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State-Owned Enterprises and Endogenous Growth

Piotr Matuszak*

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Abstract

This article analyzes the growth impact of state ownership in enterprises by introducing state-owned enterprises (SOEs) into the endogenous, Romer-type economic growth model. We build on the empirical firm-level analysis showing that SOEs underperform their privately owned counterparts and consider SOEs' inefficiency and related subsidization in the growth model. Our model predicts that the growth rate is decreasing in the SOE inefficiency and SOE shares in final goods production and R&D sectors. The model helps to shed light on the mechanisms behind empirical facts observed in European economies in the 21st century - lower growth and innovation rates in countries with larger SOE shares.

Keywords: state-owned enterprises, state ownership, Romer model, expanding variety growth model

JEL Classification: L32, O11, O30, O43

*The Polish Academy of Sciences, Institute of Economics; e-mail: pmatuszak@inepan.waw.pl; ORCID: 0000-0003-4572-3168

1 Introduction

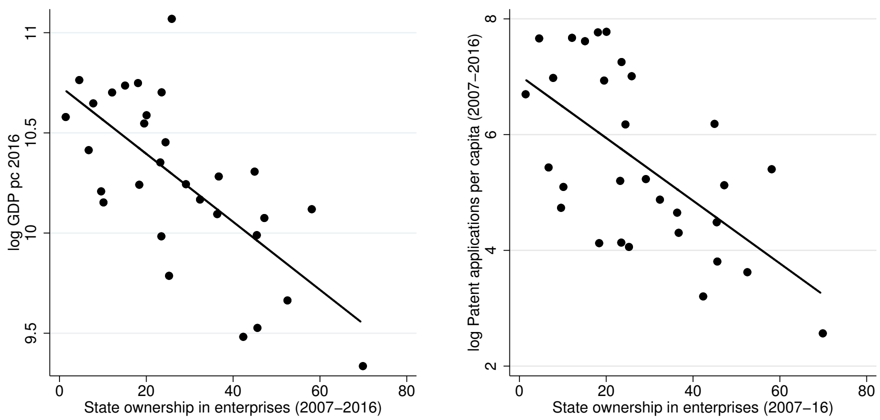
The discussion on the role of state-owned enterprises (SOEs) in market economies is an important element of contemporary economic literature thanks to the significant scale of state ownership in enterprises in many countries around the world. Szarzec et al. (2021) showed that SOEs had at least a 20% share in the group of large enterprises in 19 out of 30 European countries in 2007-16. Kwiatkowski et al. (2022) found 141 SOEs among the 500 largest enterprises worldwide in 2020. However, SOEs are most often analyzed at the microeconomic level and there is little research into their impact on economic growth, particularly from the theoretical perspective. This article aims to address this research gap.

This study builds on the Romer (1990) model and extends the basic framework by introducing SOEs in the final goods production and R&D sectors. The focus on the endogenous growth model is driven by the ongoing discussion on the relationship between state-owned enterprises and innovations. On the one hand, SOEs might be perceived as entities internalizing the positive spillovers of R&D, especially when high start-up costs and long-term investments are needed. Therefore, their presence might be beneficial for economic growth, even more so when they operate under a sufficiently good institutional environment (Mazzucato 2013, Antonelli et al. 2014, Tõnurist 2015, Clò et al. 2020, Lazzarini et al. 2021). On the other hand, SOEs are often characterized by worse performance than privately owned enterprises (POEs) and state ownership in enterprises is usually indicated as inferior to private ownership (Megginson and Netter 2001, Megginson 2017, Wang and Shailer 2018, Tihanyi et al. 2019). In fact, the scale of state ownership in enterprises is negatively associated with the income level, growth rates, and patent applications per capita in European economies in the 21st century (see Figure 1 and Section 2). Our model sheds light on the mechanisms behind these empirical facts.

Our modeling strategy builds on the firm-level empirical analysis showing that SOEs underperform POEs in European economies (see Section 2). The model features full employment and substitutability between output produced by POEs and SOEs. Growth arises from an increasing variety of inputs. Given SOEs' relative inefficiency, perfect competition in final goods production and R&D sectors and free entry, a subsidy must be granted to keep SOEs afloat. This subsidy is financed by taxes imposed on privately owned intermediate goods producers which reduces future profits from designing new intermediate goods. This in turn lowers the demand for blueprints and shifts labor from R&D to final goods production which decreases the growth rate. The model shows that the growth rate is decreasing in the SOE inefficiency and SOE share in final goods and R&D sectors in competitive equilibrium. We also consider the special case of the socially-planned SOE R&D sector in which the growth rate might be higher than in the private R&D sector in competitive equilibrium when the SOE inefficiency is sufficiently low.

