The Use of the Queueing Theory for the Analysis of Transport Processes

Marianna Jacyna,  
_Warsaw University of Technology, Faculty of Transport_

Jolanta Żak,  
_Warsaw University of Technology, Faculty of Transport_

Piotr Gołębiowski  
_Warsaw University of Technology, Faculty of Transport_

This article focuses on the application of the queueing theory for the analysis of transport processes with regard to rail traffic. Selected aspects of queueing theory application for the analysis and rail traffic flow assessment are presented. A literature review regarding process modelling in rail transport using mass service theory models was performed. In order to study rail traffic flow, the application Java Modelling Tools- JSIM graph was used. The train movement process was analysed with regard to the traffic flow. The study was conducted on the basis of selected stretches of railway lines, numbers 2 (central long-distance in Warsaw) and 448 (central suburban in Warsaw).

**Keywords:** modelling processes, rail transport, mass service theory, traffic flow.

1. **INTRODUCTION**

Methods of solving decision problems in transport systems can be separated into optimization and simulation. In order to solve optimization tasks formulated using transport process models, it is necessary to apply appropriate method of searching in the solution domain. Conducting research in terms of various goals requires a behavioural representation of a system as well as its individual elements. In order to test dynamic transport models, it is necessary to build systems which simulate their operation. These efforts are undertaken since carrying out studies on real facilities is not possible, or there would be high costs associated with performing this type of research. One of the basic methods applied to the study of transport processes is the mass service theory, or in other words, the queueing theory. The queueing theory studies processes where the need to carry out certain tasks (services) arises on one side, and the need to fulfil them arises on the other side. This theory, encompassing so-called queue systems, has the objective to search for analytic dependencies. In turn, these dependencies allow us to set parameters characteristic of service processes completed in systems. Determination of mass service theory characteristics requires knowledge about the kind and type of service requests and types of service schedules.

Due to this fact, the mass system models may only be applied in few instances, however, for the remaining instances it is necessary to apply the simulation research technique.

For the classic mass service theory (MST) areas of application, we can include:

- rail traffic engineering, including planning train traffic on a single-track railway line, planning passenger train servicing at holding
The Use of the Queueing Theory for the Analysis ...

Logistics and Transport No 1(41)/2019

stations or planning the train composition at
marshalling yards in particular [9, 10, 11, 25, 27],
− determining the number of vehicles necessary
to service a given region in variable demand
conditions for transport [16, 17],
− optimizing the run frequency of vehicles
based on the type of given route, taking into
account the cost and completion quality of
service [15, 16],
− setting the length of light signal cycle
composition [2, 14],
− assessing the consequences of infrastructure
parameter changes (for example: the number
of traffic lanes or changes in geometric road
parameters) on the delays arising during rush
hour, as well as delays and congestion caused by
traffic accidents and incidents involving elements
of linear transport infrastructure [3, 13], etc.

Many researchers applied the mass service
theory to describe railway transport processes. J.
Leszczyński in publication [10] defined the term
“process phase networks” (denoted in the form of
simple mass service system networks) for the first
time and introduced the possibility of applying this
approach to plan railway traffic on a single-track
railway line, planning passenger train servicing at
holding stations or planning train composition at
marshalling yards.

In the railway segment model presented in
publication [11], elements of nodal infrastructure
(railway stations) were modelled using TMO. A
random railway station represented a system of mass
service containing three subsystems: the station
entrance, service at the station and the station exit.

In articles [28, 30] J. Żak indicated that there exists
a possibility of applying the process phase network
method (containing TMO) in order to solve problems
arising not only in railway traffic engineering, but
also problems from other transport processes. J.
Woch also used the mass service theory to model
processes in railway transport. In publication [25],
he suggested applying mobile buffer models and the
maximum expected traffic flow.

In research papers on this topic, authors most
often assumed that requests in transport processes
have the character of Poisson’s schedule, however
Węgierski [22] and Woch used various schedules
in railway traffic engineering. In this paper, models
containing process phase networks were used for
analysis [1, 6, 11, 29].

The subject of this article is application of the
queueing theory to assess the completion of transport
processes with regard to rail traffic. The process of
train movement based on traffic flow was analysed.
The study was conducted on selected stretches of
railway lines, numbers 2 (central long-distance in
Warsaw) and 448 (central suburban in Warsaw).

