

Lies, Damned Lies, and Statistics? Examples from Finance and Economics

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Abstract

Reliable data analysis is one of the hardest tasks in sciences and social sciences. Often misleading and sometimes puzzling results arise when the analysis is done without regard for the special features of the data. In this exposition, I will focus on designing new statistical tools to deal with some prominent questions in Finance and Economics. In particular, I will talk about the following. (1) How to characterize the randomness of variables, motivated by a problem in the pricing of financial options. (2) Uncovering the relation between interest rates on different maturities, now and in the future; the "term structure of interest rates". (3) Modelling the unconventional nonlinear long-memory dynamics that arise from a general-equilibrium economic model, and their implications for exchange rates, stock market indexes, and all macroeconomic variables; with recommendations for trading in financial markets, but also for the design of macroeconomic stabilization policies by governments.

Keywords: flexible density specification, option pricing, term structure of interest rates, expectation hypothesis, nonlinear long-memory, macroeconomic dynamics

JEL Classification: C10, C5, C32, G1, E3, E6

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1 Introduction

As the saying goes, lies can be classified into the following, from bad to worse: lies, damned lies, and statistics (the worst of all). But is this saying really true? Many puzzles in finance and economics have accumulated over the years. Seemingly reasonable economic theories have been badly rejected by the data, giving rise to suspicions that something is wrong, and not just with the theories. What if the data contained some hidden features that, if neglected, could be misleading the analysis? What if these features required new statistical tools to bring them out and handle them? With a few examples collected from my research with colleagues, this paper will show how the conclusions could be quite different from what was originally thought. *Reliable* data analysis is one of the hardest tasks in sciences and social sciences. The need for it is pervasive, as this quote from John Tukey implies:

"The best thing about being a statistician is that you get to play in everyone's backyard."

A host of modern statistical tools are constantly being developed to cope with a wealth of new data coming from different disciplines, but some methods of analysis need to be tailored to cope with different environments. (Not all ailments are cured by the same medicine!) For example, econometrics has been responsible for many developments in statistical tools and methodology, especially in time series. The "backyards" that this paper will focus on are financial econometrics and empirical macroeconomics. In particular, we will touch on the following areas:

1. Statistical distribution theory: how to quantify randomness? Option-pricing requires the specification of a distribution of likely future prices. An arbitrarily-chosen functional form will give misleading results. Though from a different field of statistical analysis, the following quote from Light, Singer, and Willett (1990) illustrates what could go wrong if the chosen functional form is not flexible enough:

"You can't fix by analysis what you bungled by design."

2. The relation between interest rates on different maturities, now and in the future: the term structure of interest rates.
 - (a) Rare uncharacteristic events can generate one-off "outliers". They have distortionary effects on traditional methods of estimating relations between these interest rates.
 - (b) Short-term interest rates do *not* have the "Markovian" dynamics that prevail in the literatures on finance and time series. We first need new methods to deal with such data.

3. These new dynamics do not come out of thin air. They arise from a general-equilibrium economic model. Because of this link, it turns out that exchange rates, stock market indexes, and all macroeconomic variables are accurately characterized by this new process. This has implications for trading (momentum, cycles, etc.), but also for the ever important question of macroeconomic stabilization. The recent recession and recovery patterns have been predicted by using this approach, which has unfortunately still not made it into the toolkit of policymakers, whether central bankers or finance ministries.

2 Flexible specification of probability distributions

A flexible parametric form for probability density functions is needed to:

1. cover a wide range of possible descriptions and shapes; and
2. achieve efficiency gains over nonparametric estimation, gains that are useful for cases where there is not a lot of data.

This is needed for many statistical applications, well beyond the current context of option pricing. In fact, this work is currently being explored by the author and Michel Lubrano for another application to income distributions that are notoriously difficult to model simultaneously in the body of the distribution and its long upper tail.

A "European call option" gives you the right (but not the obligation) to buy an asset at a predetermined price at an expiration (or maturity) date. Its value depends on the distribution of likely future prices. Let S_t be the price of an asset at time t . Assume, for the moment, that the asset does not pay dividends. Suppose that this asset is underlying a European call option with expiration date T and strike price k . Then, the intrinsic value of this option at expiration is $\max\{S_T - k, 0\}$. In an arbitrage-free economy, there exists a risk-neutral density f such that the price of this call option at time t can be written as

$$c_t(k) = e^{-(T-t)r} E_t(\max\{S_T - k, 0\}) \equiv e^{-(T-t)r} \int_k^\infty (u - k) f(u) du, \quad (1)$$

where r is the continuously-compounded risk-free interest rate and E_t is the expectation taken at time t . This function c is declining in k : if k is too high, it is unlikely that the option will be worth much at expiration because $\max\{S_T - k, 0\} \approx 0$; whereas it is likely to be worth more if k is low and it is easier to achieve $S_T - k \gg 0$ at expiration. At any point in time t , we observe a series of call prices c_t , each corresponding to a different k , and we aim to find a density f in (1) such that $c_t(k)$ fits the data well.