We contribute to the very scarce theoretical literature on the effects of SOEs

Figure 1: State ownership in enterprises, income level, and patent applications



(a) State ownership and income level (b) State ownership and patent applications

Notes: The (a) figure shows the correlation between the scale of state ownership in enterprises and the income level. The (b) figure displays the correlation between the scale of state ownership in enterprises and the (log) number of patent applications per million people.
Source: The scale of state ownership in enterprises based on the SOE measure (25% ownership threshold, based on total assets) by Szarzec et al. (2021). The income level was collected from the World Bank and patents applications from the World Intellectual Property Organisation.

on economic growth. Plane (1992) analyzes state-owned enterprises within the neoclassical framework and shows that the presence of SOEs leads to a lower steady state output due to their inefficiencies. Huang et al. (2010) build on a multitask theory of state-owned enterprises by Bai et al. (2000) and analyze China's economic transition within a neoclassical framework and indicate that SOEs might positively contribute to economic growth - or at least their productive inefficiency might be offset - by a positive externality of maintaining social stability thanks to preventing large-scale unemployment. Song et al. (2011) also focus on the Chinese transition and provide a model in which SOEs have low productivity but survive thanks to better access to the credit market. To the best of our knowledge, Gylfason et al. (2001) is the sole study including SOEs within the endogenous growth framework. These authors build on the Romer (1990) model and include SOEs in the final goods production sector. State-owned enterprises in their model are characterized by lower efficiency, employ less skilled labor, and are less eager to adopt new technology. The larger SOE sector leads to lower economic growth rates in Gylfason et al. (2001). Compared to Gylfason et al. (2001), our model extends the analysis by including SOEs in both final goods and R&D sectors; taxes are imposed on intermediate goods producers instead of final goods output; the budget is balanced in each period t and we abstract from the debt accumulation growth impact which plays an important role in Gylfason et al. (2001) - therefore, we clearly emphasize the effects of SOE subsidization on the entry decisions of intermediate goods producers and shift of labor from R&D to final goods production instead of focusing on the growth impact of mounting public debt. The remainder of this article is organized as follows. We empirically analyze the relationship between state ownership in enterprises, income level, growth rates, and innovations, as well as we compare the performance of SOEs and POEs based on the firm-level data in Section 2. The model is presented in Section 3. We discuss our research and conclude in Section 4.

2 State-Owned Enterprises, Innovations, and Economic Growth: Empirical Evidence

This section presents a simple empirical analysis of the relationship between state-owned enterprises, innovations, and economic growth. We use the novel dataset on the scale of state ownership in enterprises by Szarzec et al. (2021). This source provides micro-level-based indicators of the economic weight of SOEs for 30 European countries in 2007-16 (see A.1 for a list of countries included). We check if the negative correlation presented in Figure 1 holds when controlling for socialist experience and geographical factors. It should be emphasized that the analysis presented in this section aims to motivate our theoretical model and is illustrative rather than comprehensive empirical research of the relationship between SOEs and economic growth.
Table 1 presents the regression results and shows that the scale of state ownership in enterprises was negatively and significantly correlated with the growth rates in 2010-16 and 2007-16 (columns 1-2). This association was also sizable in economic terms - a one standard deviation (SD) increase in state ownership (17.13%) was related to the average growth rate lower by 0.91 and 0.6 p.p. for the 2010-16 and 2007-16 periods. Similarly, the SOE share was also negatively correlated with the income level in 2016 (column 3) and a one SD increase in state ownership was associated with GDP per capita lower by 15.7%. Column 4 displays the results for the (log) patent applications per million people and this variable remains negatively related to the scale of state ownership in enterprises. The SOE share larger by a one SD was related to the number of patent applications being lower by 31.4%.
State-owned enterprises are usually characterized by worse financial performance than privately owned entities (e.g., Megginson and Netter 2001, Tihanyi et al. 2019) and SOE relative underperformance negatively influences growth in the theoretical models by Plane (1992), Gylfason et al. (2001), and Song et al. (2011). In the next step, we check if this association is also present in the firm-level data that are the basis for aggregate SOE measures employed above. We compare financial indicators of SOEs and POEs from the Amadeus database for entities operating in 30 European economies in 2007-16. Our sample includes more than 130,000 large non-financial

enterprises. We use the return on assets (ROA; calculated as EBITDA to total assets) and return on sales (ROS; calculated as EBITDA to operating revenues) as indicators of profitability.

