

# Integrated Maintenance-Quality Strategies Taking into Account the Impact off the System Degradation on the Quality of Output Products.

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This paper deals with integrated models joining maintenance and quality. We consider a manufacturing system composed of a single machine subject to an increased random failure rate and producing conforming and non-conforming items. In order to decrease the random failure rate and its impact on the quality of output products, a bloc type preventive maintenance strategy with minimal repair is considered. Our study consists in developing integrated analytical models joining maintenance and quality in order to determine the optimal preventive maintenance plan taking into account the progressive quality perturbation and the economic impact of reworking activities. Two strategies are developed. In fact, for the first strategy, we propose reworking activities for all non-conforming products in order to improve their quality condition and to sell all batches at the best price  $P_{max}$ . The aim of this strategy is to find the optimal number of batches produced and reworked  $N^*$  before each preventive maintenance action, maximizing the total net profit per time unit  $PT1$ . For the second strategy, the same problem is studied but for a finite horizon. Two mathematical models are developed in order to find the optimal value of the decision variable  $N^*$  for both strategies. Numerical examples are presented in order to illustrate the proposed models and a sensitivity study is developed to evaluate the influence of the variation of some parameters.

**Keywords:** Integrated Maintenance strategy, reworking activity, deterioration, optimization, quality.

## 1. INTRODUCTION

In today's highly dynamic and rapidly changing environment, companies are facing difficult challenges and they have to manage several functional areas successfully, such as production,

maintenance, quality and inventory. This increased competition calls firms to ensure an efficient coordination between these different areas. In fact, managers and researchers are increasingly recognized that one of keys of success consists to establish new management approach integrating the

interaction between parts or all of those functions. In the past, production, quality, inventory and maintenance have been treated separately. Because of their interdependence, many efforts have been made to understand and solve this problem. Developing adequate models that take into consideration more than one of these aspects became a necessity. This area of research has grown considerably in the last decade and there are many papers deals with the integration of more than one of these main aspects of any modern production process. Maintenance and quality management are among the most important components in any industrial system, and they have been widely studied separately during the last decades. Studies in maintenance were started with Barlow and Hunter (1960). They proposed a simple periodic replacement model with minimal repair. When the system fails, minimal repair is performed immediately to restore the system to its prior state before the failure. Since that, a lot of scheduled maintenance policies have been developed and considerable efforts have been allocated to developing and optimizing preventive maintenance models (Ait-Kadi et al., 2003; Barlow and Proschan, 1965; Dekker, 1996; Liao et al., 2010; McCall, 1965) which contributed to a good understanding of properties and effectiveness of preventive maintenance policies under various conditions. A huge number of contributions can be seen in a survey on maintenance models by Hongzhou (2002). Practitioners and academicians recognized that there is a strong relationship between product quality and equipment maintenance and the integration of these two aspects is beneficial to organizations. Then, more research considering simultaneously maintenance policies and quality deterioration needs to be conducted. In these perspectives, in this study we intend to develop new models joining maintenance and quality simultaneously. The literature includes several works establishing the link between all these different functions maintenance, quality, production and inventory, or a part of them.

Rahim and Ben-Daya (2001) provided an overview of the literature dealing with integrated models for production, quality and maintenance and they outlined directions for future research. According to them, treating these different aspects independently may not result overall optimization which justifies the need to develop models that take into consideration more than one of these three aspects. Ben-Daya (2002) proposed an integrated

model for the joint determination of economic production quantity and preventive maintenance level for imperfect production process. He proved that performing preventive maintenance gives way to a reduction of quality control related costs.

In the same context of integrated maintenance policies, Chelbi and Rezg (2006) consider a policy based on the age of the equipment. They determine the optimal age of the system, the most appropriate time to be submitted to preventive maintenance, and the optimal size of the buffer stock. The aim is to minimize the total cost per time unit, taking into account a minimum required system availability level.

At the same frame, Dellagi et al. (2007) studied integrated maintenance-production strategies which take into consideration subcontracting. They developed and optimized a maintenance policy incorporating subcontractor constraints and demonstrated, through a case study, the influence of the subcontractor constraints on the optimal integrated maintenance-production strategy.