Differentiating the integral by Leibniz' rule gives

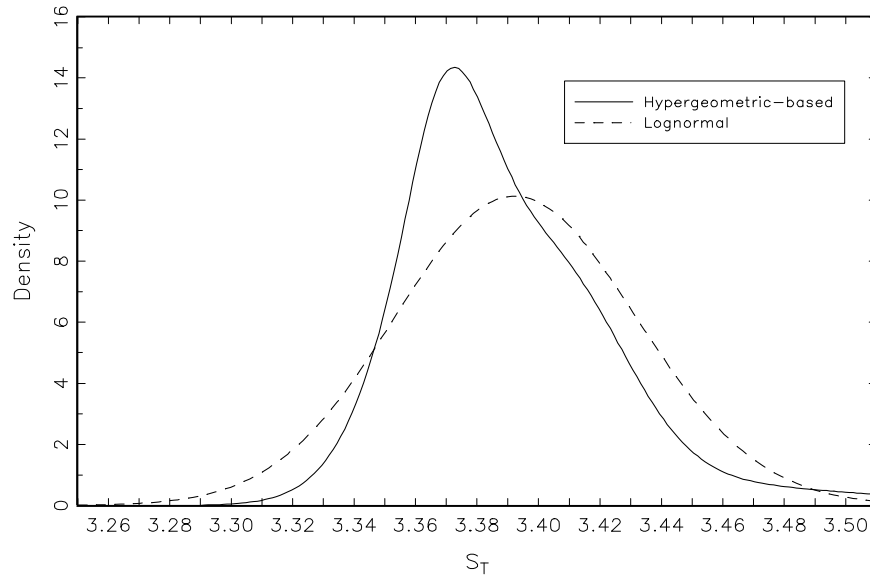
$$\frac{dc_t(k)}{dk} = -e^{-(T-t)r} \int_k^\infty f(u) du \equiv -e^{-(T-t)r} (1 - F(k)),$$

where F is the c.d.f. corresponding to the p.d.f. f . The second derivative is given by

$$\left. \frac{d^2 c_t(k)}{dk^2} \right|_{k=u} = e^{-(T-t)r} f(u),$$

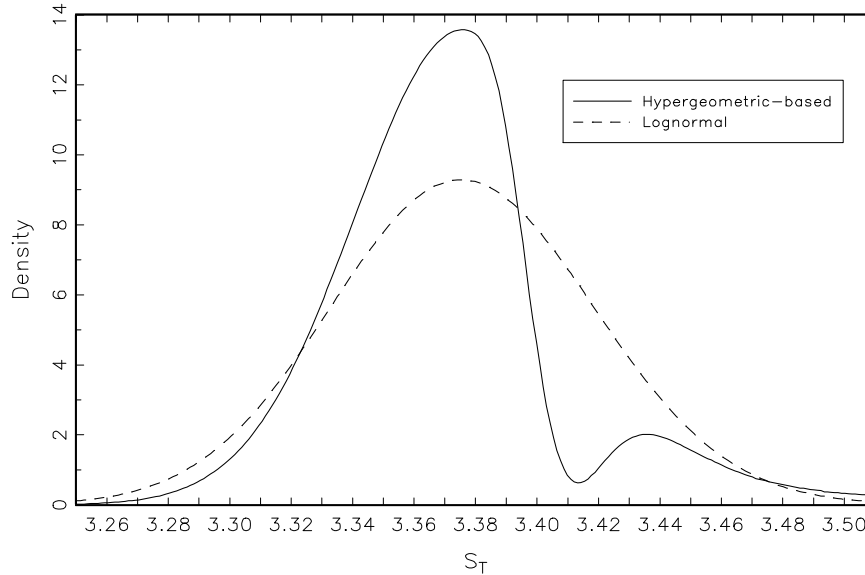
which reveals the required density f . If the asset pays a dividend yield of r^\dagger , then it should be subtracted from r and the RHSs should be multiplied by $e^{-(T-t)r^\dagger}$.

Figure 1: Lognormal and Hypergeometric-based Risk-Neutral-Densities. 17 May 96, 3-months to maturity



The Black-Scholes (1973) and Garman-Kohlhagen (1983) models assume that f is lognormal for call options on stock prices and currencies, respectively. Using an explicit general construction of f from special functions in mathematics (“hypergeometric functions”), first introduced in Abadir and Rockinger (2003), we get the following estimates of the densities f about future asset prices as implied by current option prices, by our method and by the lognormal. Figures 1 and 2 concern options on exchange rates, with the same maturities but measured at different times t . The first was during a quiet period where there was not much news coming into the market: the two shapes were not very different and both were unimodal. Then the second was after a major political announcement that left the market split on how the exchange rate would move by expiration period: the density estimate by our method suddenly became bimodal, reflecting this polarization, while the lognormal remained unimodal (by construction) and did not reveal this new feature. The finance

Figure 2: Lognormal and Hypergeometric-based Risk-Neutral-Densities. 28 April 97, 3-months to maturity



literature on the difference in beliefs and its real implications has recently received increased attention; e.g. see Beber, Breeden, and Buraschi (2010), Buraschi, Trojani, and Vedolin (2013).