Table 2 shows the results of the random effects model. SOEs had a substantially lower level of ROA and ROS indicators than POEs. This relationship holds when company size, time-fixed effects, country- and sector-specific factors are controlled for (we include separate country and sectoral dummy variables in columns 1 and 3, and their interactions in columns 2 and 4). State-owned enterprises had the ROA indicator lower by 1.07-1.11 p.p. and the ROS indicator lower by 2.18-2.35 p.p. on average, corresponding to 0.148-0.154 and 0.15-0.16 standard deviations of the dependent variables.

Table 1: Larger SOE shares are associated with lower growth rates, income levels, and number of patent applications

	Growth rate 2010-16	Growth rate 2007-16	log GDP pc 2016	log Patent applications per m 2010-16
	(1)	(2)	(3)	(4)
SOE share	-0.053***	-0.035***	-0.010***	-0.022**
	(0.011)	(0.010)	(0.001)	(0.009)
log GDP pc 2009	-1.650***			
	(0.568)			
log GDP pc 2006		-2.034***		
		(0.385)		
Post-socialist	1.891**	1.071*	-0.441***	-2.049***
	(0.796)	(0.593)	(0.070)	(0.313)
Intercept, Geo controls	yes	yes	yes	yes
N	30	30	30	30
R^2	0.737	0.707	0.778	0.759

Notes: Table displays the regression results at the country level. Conley (1999) standard errors are used to account for spatial correlation and are presented in parentheses. *** - 1% sign. level, ** - 5%, * - 10%. *N* denotes the number of observations. *SOE share* is the share of state-owned enterprises in the group of large enterprises (25% ownership threshold, based on total assets), collected from Szarzec et al. (2021). *Geo controls* include latitude and longitude. See Table A1 for variables' description, sources and summary statistics.

Table 2: State-owned enterprises underperform privately owned enterprises

	Return on Assets		Return on Sales	
	(1)	(2)	(3)	(4)
State-owned	-1.113***	-1.070***	-2.354***	-2.177***
	(0.064)	(0.067)	(0.170)	(0.176)
Privately owned	ref.	ref.	ref.	ref.
Country FE	yes		yes	
Sector FE	yes		yes	
Country-Sector interaction		yes		yes
Time FE	yes	yes	yes	yes
Controls	yes	yes	yes	yes
N	804,445	804,445	799,650	799,650
N enterprises	109,449	109,449	105,270	105,270
R_o^2	0.098	0.104	0.047	0.059

Notes: Table displays the results of the analysis with the random effects model. Robust standard errors are reported in parentheses. *** - 1% sign. level, ** - 5%, * - 10%. N denotes the number of observations. R_o^2 is the overall R-square. *Return on Assets* is calculated as EBITDA to total assets. *Return on Sales* is calculated as EBITDA to operating revenues. *Controls* include the log operating revenues and total assets. *State-owned* depicts the effect in percentage points. See Table A2 for variables' description, sources and summary statistics.

3 The Model

In this section, we extend the original Romer (1990) model with labor as an input in R&D by including SOEs in final goods (FG) production and R&D sectors. For tractability, we analyze SOEs in these sectors separately in Section 3.1 and Section 3.2, respectively. Similar to the original article (Romer, 1990), we focus on the balanced growth path (BGP) throughout the analysis. As our empirical indications are based on European countries which are (in a majority) developed economies, we focus on the invention of new varieties as a driver of growth rather than technology adoption. Building on the firm-level analysis in Section 2, we consider SOE relative inefficiency and the predictions of our model are consistent with the empirical facts documented above, that is, lower growth and innovation rates in countries with larger SOE shares. The consumption side of the economy is the same in Sections 3.1 and 3.2. The economy is populated by infinitely lived agents having the same preferences over consumption c_t:

$$U = \int_0^\infty e^{-\rho t} \ \ln c_t \ dt, \tag{1}$$

where $t \in [0, \infty)$ is time (omitted whenever no confusion arises), $\rho > 0$ is the subjective discount rate. The capital evolves as

$$\dot{k} = [R - \delta]\, k + w - c, \tag{2}$$

where k is capital per capita, R is the interest rate, δ is the depreciation rate, w is the wage rate, and agents inelastically supply labor to FG and R&D sectors. The population L is constant. Transversality condition holds with $\lim_{t\to\infty}\left[k_t e^{\int_0^t (R_s-\delta)ds}\right] = 0$. The standard utility-maximization problem leads to the intertemporal optimal condition:

$$\frac{\dot{c}}{c} = R - \delta - \rho. \tag{3}$$

3.1 State-Owned Enterprises in Final Goods Production

In this step, we analyze SOEs in the FG sector, and R&D and IG producers are POEs only.

Final Goods Sector Final goods produced by POEs, Y_t^{POE}, and SOEs, Y_t^{SOE}, are perfect substitutes, and aggregate FG production consists of production by SOEs and POEs

$$Y = Y^{POE} + Y^{SOE}, \tag{4}$$

and production functions of POEs and SOEs are

$$Y^{POE} = \left(L_Y^{POE}\right)^{1-\alpha} \int_0^A \left(x_i^{POE}\right)^{\alpha} di, \tag{5}$$

$$Y^{SOE} = (1-\sigma_{FG}) \left(L_Y^{SOE}\right)^{1-\alpha} \int_0^A \left(x_i^{SOE}\right)^{\alpha} di, \tag{6}$$

where $0 < \alpha < 1$, Y^{POE} is the FG output in the POE sector and Y^{SOE} in the SOE sector. L_Y^{POE} and L_Y^{SOE} are labor used in FG production in both sectors. $L_Y = L_Y^{POE} + L_Y^{SOE}$ is total labor in FG production. A is the measure of differentiated intermediate goods (IGs). x_i^{POE} and x_i^{SOE} are the quantities of a certain IG i used in production in the POE and SOE sectors. The final good is the numeraire and is used for consumption or transformed into capital.

As discussed above, there is empirical evidence that SOEs underperform POEs and we include the parameter $0 < \sigma_{FG} < 1$ as the measure of the SOE inefficiency in FG production. This inefficiency might be a result of, among other things, agency problems, soft budget constraints, clientelism, or the misuse of SOEs as political goods (e.g., Shleifer and Vishny 1998, Szarzec et al. 2022). One can also interpret $\sigma_{FG} Y^{SOE}$

as rents captured by political principals of SOEs and spent on wasteful consumption which does not provide any utility to households.

Let γ_{FG} denote the SOE share in labor and IGs used in FG production, such that:

$$\gamma_{FG} = \frac{L_Y^{SOE}}{L_Y^{POE} + L_Y^{SOE}} = \frac{\int_0^A x_i^{SOE} di}{\int_0^A x_i^{POE} di + \int_0^A x_i^{SOE} di}. \tag{7}$$

As FG production operates under perfect competition, production factors in the POE sector are paid their marginal products and SOEs are less efficient than POEs, a subsidy $s_{FG} = \sigma_{FG}/(1-\sigma_{FG})$ has to be granted to SOEs to equalize the rental prices of labor and IGs in FG production.

The economic literature on the role of state-owned enterprises as state aid beneficiaries is limited, but several studies emphasize the relevance of this issue in economies around the world. For instance, Shleifer and Vishny (1994) provide a theoretical model in which the subsidization of SOEs emerges as a result of political considerations and bargaining between politicians and managers. These authors also discuss the predictions of the model in relation to empirical facts from several countries. Matuszak et al. (2020) in turn show that state-owned enterprises are more likely to receive state aid than POEs in the Polish economy.