Since the reduction in equipment condition leads to reduction in product quality, an adequate maintenance policy could reduce process variation (Liao and Xi (2010)) and consequently help to increase the product quality. Many models are developed with the object of reducing total cost, decreasing non quality cost and increasing product quality.

Dealing with this subject, Ben-Daya and Duffuaa (1995) discussed the relationship between maintenance and quality and they have proposed two approaches for linking and modelling this relationship. The first approach is based on the fact that maintenance affects the failure rate of the equipment and that it should be modelled using the concept of imperfect maintenance. The second approach is based on Taguchi's approach to quality which consists on using maintenance to reduce the deviation of product quality characteristics from their target value.

Radhoui et al. (2010) studied a joint preventive maintenance and quality control policy for a manufacturing system producing conforming and non-conforming products. The number of nonconforming items is the decision variable for the maintenance intervention. A buffer stock is constructed whenever the rate of non-conformity products attains a specific limit. The objective was to determine this limit and the size of buffer stock that minimizes the overage total cost per unit time

integrated maintenance, quality and inventory cost. In the same framework, some works discussed the integration between the Statistical Process Control (SPC), as one of the most common methods for controlling product quality, and Maintenance. Ben-Daya (1999) developed an integrated model for the joint optimization of the economic production quantity, the economic design of x-control chart, and the optimal maintenance level, for a deteriorating process where the in-control period follows a general probability distribution with increasing hazard rate. In addition, Ben-Daya (1999) and Ben-Daya and Rahim (2000) proposed an integrated model based on X control chart and preventive maintenance in which the in-control time follows a probability distribution with increasing hazard rate. After some years, Linderman et al. (2005) demonstrated the economic benefits of the integration of the Statistical Process Control (SPC) and maintenance by jointly optimizing their policies to minimize the total costs.

The indicator of the presented studies in literature consists in the non-considering of the non-conforming cost and the reworking aspect as an economical profit. In the present study, we developed two strategies combining PM actions and reworking activities in order to establish an optimal integrated maintenance-quality plan.

The remainder of this paper is organized as follow: next section introduced the problem statement and the notation used. In section 3, we present two mathematical models each for giving strategy. Section 4 is dedicated to present numerical examples to illustrate our models and results obtained. In the section 5, we present a sensitivity study to show the impact of the variation of some parameters. In the last section, a conclusion and some perspectives are presented.

## 2. PROBLEM STATEMENT

We consider a manufacturing system composed of a single machine and subject to an increasing failure rate following a Weibull distribution with a shape parameter  $\beta > 1$  and a scale parameter  $\epsilon$ . The repair time distribution is exponential with a mean of  $\mu_p$ . In order to reduce the probability of failures and its impact on the final product quality, a preventive maintenance policy of the block type with minimal repair at failure is considered. A preventive maintenance (PM) action is performed after each N produced batches. We assume that, after a PM action, the manufacturing system is restored

to a state as good as new and the product's quality returns to a new quality condition. We propose reworking activities for non-conforming items in order to improve their quality condition and to be sold at the best price  $P_{max}$ . Minimal repairs take place whenever failures are detected between PM actions to restore the system into the operating state without changing its failure rate function.

In addition, we assume that preventive and corrective maintenance actions have non-negligible durations. Hence, the expected cycle length includes the average production time, the preventive maintenance time and average minimal repair time.

The increasing failure rate of the manufacturing system results an increased rate of non-conforming items. We define a quality degradation rate noted  $\alpha$  representing the ratio of conforming items in a batch. This latter one is given by the normalization method relating the quality deterioration of the product to the failure rate of the manufacturing system.

$$\alpha_i = 1 - \frac{\lambda_i}{\lambda_{N+1}} \text{ for all } i \in 1..N \quad (1)$$

With  $\lambda_i$  represents the failure rate of the manufacturing system following a Weibull distribution after the production of a batch  $i$ .

It is given by the following expression:

$$\lambda_i = \frac{\beta}{\epsilon} \times \frac{(i \times Proc)^{\beta-1}}{\epsilon} \text{ for all } i \in 1..N \quad (2)$$

Then we obtain an analytical expressing of the quality degradation ratio  $\alpha_i$ . It is a decreasing function representing the link between the machine degradation and the quality of output products.