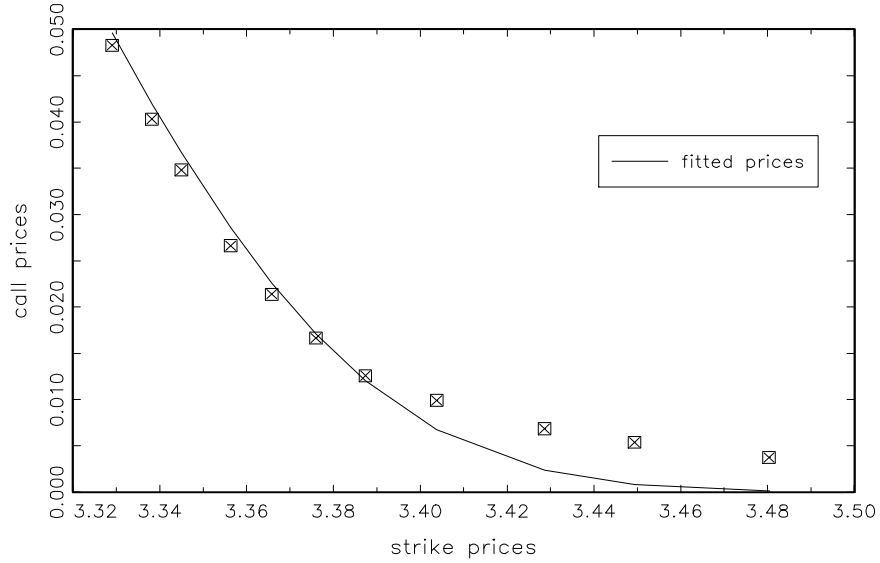
The representative fit for option prices $c_t(k)$ implied by these densities is given in Figures 3 and 4, respectively for the lognormal and hypergeometric-based functions. Similarly, when our method was applied to options on S&P 500 in Figure 5, we revealed a negative skew (due to the prevalence of large negative returns over comparable positive ones) which cannot be represented by lognormal densities that are positively skewed by construction. Note that our f can also represent positive skewness, as in our previous graphs.

There are other methods too. But they do not fit as well, and they do not detect:

1. the growing polarization in the currency market when an important event happens; and
2. the negative skew of the density of stocks (higher prevalence of negative corrections in shares).

Also, other methods force unjustified features on the estimates (e.g. number of modes, oscillatory tails, etc.) or have no explicit structure (a drawback when forecasting and/or analyzing tails, i.e. extreme events). Our method does very well, and uniformly so, for:

Figure 3: Original and fitted Garman-Kohlhagen call option prices. 28 April 97, 3-months to maturity



1. volatile days, as well as for quiet ones;
2. different maturities; and
3. options on very different underlying assets.

Our hypergeometric-based f allows for a wide range of functional forms, covering most of the commonly-used ones, and the resulting estimated functional forms (hence shapes) vary over the dates, maturities, and assets. Therefore, other parametric methods which restrict functional forms will not do uniformly well. The estimates are stable and robust, as revealed by sensitivity analysis to bid-ask spread, dependence on each data point, and so on.

Talking of robustness...

Figure 4: Original data and Hypergeometric-based fit. 28 April 97, 3-months to maturity

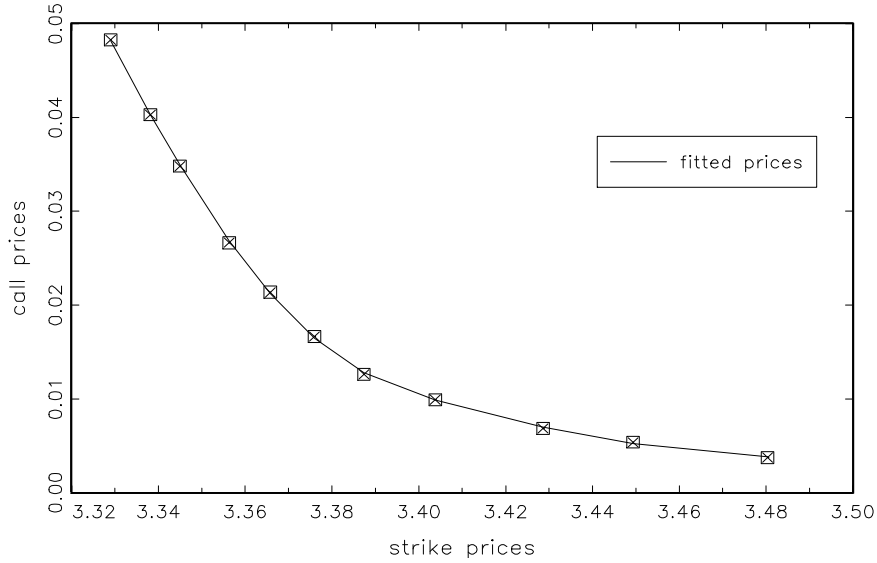
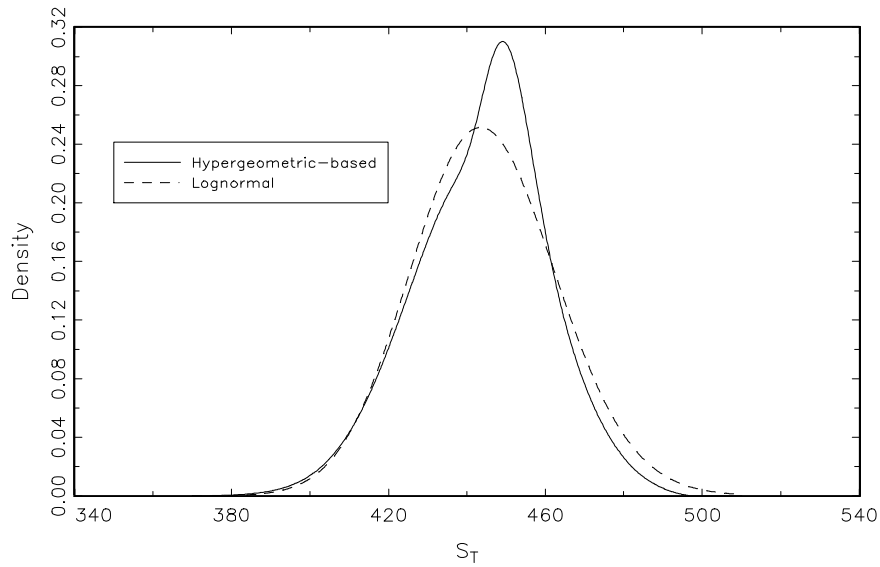