Given a subsidy s, the wage level in FG, w_Y, and the price for IG i, p_i, are

$$\begin{aligned} w_Y &= (1-\alpha)\left(L_Y^{POE}\right)^{-\alpha} \int_0^A \left(x_i^{POE}\right)^{\alpha} di = \\ &= (1+s_{FG})(1-\alpha)(1-\sigma_{FG})\left(L_Y^{SOE}\right)^{-\alpha} \int_0^A \left(x_i^{SOE}\right)^{\alpha} di, \end{aligned} \tag{8}$$

$$\begin{aligned} p_i &= \alpha\left(L_Y^{POE}\right)^{1-\alpha}\left(x_i^{POE}\right)^{\alpha-1} = \\ &= (1+s_{FG})\,\alpha\,(1-\sigma_{FG})\left(L_Y^{SOE}\right)^{1-\alpha}\left(x_i^{SOE}\right)^{\alpha-1}. \end{aligned} \tag{9}$$

Subsidy s_{FG} is financed by a tax τ_{FG} imposed on the profit of IGs producers (see below).

Intermediate Goods Sector IG producers operating under monopolistic competition purchase blueprints from the R&D sector and rent capital at price R, which is transformed one-to-one into the IG. IG producer i maximizes her operating profits by

$$\max_{\{x_i\}} \pi_y = p_i x_i - R x_i = p_i k - R k = \alpha L_Y^{1-\alpha} x_i^{\alpha} - R x_i, \tag{10}$$

which leads to (implementing (9) into the related FOC with respect to x_i)

$$p_i = \frac{R}{\alpha}. \tag{11}$$

All firms i charge $p_i = p_x$ and therefore IGs are bought to the same extent $x_i = x$. Thus, (9) becomes:

$$p_x = \alpha L_Y^{1-\alpha} x^{\alpha-1}. \tag{12}$$

IG producers realize operating profits (combining (10), (11), (12)):

$$\pi_y = (1-\alpha)\,\alpha L_Y^{1-\alpha} x^{\alpha}. \tag{13}$$

As the number of IG producers is given by A, aggregate realized operating profits are

$$\Pi_y = (1-\alpha)\,\alpha L_Y^{1-\alpha} x^{\alpha} A. \tag{14}$$

These operating profits are taxed with a tax rate $0 < \tau_{FG} < 1$ in order to subsidize SOEs. The budget is balanced in each period t, such that aggregate taxes equal aggregate subsidies:

$$\Pi_y \tau_{FG} = \sigma_{FG} \gamma_{FG} A L_Y^{1-\alpha} x^{\alpha}, \tag{15}$$

(Proof in Appendix B.1). Using (14) into (15), the tax rate τ_{FG} is given by

$$\tau_{FG} = \frac{\gamma_{FG}\sigma_{FG}}{(1-\alpha)\,\alpha}. \tag{16}$$

Therefore, post-tax income of the IG producer is

$$(1-\tau_{FG})\,\pi_y = \left(1 - \frac{\gamma_{FG}\sigma_{FG}}{(1-\alpha)\,\alpha}\right)\pi_y, \tag{17}$$

where $\gamma_{FG}\sigma_{FG} < (1-\alpha)\,\alpha$ must hold because $0 < \tau_{FG} < 1$.

R&D Sector The R&D sector operates under perfect competition and the technological frontier, A, evolves according to:

$$\dot{A} = \eta A L_A, \tag{18}$$

where $L_A = L - L_Y$ is labor input in R&D and η is R&D productivity. Prices for labor, w_A, and for new blueprints, p_A, are taken as given and R&D firms solve the problem:

$$\max_{\{L_A\}} \pi_A = p_A \eta A L_A - w_A L_A \tag{19}$$

with

$$w_A = p_A \eta A. \tag{20}$$

Intermediate Goods Producer Market Entry The tax imposed on IG producers decreases their present value of operating profits and with the free entry condition, the no-arbitrage condition holds:

$$p_{A,t} = \int_t^{\infty} e^{-\int_t^{\vartheta} r_\mu d\mu} \left(1 - \tau_{FG}\right) \pi_{y,\vartheta} d\vartheta. \tag{21}$$

where ϑ is the time in the market with a blueprint. Along the BGP, $\dot{\pi}_y = 0$, and derivative of (21) with respect to t leads to

$$r = \frac{\dot{p}_A}{p_A} + \frac{\left(1 - \tau_{FG}\right) \pi_y}{p_A}. \tag{22}$$