2. APPLICATION OF MASS SERVICE
THEORY TO STUDY RAILWAY
TRANSPORT PROCESSES

Creating queues in railway transport results from
factors depending on:
− the infrastructure: implementing interim speed
limits (not included in the timetable- exceeded
traffic product at the train crossing , damage
to the track, track bed and other structures),
introducing intensive modernization work and
emergency track repairs, traffic control device
malfunction, a break in the conductor rail,
among others,
− the superstructures: vehicle with different
parameters than in the timetable intended
for servicing a train, no possibility to make
use of the full power of the vehicle, vehicle
malfunction, driving the vehicle against the
style imposed in the timetable (prolonged
driving time),
− the carrier: prolonged exchange of travellers,
unplanned exchange of conductor or contact
line crews, prolonged customs clearance at
the border, realization of communication
conditions, delayed notification of readiness
to depart, foreign train delay,
− other reasons: associated with traffic
engineering (late permission signal projection
on the semaphore, incorrect traffic organization,
lack of crew at the post, etc.), associated
with the timetable (incorrect construction,
transport of oversize or hazardous cargo),
accidents (train, with vehicles, with people),
elements (atmospheric events, fires, floods
and others) and other events (emergency
service intervention, strikes, terrorist threats),
thefts, devastations and others.

The factors listed above are the reason why
it is essential to use stochastic parameters and
characteristics- variable in time, on the base of which
steering algorithms are constructed and which are
used to model infrastructure and transport processes to define queue problems in railway transport systems. One of the methods which considers the stochastic character of events is the theory of mass service, or in other words, queueing theory.

The essence of the mass service theory is made up of the following elements:

1. Request – the queueing theory assumes a random nature of service request creation, i.e. the time interval between adjacent moments of incoming requests into the system is a positive random variable. Other studied quantities are similar in nature.

2. Input request stream – the sequence of consecutive time intervals between adjacent moments of requests received by the system.

3. Service device – a person or device which performs services. The service time is also a positive random variable, and similarly to the input request stream, it is assumed that service devices work independently of each other and the service time of each device has the same timetable.

4. Queue – the queue in the service system arises when it is not possible to service all requests at a given time since all service devices are occupied at that time.

The basic condition for applying analytical methods is the assumption that the request stream is:

- a stationary process – the probability of incoming requests depends only on the length of time intervals, but does not depend on their position on the timeline,
- memoryless – the probability of k requests in a given time interval does not depend on the number of requests and how they occurred up to that point,
- single – two or more requests cannot appear at the same time,

Formulating the queue model, the following should be defined:

- type of probability distribution of random variables;
- dependence or independence of random variables of request waiting time and service time;
- limited or unlimited value of the number of service stations, queue length;
- the system’s service discipline.

The random variables which appear in the analysed model are of the type:

1. the time passed between the input requests;
2. the service time of one request by the service station;
3. the number of service stations;
4. the number of spots in the queue awaiting service.

A set of interconnected queue systems, where requests are transferred realizing the service demand, form a network of simple systems of mass service. It should be noticed that analogous elements to the transport network appear in the queue network, which makes it possible to apply concepts and claims from transport networks to describe the properties of queue networks (Table 1).

The most commonly used measures to assess the characteristics of a queue system in application to transport systems are:

- the side of flow determined on the basis of client service intensity within the system,
- indicators of congestion described as separation among clients or traffic density,

Table 1. Comparison of the transport network with queue network

<table>
<thead>
<tr>
<th>Elements of the network</th>
<th>Queue network</th>
<th>Transport network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>input request (source)</td>
<td>vertex with only outgoing edges</td>
</tr>
<tr>
<td>Output</td>
<td>output request</td>
<td>vertex with only incoming edges</td>
</tr>
<tr>
<td>Flowrate</td>
<td>request stream</td>
<td>flow volume</td>
</tr>
<tr>
<td>Vertex</td>
<td>queue system</td>
<td>transport node</td>
</tr>
<tr>
<td>Edge</td>
<td>possible transfer</td>
<td>are between nodes</td>
</tr>
<tr>
<td>Direction of the edge</td>
<td>direction of transfer</td>
<td>flow direction</td>
</tr>
<tr>
<td>Value describing the edge</td>
<td>probability of transfer 0&lt;p≤1</td>
<td>capacity</td>
</tr>
</tbody>
</table>

Source: developed based on [4]
3. MODEL OF TRAINS MOVEMENT MAPPING ON THE RAILWAY NETWORK

The queuing theory can be used to analyse the train traffic on a selected part of the rail network for its liquidity - the train’s ability to overcome the analysed routes without commercial stops (mainly for travellers exchange) For the purposes of the study, it is assumed that:

− travelling unit in the network is a train,
− trains begin their route on the initial station and terminate on the final station,
− elements of the model can be forwarding offices (stations and passenger stops), where trains can stop,
− analysis is carried out for a specified period of time - e.g. 24 hours, and adopts the following limitations:
  − related to the track layout occurring at the stations and on the open lines (the number of tracks should be mapped at the operating control points and lines),
  − travel time of trains between forwarding offices should be specified,
  − stopping time in forwarding offices should be specified,
− traffic dependencies should also be appropriately mapped (control command and signalling on the stations and on the open lines, non-simultaneity of arrival and departure, the existence of track closures and closures of traffic stations, etc.).