Figure 5: Lognormal and Hypergeometric-based Risk-Neutral-Densities. S&P500 index options, 3 May 93, 46-days to maturity



3 The term structure of interest rates

3.1 The expectation hypothesis

This subsection is based on Abadir and Atanasova (2013). Consider a 1-year bond and a 2-year one. In long-run equilibrium, it is reasonable to expect that what you'll make over the 2 years of the longer bond should be the same as what you would make by buying the 1-year bond now then buying another 1-year bond next year. In other words, today's difference between the total yield of the 2 and 1 year bonds is what you would expect 1-year bonds to offer next period. This is the essence of the expectation hypothesis (EH), more generally over all maturities.

Time and time again, the EH has been rejected empirically. Up to today, many papers treat deviations from the EH as specification errors of the economic model, and try to fix them. But is this really the case? Could it be a statistical artifact arising from problems with estimation and testing? And to what extent is this the case: how bad is the failure of the EH? It will turn out that the empirical rejection of the EH is largely due to a handful of outliers, corresponding to a couple of rare events that are mostly politically-driven. Once these are accounted for, there is a dramatic change in the estimates of term-structure regressions, bringing them much closer to the EH.

Let us focus on the following version of the EH:

forward rates are unbiased predictors of future rates.

Recall the example of the 1-period and 2-period bonds available *today*. The forward rate is the difference between the total yields of these two: a bank would buy the long bond and sell the short one, offering you the difference as the forward rate (by arbitrage), without any uncertainty. Then, the expectation hypothesis states that this forward rate is what you would expect the 1-period interest rate to be next period. Define $f(t, u)$ as the instantaneous forward rate that satisfies the pricing

$$p(t, T) = \exp\left(-\int_t^T f(t, u)du\right), \quad T > t,$$

for a zero-coupon bond with face value 1. Let $\pi(t, u) := f(t, u) - E_t[r(u)]$, where $r(u)$ is the spot rate at $u > t$ and E_t is the expectation taken at time t . Campbell (1986) shows that the EH with constant risk premium gives $\pi(t, u) = \pi_{u-t}$ and

$$y(t, T) = \frac{\int_t^T (\pi_{u-t} + E_t[r(u)]) du}{T-t} = \bar{\pi}_n + \frac{\int_t^T E_t[r(u)] du}{n}, \quad (2)$$

where $y(t, T)$ is the yield at time t of a bond maturing at time T , and $n := T - t$ is the time to maturity. The restrictive version where $\pi_{u-t} = 0$ (for all $u > t$) is called the "pure" EH.

A discretization of (2) gives a restriction that has been extensively tested:

$$y(t, t+n) = \bar{\pi}_n + \frac{\sum_{i=1}^n E_t[y(t+i-1, t+i)]}{n}, \quad (3)$$

where we see that the n -period rate is the average of a series of one-period expectations of future rates (the "pure" component), plus a constant risk premium $\bar{\pi}_n$ varying only with the maturity n . Here, changes in yields can only be explained by changes in the expectation of the future interest rates. If the risk premium were time-varying, yields would also depend on changes of attitudes towards risk or changes in interest rate volatility.

Interest rates are highly persistent, as we shall see in the next subsection. To reduce the possibility of spurious results, most empirical tests of the EH focus on interest-rates' differences or spreads, rather than levels. Two relations can be derived from (3). The first of these gives the seemingly worst violation of the EH, and is

$$E_t[y(t+1, T)] - y(t, T) = \alpha_n + \beta_n \frac{y(t, T) - y(t, t+1)}{n-1},$$

where $\beta_n = 1$ if the EH is satisfied. Under rational expectations, $E_t[y(t+1, T)] = y(t+1, T) - \varepsilon_{n,t+1}$ with $\varepsilon_{n,t+1}$ an unsystematic error centered around 0. The equation relates future changes in long-term interest rates $y(t+1, T) - y(t, T)$ (call this LHS z) to the slope of the yield curve $[y(t, T) - y(t, t+1)] / (n-1)$ (call this RHS fraction x) after correction for a constant term premium: $z = \alpha_n + \beta_n x + \varepsilon$.

Least squares (LS) has been used to check this hypothesis. But the function h that minimizes the squares $E(z - h(x))^2$ is $h(x) = E(z | x)$, which may or may not be $h(x) = \alpha_n + \beta_n x$ in general, and we can calculate this conditional expectation h unrestrictedly from the joint density of (x, z) . Conditional expectations are linear for distributions like elliptical ones (including normal and Student t), but nonlinear for most others; and we'll see that the scatter plot implies deviations from elliptical shapes. Nadaraya-Watson nonparametric (NP) kernel regression reveals the functional form of the conditional expectation and how far it deviates from linearity and from $\beta_n = 1$. Being a local-LS device, it also reveals the extent to which earlier results in the literature depended on a handful of extreme observations driven by unusual events: in 1981/1982 (the Volcker experiment) and 1987 (the stock market crash).