Equilibrium Labor is assumed to be perfectly mobile between sectors, with no differences in productivity in FG and R&D. Along the BGP:

$$L_A = L - \frac{(R - \delta)}{\left(1 - \tau_{FG}\right) \alpha\eta} = L - \frac{(R - \delta)}{\left(1 - \frac{\gamma_{FG}\sigma_{FG}}{(1-\alpha)\alpha}\right) \alpha\eta}, \tag{23}$$

L_A decreases in γ_{FG} and σ_{FG}. The growth rate along the BGP in the economy with SOEs in FG production is

$$g^{SOE,\ FG} = \frac{L\eta \left(\alpha - \frac{\gamma_{FG}\sigma_{FG}}{(1-\alpha)}\right) - \rho}{\alpha - \frac{\gamma_{FG}\sigma_{FG}}{(1-\alpha)} + 1}. \tag{24}$$

Proof. See Appendix B.2.

Comparative Statics

$$\frac{g^{SOE,FG}}{\partial \sigma_{FG}} = -\frac{(1-\alpha)\gamma_{FG}(L\eta + \rho)}{\left(\alpha^2 - 1 + \gamma_{FG}\sigma_{FG}\right)^2} < 0 \text{ for } \gamma_{FG} > 0, \tag{25}$$

$$\frac{g^{SOE,FG}}{\partial \gamma_{FG}} = -\frac{(1-\alpha)\sigma_{FG}(L\eta + \rho)}{\left(\alpha^2 - 1 + \gamma_{FG}\sigma_{FG}\right)^2} < 0. \tag{26}$$

Proposition 1. *In an economy with SOEs in final goods production, the growth rate along the BGP is decreasing in the SOE inefficiency and SOE share.*

It can be easily shown that the growth rate with no SOEs, $\gamma_{FG} = 0$, is larger than $g^{SOE,\ FG}$ with $\gamma_{FG} > 0$:

$$g^{\gamma_{FG}=0} = \frac{L\eta\alpha - \rho}{\alpha + 1} > \frac{L\eta \left(\alpha - \frac{\gamma_{FG}\sigma_{FG}}{(1-\alpha)}\right) - \rho}{\alpha - \frac{\gamma_{FG}\sigma_{FG}}{(1-\alpha)} + 1} = g^{SOE,FG} \text{ for } L\eta\alpha > \rho. \tag{27}$$

$L\eta\alpha > \rho$ ensures that the growth rate in an economy with no SOEs is larger than zero, which is a standard assumption in the endogenous, expanding variety models. When this condition is not satisfied, all workers are employed in the production of final goods.

3.2 State-Owned Enterprises in the R&D Sector

In this step, we analyze SOEs in the R&D sector, and FG and IG producers are POEs only.

Final Goods and Intermediate Goods production The production function in the FG sector is

$$Y = L_Y^{1-\alpha} \int_0^A {x_i}^{\alpha} di. \tag{28}$$

The wage and IG price levels are

$$w_Y = (1-\alpha)\, L_Y^{-\alpha} \int_0^A {x_i}^{\alpha} di = (1-\alpha)\, \frac{Y}{L_Y}, \tag{29}$$

$$p_i = \alpha L_Y^{1-\alpha} {x_i}^{\alpha-1}. \tag{30}$$

The IG producer maximizes the operating profit as in (10) and the price for IG i is (11). As IGs are bought to the same extent $x_i = x$, we can present (28) as

$$Y = AL_Y^{1-\alpha} x^{\alpha}. \tag{31}$$

Combining (10), (11), (30), and (31), IG producers realize operating profits:

$$\pi_y = (1-\alpha)\, \alpha \frac{Y}{A}. \tag{32}$$

Aggregate operating profits are (the number of IG producers is A)

$$\Pi_y = (1-\alpha)\, \alpha Y. \tag{33}$$

These aggregate operating profits will be taxed in order to subsidize inefficient SOEs in the R&D sector (see below).