Model of trains movement mapping can be created for different configurations of the stations and open lines. Fig. 1 shows an example of a section of a railway line which is analysed.

Fig. 1 shows a diagram of a section of a single track railway line. The model includes the beginning, final and two intermediate station and three single track open lines. Trains to pass the analysed segment can appear both at the beginning and final station according to the specified timetable. Individual elements of the line system may be reproduced using the simple queue systems.

For the single-way railway line there is a characteristic way of riding on the open line. At the moment there can be only one train on the open line. There are exceptions where self-locking devices are installed, where there may be more than one train on the open line but only in the same direction. There is always the possibility to ride trains from the opposite direction. Thus, the model of train traffic on a single track railway line, which is not equipped with an automatic line block system, can be presented as in Fig. 2.

For the analysed scheme the initial condition must be assumed: expedition of the train in one direction prevents expedition of the train from the opposite direction. In the real system it is done by telephone pre-announcement of trains or handling of automatic line block system. In the model shown in Fig. 2, the diagram traffic conduction from beginning station to station A is illustrated with the red arrows. The first queue station is the exit semaphore B from the beginning station. The train is waiting near it.

![Diagram of a section of a single track railway line](source: own work)
for departure to the open line (1). Upon receiving the signal, the train will be operated in the queue system B (2) and prevention from leaving train from the opposite direction (hence the connection (2) from the system B will cause the occupancy of the service station in B and in D). The departure of the train begins (3), which terminates at the moment of covering the route by signal prohibiting driving on the semaphore. The train follows the route (it is operated) and after reaching the entrance semaphore of station A (4) begins its operation in the queue system E (5). Train operation, i.e. the display on the entry semaphore permit signal, is the same as the release of the operating position in the D system. This means that there is a possibility to pass the train in the opposite direction. After the service (covering the train by a forbidden signal), the train occupies the platform (6). The same sequence of actions can be distinguished for the opposite direction.

Fig. 3 shows the section of the railway line from Fig. 1 presented using simple queue systems. The set of elements shown in Fig. 2 is labelled by A in the square.

4. TESTING OF TRAFFIC FLUIDITY OF TRAINS ON THE SELECTED RAILWAY LINES USING THE QUEUE THEORY

As mentioned earlier, the subject of the article is the application of the queue theory to assess the implementation of transport processes on the example of railway traffic - the analysis was carried out on the process of trains moving due to traffic fluidity. The problem is searching for the minimal time for train calls from the Warsaw Grochow station and from the east lines, for which no queue is being constructed on the analysed railway network. In the article, the following open lines were conducted:

- Warsaw East – Warsaw West – railway line 448 (Warsaw West – Warsaw Rembertow) /tracks dedicated for suburban passenger traffic, direction opposite to the fundamental/,
- Warsaw East – Warsaw Central and Warsaw Central – Warsaw West – section of railway line 2 – Warsaw West – Terespol) / tracks dedicated for domestic passenger traffic, direction opposite to the fundamental/.
The diagram of the analysed sections of railway lines is shown in Fig. 4.

The analysed open lines are characterized by the following parameters:

1. Open line Warsaw East – Warsaw West (section of railway line 448):
   - Length of the open line – 7,189 km;
   - Average typical train travel time of the open line – 14 min (including stops on the passenger stops: Warsaw Stadium, Warsaw Powisle, Warsaw Srodmiescie and Warsaw Ochota);
   - Type of linear motion control devices – four-aspect automatic line block system (split of the track at 11 spacing);
   - Average length of the one spacing – 654 m;
   - Average travel time of the one spacing – 1.3 min;
   - Number of platform edges at beginning station – 3;
   - Number of platform edges at final station – 3;

2. Open line Warsaw East – Warsaw West (section of railway line 2):
   - Length of the open line – 4,254 km;
   - Average typical train travel time of the open line – 6 min;
   - Type of linear motion control devices – three-aspect automatic line block system (split of the track at 6 spacing);
   - Average length of the one spacing – 709 m;
   - Average travel time of the one spacing – 1 min;
   - Number of platform edges at beginning station – 5;
   - Number of platform edges at final station – 4.