Two representative graphs for the same NP regression will follow: one showing all the data points, the other zooming into the same picture to show in more detail the fitted NP regression curve for the bulk of the data. In Figure 6, the NP regression *seems* like a flat line, except for a handful of outliers at each extremity. At the left extremity, we get the usual sensitivity of kernels to few points: the slope is positive then negative, with not much direction on average. At the right extremity, the story is different, with a noticeable drop in the level of the regression.

Now let us zoom into the body of the graph and stretch it in such a way that the two axes are of comparable scales, as the vertical axis was previously too compressed. Figure 7 shows that the NP regression line does not have a zero slope, as it seemed from the previous graph. For the x-axis ranging from 0.05 to 0.12, the NP regression is amazingly linear (this has not been imposed at the outset) with a slope of just

Figure 6: Nonparametric regression, 1-year maturity

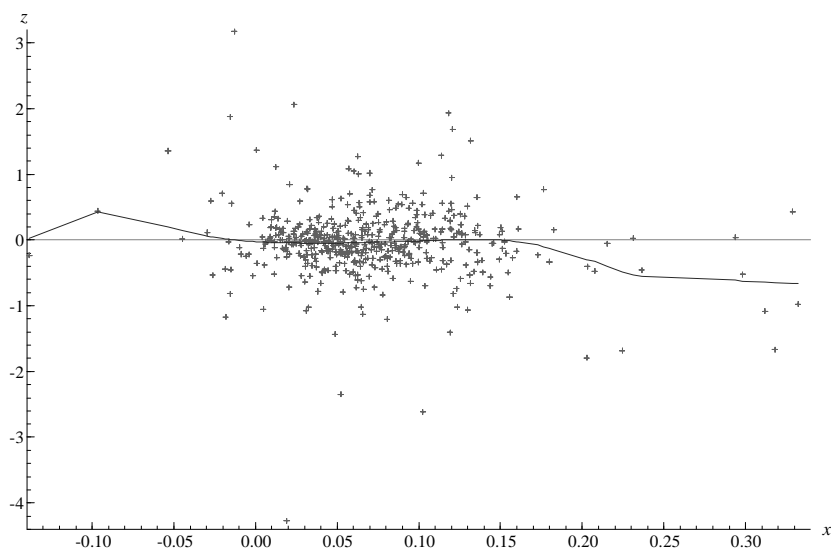
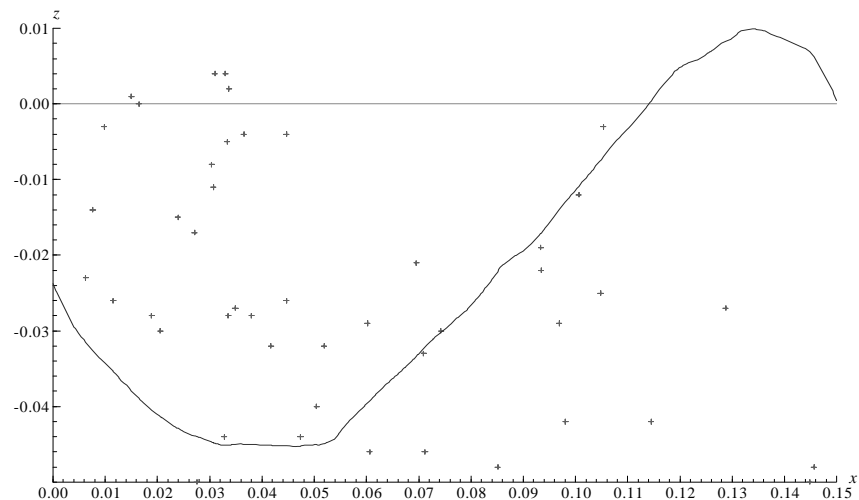


Figure 7: Same nonparametric regression as in Figure 6, zoomed-in and rescaled axes



over 0.7. Even when taking all the points on this graph, the “average” slope is little changed from 0.7. This is in sharp contrast to the massively negative number obtained by LS regression, namely -1.2 with a standard error of 0.45 leading to the rejection of the EH. We can now see that the puzzling LS result was due to the handful of extreme

values (in Figure 6) that are driving the “average” slope down, and that the slope for the bulk of the data is statistically indistinguishable from the unit coefficient that is implied by the EH. Abadir and Atanasova (2013) go beyond this descriptive NP result, by means of a new recursive approach, and find that the rejection of the EH is due to only a few observations corresponding to extremely high values of x . They quantify the point where the EH is rejected and should be supplemented by peso-type models, showing that the EH holds most of the time if such extreme x -values are properly accounted for.

3.2 Up and down it goes (dynamics of the short rate)

It is well known that interest rates are persistent. Further, it has been acknowledged relatively recently that the short-rate process contains nonlinearities. For example, Aït-Sahalia (1996) finds nonparametrically that the process is:

1. very persistent when rates are close to their historical norm; but
2. more mean-reverting when rates are far from their norm.

