The shortest path problem is to find the shortest path and the lowest possible cost between two vertices. Mathematically the shortest path is a straight line, except that in practice it is not always possible to translate this result in specific cases. When you want to move from one city X to another city Y is trying to follow the least expensive roads and having a short distance, the problem in this case is how to find the shortest path between two cities. In graph theory, this problem can be modelled by applying fast algorithms for finding the shortest path and in our case study applied the algorithm is the Dijkstra [9].

4.2. MAINTENANCE OPTIMIZATION

For this policy, the decision variable is the optimal number of preventive maintenance intervals N*. Analytically, it is difficult to determine the optimal solution due to the complexity of the maintenance cost equation. Normally, to search the optimal solution is to solve the following equation:

$$\frac{\partial \Gamma(Y, N)}{\partial N} = 0 \tag{22}$$

From [3], they have shown that exist an optimal number of preventive maintenance, and to obtain this optimal number, they performed a numerical procedure.

5. NUMERICAL EXAMPLE

In this section, we will present an illustrative case in order to apply the analytical study presented in the previous sections.

Our example composed of 3 distribution centres and 3 manufactories. The following table presents the solution for the logistics model together optimal quantities:

Table 1. Delivery quantity from manufactory *K* to distribution centre *J*.

	Distribution Centre			
factories	C1	C2	C3	Factories capacities
M1	67	80	80	400
M2	67	80	80	560
M3	66	80	80	560
Demands of the distribution centres	200	240	240	

Similarly, we will determine the quantities requested by clients. The following table shows the quantities delivered by each distribution centre:

Table 2. Delivery quantity from distribution centre *J* to customer *I*.

	Customers			
Distribution centres	Cl1	Cl2	Cl3	Capacities Of C.D
C1	33	0	40	300
C2	33	120	40	350
C3	34	0	40	350
Customers' demands	100	120	120	

After completing the first stage, the second objective is to determine the paths that will eventually be covered by the chosen transportation means to perform the all of these delivery roads. For information in our case we used a single means of transport that will accomplish all tours.

Recall that to determine the levels of influence factor, we have:

$$Z_i = \begin{cases} 1 & d_{q_v} \in \left]a,b\right] \\ 2 & d_{q_v} \in \left]b,c\right] \\ 3 & d_{q_v} \in \left]c,d\right] \end{cases}$$

with *a*=0; *b*=950; *c*=1500; *d*=30000

The table below indicated the factor with coding the levels of influence factor

Table 3. Coding and model values for types of path.

Factor	Travelled distance d_{qv}		
Coding *Zi*	1	2	3
Distance	]0,950]	]950,1500]	]1500,30000]
Types of road	Short (S)	Medium (M)	Long (L)

The following table presents the distance and these path types together that the duration for each road:

Table 4. Distance and type of road and duration of each round.

Road (q_v)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Duration δ_{qv}	**44**	**72**	**93**	**94**	**95**	**123**	**124**	**140**	**152**
Distance in km	**565**	**950**	**1265**	**1267**	**1282**	**1690**	**1698**	**1926**	**2092**
Type of road	**S**	**S**	**M**	**M**	**M**	**L**	**L**	**L**	**L**

In order to determine the coefficient of influence factor, we computed the partial derived of equation (12). The numerical solution of this equation (11) gives the following value:

Coefficient associated of travelled distance: 0,077

According to the equation (10), the expression of risk function has become:

$$g(z_i) = e^{0.077 \times z_i}$$

Point of view reliability, we suppose that the failure time of delivery means has a degradation law characterized by a Weibull distribution. The Weibull scale and shape parameters are respectively $\beta=600$ and $\gamma=2$. The corrective and preventive maintenance cost are respectively $M_{cv}=500$ mu, $M_{pv}=350$ mu .

The above figure illustrates the minimum total maintenance cost for different values of the number, *N*, of PM actions to be performed.

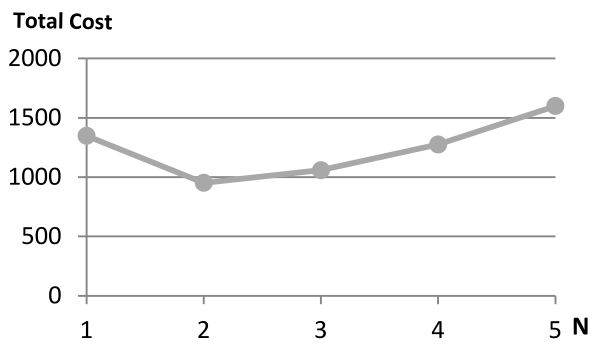

Fig. 3. Minimal total cost according to *N*.

Fig. 3 shows the curve of the total cost of maintenance according to N (Number of preventive maintenance actions). We conclude that the optimal number of preventive maintenance actions that minimizes the total cost of maintenance is N*=2. Hence, the cumulated distance after which we will make the intervention for preventive maintenance equal to 6637km through the transported means.

6. CONCLUSIONS

In this paper, we consider a maintenance optimization for delivery means in the supply chain. We made a linear optimization problem in the supply chain, taking into account the distance between the various elements building the logistics network and maintenance costs related to transportation which distributes the goods. The objective function contains three different costs which aim to minimize all these costs, transport costs, road costs and maintenance costs.

However we must be aware of the need to ensure the availability and reliability of transportation during the implementation of a tour to avoid any costs related to the customer satisfaction.

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