The expected number of failures during  $[0, t]$  is given by equation (3). This expected number of failures is based on the convolution of the failure distribution and the repair distribution (Lim et al. (2005)):

$$Nb_{cm} = -Log[1 - \int_0^t F(t-x) \times g(x)dx] \quad (3)$$

with

- $F(t)$ : Weibull probability distribution function associated with the time to failure.
- $g(t)$ : Exponential probability density function associated with the length of repair time.

2. 1. DESCRIPTION OF DIFFERENT STRATEGIES

As described above, the product quality is progressively deteriorated according to natural machine degradation. For the first strategy, we propose reworking activities in order to improve the product quality of all produced batches to be sold at the best price  $P_{max}$ . The rework task generates an additional cost  $C_{rk}$ , but, on the other hand, all batches will be sold at  $P = P_{max}$ . The preventive maintenance action is performed every  $N^*$  produced and reworked batches. For the second strategy, we propose to study the same problem over a finite horizon  $H$  and we integrate a setup cost  $C_{setup}$ . Preventive maintenance actions are performed every  $N^*$  produced and reworked batches during the finite horizon.

2. 2. OBJECTIVES

The objective is to maximize the total profit per time unit under different strategies by determining the optimal number of batches  $N^*$  before each preventive maintenance action for every strategy.  $N^*$  is obtained by maximizing the total profit by time unit respectively for the first and the second strategy (PT1 and PT2). The total profit is integrating selling price, production cost, preventive and corrective maintenance costs, reworking cost and setup cost adopted in the second strategy. It means that the PM action will be undertaken after each  $N^*$  batches. The analytical study developed in order to solve the proposed problem is detailed in next section.

3. ANALYTICAL STUDY

3. 1. NOTATIONS

In order to develop analytical models related to our problems, we present the following notation that will be used throughout the paper:

- T: Average duration of a full cycle
- $T_1$ : The remaining period after the last preventive maintenance action performed during the planning horizon
- H: The planning horizon
- Proc: Average processing time

- $\mu_p$  : Average duration of preventive maintenance action
- $\mu_c$  : Average duration of corrective maintenance action
- $P_{Ti}$  : Average total profit per time unit of a full cycle for the  $i$ th strategy. ( $i \in \{1,2\}$ )
- SP : Total selling price of a full cycle T
- SP1 : Total selling price of an extra period T1
- $M_p$  : Unit preventive maintenance cost
- $M_c$  : Unit corrective maintenance action cost
- $C_{cm}$  : Total corrective maintenance cost of a full cycle T
- $C_{cm1}$  : Total corrective maintenance cost of an extra period T1
- $M_{c1}$  : Corrective maintenance action cost of an extra period T1
- $N_{bcm}$  : Average number of failure per time unit during the cycle T
- $N_{bcm1}$  : Average number of failure per time unit during the extra period T1
- $g(t)$ : Probability density function associated with the length of repair time
- $F(t)$  : probability distribution function associated with the time of failure
- $P_{max}$  : The selling price of the first batch
- $C_{rk}$  : Total reworking cost of a full cycle T
- $C_{rk1}$  : Total reworking cost of an extra period T1
- $C_{rku}$  : Unit reworking cost.
- $C_p$  : Total production cost of a full cycle T
- $C_{p1}$  : Total production cost of an extra period T1
- $C_{pu}$  : Unit production cost
- $C_{setup}$  : Setup Cost
- $\alpha_j$  : Degradation ratio of quality
- E : The integer part

3. 2. MATHEMATICAL MODEL FOR THE FIRST STRATEGY.

The expected total profit by time unit (PT<sub>1</sub>) for the first integrated model includes:

- Total selling price
- Corrective maintenance cost
- Preventive maintenance cost
- Production cost
- Reworking cost

The expected total profit by time unit for this integrated model is given by (4):

$$PT_1 = \frac{SP - (M_p + C_{cm} + C_p + C_{rk})}{T}$$

with

$$C_{p,i} + C_{rk,i} \leq SP_i \text{ for all } i \in 1..N$$

$$\text{for all } i \in 1..N \quad (4)$$

We develop the expression of different costs presented in equation (4).