NP provides an exploratory tool, but it does not help with forecasts: NP is a "local" smoothing tool and is too flexible for projections into the future. We need a parametric model to be able to do so. To add to the troubles, we will see that the process is not Markovian:

1. one cannot summarize the bulk of the dynamics by conditioning on a fixed number of previous values; and
2. the representation of the conditioning structure evolves over time if (as is always the case) the process has started at a fixed point in the past.

So we cannot use such models, which are currently dominating the field. Furthermore, the process does not look like any existing non-Markovian model that we can pull off the shelf: we need new technology!

Here is a picture in Figure 8 illustrating what we’re up against. What a horrible animal! All jagged and seemingly patternless. But is this really the case?! What if we view its features in the auto-correlation domain? Figure 9 shows how each point correlates with the previous values, the auto-correlation function (ACF).

It is a smooth one, showing that interest rates go in long cycles, superimposed with shorter business cycles. There are no prizes for guessing where nominal interest rates are going next in the original Figure 8! The long cycles and the shorter ones are visible there too, though less clearly.

It turns out that this interest-rate process has a parsimonious representation in the ACF and frequency domains. The ACF is

$$\rho_\tau := \frac{\text{COV}(z_t, z_{t-\tau})}{\sqrt{\text{var}(z_t)\text{var}(z_{t-\tau})}} \approx \frac{\cos(\omega\tau)}{(1+b\tau)^c} \quad (b, c > 0). \quad (4)$$

Figure 8: United States, Policy Rates, Fed Funds Effective Rate, Average, USD

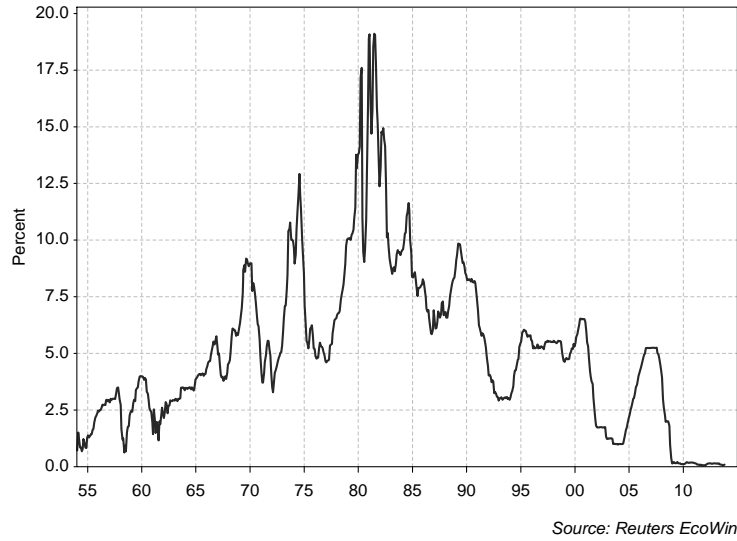
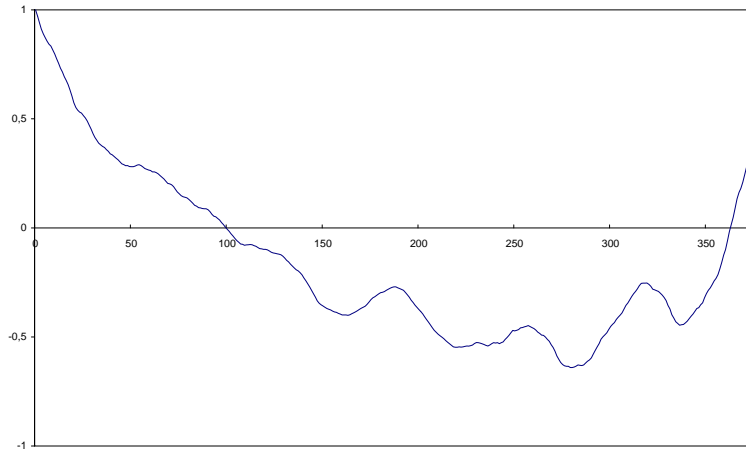


Figure 9: Auto-correlation function of short-run interest rates



Its Fourier-inversion produces a spectrum $f(\lambda)$ that is proportional to $|\lambda - \omega|^{c-1}$; that is, at frequency ω , there is a singularity when $c \in (0, 1)$. For linear ARIMA(p, d, q) processes having $d \in (0, \frac{1}{2})$, the spectrum has a singularity at the origin that is proportional to $|\lambda|^{-2d}$, giving the correspondence $c = 1 - 2d$ if $\omega = 0$ but not otherwise; see Abadir and Talmain (2011). Persistent cycles *cannot* occur if $\omega = 0$ is where the