For the purposes of the study, the model for the selected part of the real railway transport system was constructed for the above mentioned data. It is assumed that point elements of the railway infrastructure will be mapped by vertices, while elements of linear infrastructure by edges. The elements of the model mapping the elements of the railway network are presented in Table 2.

Fig. 5 shows graphical model of selected railway open lines in the form of simple mass-service network systems.

The parameters of the individual components of the model (see Fig. 5) in the basic state are presented in Table 3. The distribution of the request flow and service time was based on the analysis of the available train timetable (values were expressed in minutes).
Table 2. Elements of the model mapping the elements of the railway network

<table>
<thead>
<tr>
<th>Type of element</th>
<th>Number of element ( v )</th>
<th>Name of element</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source no 1</td>
<td></td>
<td>flow of trains reporting to the Warsaw East from the Warsaw Grochow station and from the east lines</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>suburban platforms at the station Warsaw East</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>1(^{st}) interval of automatic line block system on the Warsaw East - Warsaw West open line</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>2(^{nd}) interval of automatic line block system on the Warsaw East - Warsaw West open line</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>3(^{rd}) interval of automatic line block system on the Warsaw East - Warsaw West open line</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>4(^{th}) interval of automatic line block system on the Warsaw East - Warsaw West open line</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>5(^{th}) interval of automatic line block system on the Warsaw East - Warsaw West open line</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>6(^{th}) interval of automatic line block system on the Warsaw East - Warsaw West open line</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>7(^{th}) interval of automatic line block system on the Warsaw East - Warsaw West open line</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>8(^{th}) interval of automatic line block system on the Warsaw East - Warsaw West open line</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>9(^{th}) interval of automatic line block system on the Warsaw East - Warsaw West open line</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>10(^{th}) interval of automatic line block system on the Warsaw East - Warsaw West open line</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>interval of automatic line block system before entering the group of suburban tracks on the station Warsaw West</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>suburban platforms at the station Warsaw West</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>long-distance platforms at the station Warsaw East</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>1(^{st}) interval of automatic line block system on the Warsaw East - Warsaw Central open line</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>2(^{nd}) interval of automatic line block system on the Warsaw East - Warsaw Central open line</td>
</tr>
<tr>
<td>17</td>
<td></td>
<td>3(^{rd}) interval of automatic line block system on the Warsaw East - Warsaw Central open line</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>4(^{th}) interval of automatic line block system on the Warsaw East - Warsaw Central open line</td>
</tr>
<tr>
<td>19</td>
<td></td>
<td>5(^{th}) interval of automatic line block system on the Warsaw East - Warsaw Central open line</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>interval of automatic line block system before entering to the station Warsaw Central</td>
</tr>
<tr>
<td>21</td>
<td></td>
<td>platforms at the station Warsaw Central</td>
</tr>
<tr>
<td>22</td>
<td></td>
<td>1(^{st}) interval of automatic line block system on the Warsaw Central - Warsaw West open line</td>
</tr>
<tr>
<td>23</td>
<td></td>
<td>2(^{nd}) interval of automatic line block system on the Warsaw Central - Warsaw West open line</td>
</tr>
<tr>
<td>24</td>
<td></td>
<td>interval of automatic line block system before entering to the station Warsaw West</td>
</tr>
<tr>
<td>25</td>
<td></td>
<td>long-distance platforms at the station Warsaw West</td>
</tr>
<tr>
<td>Estuary no 1</td>
<td></td>
<td>flow of trains leaving Warsaw West to Warsaw Ochota and on the lines in the west side</td>
</tr>
</tbody>
</table>

Source: own work
5. APPLICATION OF THE JAVA MODELING TOOLS COMPUTER PACKAGE - JSIM GRAPH FOR TRAFFIC FLUIDITY TESTS ON THE RAILWAY LINE

The Java Modelling Tools - JSIMgraph application was used to analyse the fluidity of train traffic on the analysed open lines of selected railway lines. Fig. 6 shows a model of the analysed routes on selected railway lines in the form of simple mass-service network systems using Java Modelling Tools.

The model entered into the program was parameterized on the basis of available data (e.g. train timetable). The values of parameters introduced into the model are presented in Table 3. Due to the specificity of the railway traffic, the distribution of train calls from the Warsaw Grochow station and the east lines was described by the Erlang distribution. To the distribution of the service time on individual stations we used the normal distribution.

The results of the performed simulations for different call distributions of trains are shown in Table 4.