- Total selling price:

$$N \times P_{max}$$

- Total corrective maintenance cost:

$$C_{cm} = M_c \times Nb_{cm}$$

- Mean number of failures :

$$Nb_{cm} = -Log[1 - \int_0^t F(t-x) \times g(x)dx]$$

- Total production cost:

$$C_p = C_{pu} \times N$$

- Total reworking cost:

$$C_{rk} = \sum_{i=1}^N C_{rku} \times \left(\frac{\lambda_i}{\lambda_{N+1}}\right)$$

- The average duration of a full cycle :

$$N \times Proc + \mu_p + \mu_c \times Nb_{cm}$$

Then, the expected total profit per time unit for this first strategy ( $PT_1$ ) is given by the equation (5):

### 3. 3. MATHEMATICAL MODEL FOR THE SECOND STRATEGY.

$$PT_1(N) = \frac{N \times P_{max} - (M_p + M_c \times Nb_{cm} + C_{pu} \times N + \sum_{i=1}^N C_{rku} \times \left(\frac{\lambda_i}{\lambda_{N+1}}\right))}{N \times Proc + \mu_p + \mu_c \times Nb_{cm}} \quad (5)$$

With

$$C_{pu} + C_{rku} \times \left(\frac{\lambda_i}{\lambda_{N+1}}\right) \leq P_{max} \text{ for all } i \in 1..N$$

$$PT_{2case1} = \frac{SP - (M_p + C_{cm} + C_p + C_{rk} + C_{setup})}{T} \times E\left(\frac{H}{T}\right) + \frac{SP_1 - (M_p + C_{cm1} + C_{p1} + C_{rk1})}{T} \quad (6)$$

The expected total profit by time unit ( $PT_2$ ) for the second integrated model includes:

- Total selling price
- Corrective maintenance cost
- Preventive maintenance cost
- Production cost
- Reworking cost
- Setup cost

Looking at the remaining period  $T_1$  after the end of the last preventive maintenance action performed during the planning horizon, we distinguish two different models corresponding to each case. These two different cases are described in figure (1).

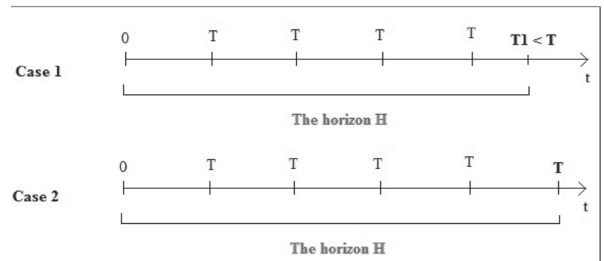


Fig.1. Description of the first and second cases

**Case 1:** The end of the last preventive maintenance action is performed before the end of the planning horizon.

$$\text{Formally : } T_1 = H - E\left(\frac{H}{T}\right) \times T \neq 0$$

The expected total profit by time unit for this integrated model is given by (6):

Using expressions of  $SP$ ,  $C_{cm}$ ,  $C_p$ ,  $C_{rk}$  and  $T$  developed in the section (3.2), the total profit per time unit  $PT_{2case1}$  is given by (7):



$$PT_{2case1} = \left( \frac{N \times P_{max} - \left( M_p + M_c \times Nb_{cm} + C_{pu} \times N + \sum_{i=1}^N C_{rku} \times \left( \frac{\lambda_i}{\lambda_{N+1}} \right) + C_{setup} \right)}{N \times Proc + \mu_p + \mu_c \times Nb_{cm}} \right) \times E \left( \frac{H}{T} \right) \\ + \frac{N1 \times P_{max} - \left( M_p + M_c \times Nb_{cm1} + C_{pu} \times N1 + \sum_{i=1}^{N1} C_{rku} \times \left( \frac{\lambda_i}{\lambda_{N1+1}} \right) \right)}{N1 \times Proc + \mu_p + \mu_c \times Nb_{cm1}} \quad (7)$$

With N1 is the number of batched produced during the extra period T1. N1 is given by (8):

$$N1 = E \left( \frac{T1 - \mu_c \times Nb_{cm}}{Proc} \right) \quad (8)$$

**Case 2:** The end of the last preventive maintenance action coincides with the end of the finite horizon H.

Formally :  $T1 = H - E \left( \frac{H}{T} \right) \times T = 0$

The expected total profit by time unit for this integrated model is given by (9):

$$PT_{2case2} = \frac{SP - (M_p + C_{cm} + C_p + C_{rk} + C_{setup})}{T} \times E \left( \frac{H}{T} \right) \quad (9)$$