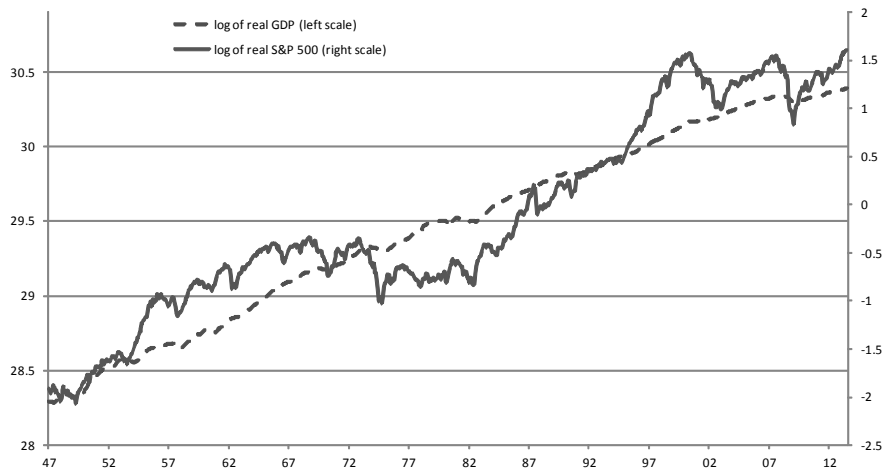
peak of the spectrum is, as is the case for a unit-root or even fractionally-integrated series. A cyclical frequency is, by definition, a strictly positive ω in the spectral representation. Frequency $\omega = 0$ is where *permanent* noncyclical components show up, including trends. A spectral peak at $\omega = 0$ may be due to the omission of the possibility of a trend when calculating a series' memory, a point that we shall return to in the next section.

The representation (4) is not arbitrarily chosen. It arises from a general-equilibrium (GE) economic model that allows for the heterogeneity of firms, introduced by Abadir and Talmain (2002). If individual units optimize their behaviour to give rise to dynamic processes that are typical in simpler representative-firm models, aggregating the result will give a very different dynamic process for the economy as a whole, similar to what we have in Figure 9 and the corresponding equation (4). Not surprisingly, most macroeconomic series and financial aggregates share this common dynamic structure, to which we now turn.

4 The new evolution

Once these nonlinear and persistent dynamics are accounted for, gone is the apparent unit root that has dominated econometrics for the past 30 years! This applies to exchange rates, stock market indexes, and all macroeconomic variables.

Figure 10: Logarithms of real US GDP and real S&P 500 over time.



First, we can illustrate this in the time domain in Figure 10, where we see that the variables are evolving around a time trend, well within finite variance bounds that don't expand over time (unlike unit-root processes). For these two variables, real GDP

& real S&P 500, Talmain (2006) shows that the aforementioned GE model implies that there is a linear relation between them in the steady state: the stock market cannot deviate indefinitely from the trendline given by the state of the economy in the long run. The graph shows that both are seen to have variance bounds that do not explode over time, once a time trend is allowed for. This explains why Cavaliere (2001) found nonparametrically that S&P 500 had less memory than a unit root would imply, once the possibility of a time trend is considered as an extension of the work of Lo (1991).

Second, we represent this evolution parametrically and give the following sample pictures, using ACFs as viewing glasses, based on Abadir, Caggiano, and Talmain (2013), ACT for short. They use a 4-parameter ACF, slightly more general than the one given in (4) where their amplification/dampening parameter turned out to be $a \approx 1$ and hence was not needed in the previous section. The selected series in Figure 11 reflect income, money, stock prices, and consumer prices. The irregular curve is the data, ACT's fitted ACF is the curve tracking it almost perfectly, and the nearly-straight line is the best fitted $AR(p)$ whose shape misguidedly implies a unit root. Clearly, the latter is not a good fit for the dynamic properties of the series, as revealed by the ACFs. There is a lot of persistence initially in the data, usually followed by a substantial loss of memory that cannot be picked up by linear ARIMA models. Furthermore, the persistent cycles are missed by ARs, as they can only represent stationary transient cycles, or nonstationary ones whose variance explodes over time, neither of which are indicated as main features of the data.