On the basis of the obtained results, it can be stated that the number of serviced trains is not greater than unity at the time of train reporting of 5 minutes at any station. At stations (Warsaw East, Warsaw Central and Warsaw West) appeared queue times greater than zero. However, it is not greater than 0,05 minutes. This means that the fluidity of

Table 3. Model parameters of the railway network developed using simple systems of mass service

<table>
<thead>
<tr>
<th>Model element</th>
<th>Distribution of applications</th>
<th>Waiting capacity</th>
<th>Rules of the queue</th>
<th>Number of service stations</th>
<th>Distribution of service time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = 1$</td>
<td>erl(5,25)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$v = 1$</td>
<td>–</td>
<td>0</td>
<td>FIFO</td>
<td>3</td>
<td>norm(1,0,5)</td>
</tr>
<tr>
<td>$v = 2$</td>
<td>–</td>
<td>0</td>
<td>FIFO</td>
<td>1</td>
<td>norm(2,0,5)</td>
</tr>
<tr>
<td>$v = 3, 4, 5, 6, 7$</td>
<td>–</td>
<td>0</td>
<td>FIFO</td>
<td>1</td>
<td>norm(1,0,5)</td>
</tr>
<tr>
<td>$v = 8$</td>
<td>–</td>
<td>0</td>
<td>FIFO</td>
<td>1</td>
<td>norm(1,5,0,5)</td>
</tr>
<tr>
<td>$v = 9, 10, 11, 12$</td>
<td>–</td>
<td>0</td>
<td>FIFO</td>
<td>1</td>
<td>norm(1,0,5)</td>
</tr>
<tr>
<td>$v = 13$</td>
<td>–</td>
<td>0</td>
<td>FIFO</td>
<td>3</td>
<td>norm(1,0,5)</td>
</tr>
<tr>
<td>$v = 14$</td>
<td>–</td>
<td>0</td>
<td>FIFO</td>
<td>5</td>
<td>norm(3,1)</td>
</tr>
<tr>
<td>$v = 15, 16, 17, 18, 19, 20$</td>
<td>–</td>
<td>0</td>
<td>FIFO</td>
<td>1</td>
<td>norm(1,0,5)</td>
</tr>
<tr>
<td>$v = 21$</td>
<td>–</td>
<td>0</td>
<td>FIFO</td>
<td>4</td>
<td>norm(4,1)</td>
</tr>
<tr>
<td>$v = 22, 23, 24$</td>
<td>–</td>
<td>0</td>
<td>FIFO</td>
<td>1</td>
<td>norm(1,0,5)</td>
</tr>
<tr>
<td>$v = 25$</td>
<td>–</td>
<td>0</td>
<td>FIFO</td>
<td>4</td>
<td>norm(1,0,5)</td>
</tr>
<tr>
<td>$b = 1$</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Source: own work
traffic is preserved. Under the time of train reporting of the 4 minutes also the maximum number of simultaneously serviced trains at each station was not exceeded, while the time of the queue was increased and at one of the stations was equal to 0.37 minutes. It can be assumed that this time is acceptable and the flow of traffic is preserved. On the other hand, a queue of 82.8 trains and a queue time of 12 minutes took place at one reporting time of 2 minutes. This means that the movement is not smooth. In summary, the rational reporting time is a 3 minute stream.
6. SUMMARY

The mass service theory is often used to study processes in transport where queues are formed. One of the areas in which these types of problems arise is railway transport. The organization of railway traffic developed by the infrastructure manager assumes that all processes are completed according to a previously prepared plan. However, in real operation, many different disruptions may arise- for example, rail traffic control, command and signalling device malfunction. Using the queueing theory, it is possible to predict the results of these types of occurrences, and at the same time prepare loss-absorbing operations.

This article presented the application of the queueing theory to assess the realization of transport processes on the example of train movement based on traffic flow. The study was conducted on the basis of selected stretches of railway lines, numbers 2 (central long-distance in Warsaw) and 448 (central suburban in Warsaw). An ideal situation was considered, where all devices are functioning and there are no disruptions in traffic flow. The studies showed that the traffic on the railway line was continuous at the time. The average number of trains serviced at each respective station does not exceed 0.5, queues do not form and the average number of serviced trains at once in the system was near 4.85.

REFERENCES:


[5] Han, S.: Dynamic traffic modelling and dynamic stochastic user equilibrium assignment for general road networks, Transportation Research 37B.


The Use of the Queueing Theory for the Analysis ...


Date submitted: 5-03-2019
Date accepted for publishing: 2019-04-23

Marianna Jacyna,
Warsaw University of Technology, Poland
maja@wt.pw.edu.pl

Jolanta Żak,
Warsaw University of Technology, Poland
logika1@wt.pw.edu.pl

Piotr Gołębiowski
Warsaw University of Technology, Poland
pgolebiowski@wt.pw.edu.pl