Using expressions of SP, Ccm, Cp, Crk and T developed in the section (3.2), the total profit per time unit PT2case1 is given by (10):

$$PT_{2case2} = \left( \frac{N \times P_{max} - \left( M_p + M_c \times Nb_{cm} + C_{pu} \times N + \sum_{i=1}^N C_{rku} \times \left( \frac{\lambda_i}{\lambda_{N+1}} \right) + C_{setup} \right)}{N \times Proc + \mu_p + \mu_c \times Nb_{cm}} \right) \times E \left( \frac{H}{T} \right) \quad (10)$$

Using the indicator function, we can deduce the expression of the total cost per time unit for the second strategy PT<sub>2</sub>:

$$PT_2 = PT_{2case1} \times I_{(T1 = H - E \left( \frac{H}{T} \right) \times T \neq 0)} + PT_{2case2} \times I_{(T1 = H - E \left( \frac{H}{T} \right) \times T = 0)} \quad (11)$$

with

$$I_{(T1 = H - E \left( \frac{H}{T} \right) \times T \neq 0)} = 1 \text{ if } T1 = H - E \left( \frac{H}{T} \right) \times T \neq 0 \text{ and equal to zero otherwise.}$$

$$I_{(T1 = H - E \left( \frac{H}{T} \right) \times T = 0)} = 1 \text{ if } T1 = H - E \left( \frac{H}{T} \right) \times T = 0 \text{ and equal to zero otherwise.}$$

Then, the expected total profit per time unit for this strategy is written as:

$$\begin{aligned}
 PT_2 = & \left[ \left( \frac{N \times P_{max} - \left( M_p + M_c \times Nb_{cm} + C_{pu} \times N + \sum_{i=1}^N C_{rku} \times \left( \frac{\lambda_i}{\lambda_{N+1}} \right) + C_{setup} \right)}{N \times Proc + \mu_p + \mu_c \times Nb_{cm}} \right) \times E \left( \frac{H}{T} \right) \right. \\
 & + \left. \frac{N1 \times P_{max} - \left( M_p + M_c \times Nb_{cm1} + C_{pu} \times N1 + \sum_{i=1}^{N1} C_{rku} \times \left( \frac{\lambda_i}{\lambda_{N1+1}} \right) \right)}{N1 \times Proc + \mu_p + \mu_c \times Nb_{cm1}} \right] \\
 & \times 1_{(T1 = H - E \left( \frac{H}{T} \right) \times T \neq 0)} \\
 & + \left[ \left( \frac{N \times P_{max} - \left( M_p + M_c \times Nb_{cm} + C_{pu} \times N + \sum_{i=1}^N C_{rku} \times \left( \frac{\lambda_i}{\lambda_{N+1}} \right) + C_{setup} \right)}{N \times Proc + \mu_p + \mu_c \times Nb_{cm}} \right) \right. \\
 & \left. \times E \left( \frac{H}{T} \right) \right] \times 1_{(T1 = H - E \left( \frac{H}{T} \right) \times T = 0)}
 \end{aligned}$$

with

$$C_{pu} + C_{rku} \times \left( \frac{\lambda_i}{\lambda_{N+1}} \right) \leq P_{max} \text{ for all } i \in 1..N \tag{12}$$

#### 4. NUMERICAL STUDY

Numerical examples are presented in this section for both first and second strategy in order to illustrate the use of these analytical models developed in the previous section.

The input data for the numerical examples are summarized in table (1) below (“mu” stand for monetary units and “tu” for time unit).

Table 1. Numerical Data

System failure	Weibull distribution Shape parameter β=2 Scale parameter c=100
Repair time	Exponential distribution Rate parameter η = 0.06
P <sub>max</sub> (mu)	450
M <sub>p</sub> (mu)	500
M <sub>c</sub> (mu)	1200
Proc (tu)	3
C <sub>pu</sub> (mu)	100
C <sub>rku</sub> (mu)	150
C <sub>setup</sub> (mu)	150
μ <sub>p</sub> (mu)	10
μ <sub>c</sub> (mu)	$\frac{1}{\eta} = 16.66$
H	400

We use a numerical procedure in order to solve problems formulated by equations (5) and (12). MATHEMATICA software has been used to perform calculations and obtain the optimal solution for any given instance of the problem. Optimal results, for the first approach, are presented in table 2. This table shows that, for the first strategy, the preventive maintenance action must be taken every 18 batches.