R&D Sector The technological frontier evolves according to

$$\dot{A} = \eta A L_A^{POE} + (1-\sigma_{R\&D})\, \eta A L_A^{SOE}, \tag{34}$$

where $L_A = L_A^{POE} + L_A^{SOE}$ is total labor in R&D production, $\sigma_{R\&D}$ is the SOE inefficiency in R&D. Let $\gamma_{R\&D}$ denote the SOE share in labor used in R&D production:

$$\gamma_{R\&D} = \frac{L_A^{SOE}}{L_A^{POE} + L_A^{SOE}}. \tag{35}$$

Then, (34) can be rewritten as

$$\dot{A} = \eta A \left(1-\gamma_{R\&D}\right) L_A + \left(1-\sigma_{R\&D}\right) \eta A \gamma_{R\&D} L_A = \left(1-\gamma_{R\&D}\sigma_{R\&D}\right) \eta A L_A. \quad (36)$$

Under perfect competition, production factors in the R&D POE sector are paid their marginal products and as SOEs are less efficient than POEs, a subsidy $s_{R\&D} = \sigma_{R\&D}/(1-\sigma_{R\&D})$ has to be granted to SOEs in order to equalize the rental prices of labor in R&D production. Therefore, the wage level in R&D, w_A, is given by

$$w_A = p_A \eta A = \left(1+s_{R\&D}\right) p_A \left(1-\sigma_{R\&D}\right) \eta A, \quad (37)$$

The aggregate subsidy for SOEs in R&D is

$$\gamma_{R\&D}\sigma_{R\&D} L_A p_A \eta A \quad (38)$$

(Proof in Appendix B.3).

Intermediate Goods Producer Market Entry Wage levels in FG, (29), and R&D, (37), are equal, aggregate operating profits of IG producers are (33), the aggregate subsidy is (38) and the budget is balanced, therefore, the tax rate, $\tau_{R\&D}$, imposed on IG producers is

$$\tau_{R\&D} = \gamma_{R\&D}\sigma_{R\&D} \frac{L_A}{\alpha L_Y}. \quad (39)$$

Post-tax income of the IG producer is

$$\left(1-\tau_{R\&D}\right) \pi_y = \left(1-\gamma_{R\&D}\sigma_{R\&D}\frac{L_A}{\alpha L_Y}\right) \pi_y. \quad (40)$$

The tax imposed on IG producers decreases their present value of operating profits and with the free entry condition:

$$p_{A,t} = \int_t^{\infty} e^{-\int_t^{\vartheta} r_\mu d\mu} \left(1-\tau_{R\&D}\right) \pi_{y,\vartheta} d\vartheta, \quad (41)$$

where ϑ is the time in the market with a blueprint. Along the BGP, $\dot{\pi}_y = 0$, and derivative of (41) with respect to t leads to

$$r = \frac{\dot{p}_A}{p_A} + \frac{\left(1-\tau_{R\&D}\right)\pi_y}{p_A}. \quad (42)$$

Equilibrium Along the BGP:

$$L_A = \frac{L\alpha\eta - (R-\delta)}{\eta\left(\alpha+\gamma_{R\&D}\sigma_{R\&D}\right)}, \quad (43)$$

L_A decreases in $\gamma_{R\&D}$ and $\sigma_{R\&D}$. The growth rate along the BGP in the economy with SOEs in R&D production is

$$g^{SOE,R\&D} = \frac{L\eta\alpha - \rho}{(\alpha + \gamma_{R\&D}\sigma_{R\&D})/(1 - \gamma_{R\&D}\sigma_{R\&D}) + 1}. \tag{44}$$

Proof. See Appendix B.4.

Comparative Statics

$$\frac{\partial g^{SOE,R\&D}}{\partial \sigma_{R\&D}} = -\frac{\gamma_{R\&D}\,(L\eta\alpha - \rho)}{\alpha + 1} < 0 \text{ for } \gamma_{R\&D} > 0 \text{ and } L\eta\alpha > \rho\ , \tag{45}$$

$$\frac{\partial g^{SOE,\ R\&D}}{\partial \gamma_{R\&D}} = -\frac{\sigma_{R\&D}\,(L\eta\alpha - \rho)}{\alpha + 1} < 0 \text{ for } L\eta\alpha > \rho. \tag{46}$$

Proposition 2. *In an economy with SOEs in R&D, the growth rate along the BGP is decreasing in the SOE inefficiency and SOE share.*