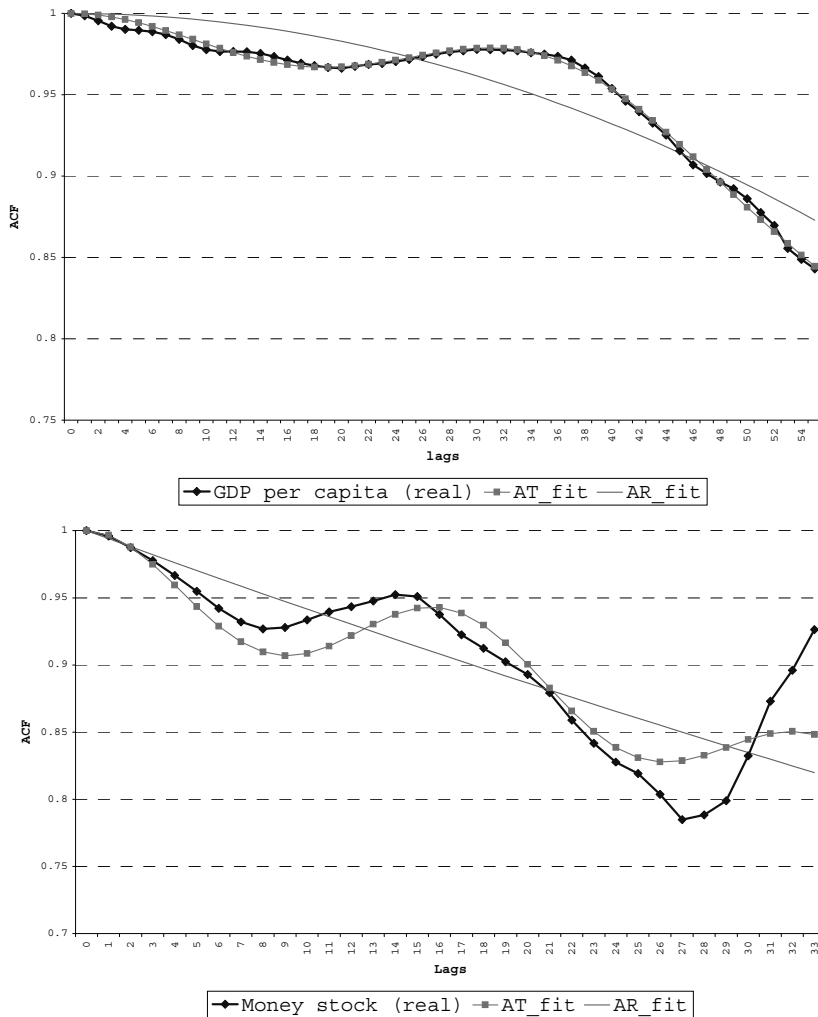
ACT also tested formally the difference in fit by using a Vuong (1989) test. They also did this for $ARI(p, d)$ models, allowing for fractional d but omitting the $MA(q)$ component which adds explanatory power only at q points in the graph and hence is not parsimonious. They considered twice as many series as the original Nelson and Plosser set. The result was a clear dominance of ACT's ACF over traditional models, and this remained the case even when checking the possibility of structural breaks, seasonality, different sample periods, and so on. Standard econometric models ignore the nonlinearities and long-memory dynamics implied by these ACFs, thus erroneous conclusions arise. The intermediate persistence of the cycles *and* the eventual turning points are completely missed by these standard models. Through these ACFs, we can see that impulses decay very slowly and these variables takes a long time to alter directions; once they do, the change is abrupt and accelerates. In the context of financial variables, these features show up as the (short-run) momentum and (medium-run) cycles that are often noted empirically, but not captured within a single model; e.g. for momentum see Jegadeesh and Titman (1993, 2001), Moskowitz, Ooi, and Pedersen (2012), and for cycles see De Bondt and Thaler (1985, 1987). In the context of macroeconomic stabilization, the features imply that, if an intervention takes place (e.g. change in interest rates), it should:

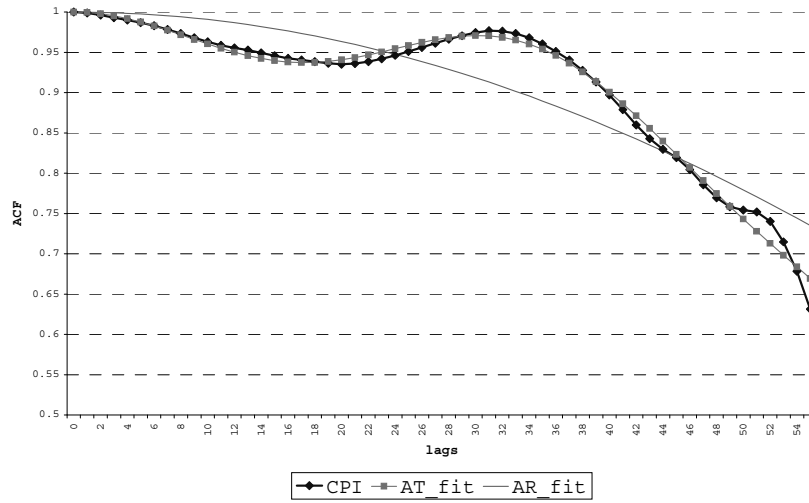
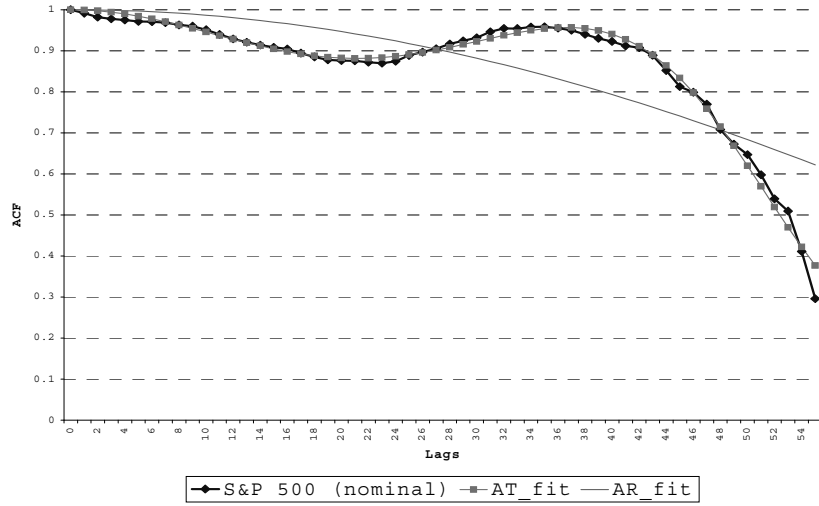
1. occur as soon as possible to give time to the policy to operate;

2. impart a stimulus sufficiently large to achieve the objective, but also taking into account the subsequent increments due to the persistent dynamics; and
3. revert to a neutral stance well before its objective is achieved, letting the economy ease onto its intended path.

Unfortunately, counterexamples to this course of action still exist; see Abadir (2011).

Figure 11: Empirical ACFs and their fits by AR and ACT models





Acknowledgements

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