Table 2. Optimal solution for the first strategy

N*	PT <sub>1</sub> *
18	76.06

The next step consists in optimizing the total net profit by time unit PT2 for a finite horizon H. Optimal results for the second strategy are presented in table 3. This table shows that for the second strategy the preventive maintenance action must be taken every 6 batches.

Table 3. Optimal solution for the second strategy

N*	PT <sub>1</sub> *
6	750.269

### 5. SENSITIVITY STUDY

In this section, we study the sensitivity of models to the variation of some parameters including the selling price, reworking cost and setup cost for different strategies.

Detailed results of the variation of models parameters are presented in table (4), (5), (6) (7) and (8).

Table 4. Result of the variation of  $P_{max}$  for the first strategy

$P_{max}$	$N^*$	$PT_1^*$
250	14	23.14
450	18	76.06

Table 5. Result of the variation of  $C_{rku}$  for the first strategy

$C_{rku}$	$N^*$	$PT_1^*$
150	18	76.06
250	14	72.3

Given the results shown in tables (4) and (5), we note that when we increase the selling price in the first strategy, for a fixed reworking cost, the number of batches produced before making a preventive maintenance increases and the total expected profit for a full cycle by time unit increases. On the other hand, the increase of the unit reworking cost  $C_{rku}$  results the decrease of the number of batches produced before preventive intervention as well as the total profit.

Table 6. Result of the variation of  $C_{setup}$  for the first strategy

$C_{setup}$	$N^*$	$PT_2^*$	Number of PM interventions
150	5	750.269	15
600	8	548.867	12

Table 7. Result of the variation of  $C_{rku}$  for the second strategy

$C_{setup}$	$C_{rku}$	$N^*$	$PT_2^*$
700	150	9	529.506
700	400	8	489.719

For the second strategy, we note that when we increase the unit reworking cost, the optimal number of batches produced before preventive maintenance actions decreased as well as the total profit per time unit. In addition, when we increase the setup cost, the number of batches produced before the preventive maintenance intervention increased but the total profit decreased as well as the number of preventive maintenance interventions during the horizon H.

We summarize the sensitivity study results as following:

Table 8. Impact of the variation of  $P_{max}$ ,  $C_{rku}$  and  $C_{setup}$ .

The first strategy	$P_{max}$	↗	$N^*$	↗
	$C_{rku}$	↗	$N^*$	↘
The second strategy	$C_{rku}$	↗	$N^*$	↗
	$C_{setup}$	↗	$N^*$	↗

### 6. CONCLUSION

Integrated maintenance has recently become an important research area and integrated models are expected to provide cost savings compared to disjoint models. This paper investigates integrated models joining maintenance and quality. We consider a single machine subject to random failure rate and producing conforming and non-conforming products. A preventive maintenance policy with minimal repair is applied with non-negligible durations of preventive and corrective maintenance actions. Two analytical models are developed in order to determine the optimal number of batches produced before the preventive maintenance action under two different strategies. For the first strategy, we propose a rework task for deteriorated products in order to ameliorate their quality condition and sell all batches at the best price  $P_{max}$ . Our objective in this strategy is to determine the optimal number of batches produced maximizing the total net profit per time unit  $PT_1$  over an infinite horizon. For the second strategy, we studied the same problem but over a finite horizon H. We integrate a setup cost and we aim to determine also the optimal number of batches produced before the preventive maintenance as well as the number of PM intervention during the finite horizon. The first contribution of our study, comparing to the literature, is the planning of the preventive maintenance action after a number of batches produced contrarily to previous works in which the preventive maintenance action is related to the time. In addition, the preventive and corrective maintenance actions have non negligible durations with a random repair time following the exponential distribution. The second contribution consists on integrating reworking activities to improve product quality in order to be sold at the best price. Numerical examples and sensitivity studies are presented in order to illustrate analytical models proposed and understand the impact of the variation of some key parameters on the optimal solution. Extensions to



this work are under consideration. One of them consists in the investigation of situations in which preventive maintenance actions are imperfect.

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