The growth rate with no SOEs, $\gamma_{R\&D} = 0$, is larger than $g^{SOE,\ R\&D}$ with $\gamma_{R\&D} > 0$:

$$g^{\gamma_{R\&D}=0} = \frac{L\eta\alpha - \rho}{\alpha + 1} > \frac{L\eta\alpha - \rho}{(\alpha + \gamma_{R\&D}\sigma_{R\&D})/(1 - \gamma_{R\&D}\sigma_{R\&D}) + 1} = g^{SOE,\ R\&D} \quad \text{for } L\eta\alpha > \rho. \tag{47}$$

3.3 Social Planning Optimum with State-Owned Enterprises in the R&D sector

Some studies suggest that under certain conditions SOEs might be beneficial for economic growth - especially when engaged in R&D activities and under sufficiently good institutions (Mazzucato 2013, Antonelli et al. 2014, Tõnurist 2015, Lazzarini et al. 2021). In Appendix B.5, we solve a social planning problem with the level of labor in the SOE R&D sector as a control variable (with no POEs in R&D). The balanced growth solution to the first-order necessary conditions leads to

$$g^{SOE^*,R\&D} = L\eta\,(1 - \sigma_{R\&D}) - \rho. \tag{48}$$

Compared to the BGP growth rate with $\gamma_{R\&D} = 0$ from competitive equilibrium, $g^{\gamma_{R\&D}=0} = (L\eta\alpha - \rho)/(\alpha + 1)$, $g^{SOE^*,R\&D} > g^{\gamma_{R\&D}=0}$ holds for $\sigma_{R\&D} < (L\eta - \alpha\rho)/(L\eta\,(\alpha + 1))$. The analysis of this special case shows that for sufficiently low SOE inefficiency, SOEs might foster economic growth when engaged in R&D activities.

In Appendix B.6, we also solve the social-planner problem with SOEs in FG production. The growth rate along the BGP is the same as with the POE FG sector and does not depend on the SOE inefficiency, σ_{FG}. This solution leads to a smaller share of labor in FG and a larger share of labor in R&D than for the scenarios with SOEs in competitive equilibrium. As SOEs are often indicated as associated with overemployment and keeping entities in the sunset industries, we perceive limiting the scope of SOE activities in FG production from this solution as contradicting empirical indications.

4 Conclusions

This article analyzes the growth impact of state-owned enterprises by introducing SOEs into the endogenous growth model with expanding input varieties. We consider enterprises owned by the state in the final goods production and R&D sectors. We show that inefficient SOEs must be subsidized to be kept afloat and these subsidies are financed by taxes imposed on intermediate goods producers. This in turn decreases future profits from designing new intermediate goods and lowers the demand for blueprints, which results in reduced shares of labor in R&D and lower growth rates.
The predictions of the model are in line with previous empirical research showing a negative impact of SOEs on growth rates (Plane 1992, Gylfason et al. 2001). However, the recent study by Szarzec et al. (2021) indicated that state-owned enterprises are not negative for growth *per se* and their impact improves with institutional quality. As SOE relative underperformance (compared to POEs) diminishes with better institutions (Borghi et al. 2016, Estrin et al. 2016, Castelnovo et al. 2019), we argue that our model might also be useful in explaining the conditional relationship between SOEs and growth revealed in Szarzec et al. (2021) - SOE inefficiency, σ_{FG} and $\sigma_{R\&D}$, can be considered as a function of institutional quality with the lower inefficiencies when SOEs operate in countries with good institutions, $\partial\sigma_{FG}/\partial Instituions < 0$ and $\partial\sigma_{R\&D}/\partial Instituions < 0$. This in turn would lead to a less detrimental impact of SOEs on economic growth. Szarzec et al. (2021) also show that the effect of SOEs can turn into positive in the right tail of the sample distribution of institutional quality. The solution presented in Section 3.3 shows that it might the case in our model when state-owned enterprises are employed in the R&D sector and their inefficiency is sufficiently low (which is more likely to hold with better institutions).
Our model focuses on the growth impact of SOEs through the lens of the endogenous, expanding variety economic growth framework, nevertheless, it by no means rules out alternative channels through which state ownership in enterprises might influence growth rates. For tractability, our model also assumes that state-owned enterprises operate in competitive environments in the FG and R&D sectors and abstracts from institutional features such as market power that might play a relevant role in state-owned enterprises. This is an important limitation of our analysis as SOEs often hold a position of (natural) monopolies. By focusing on the economic growth effects of state ownership in enterprises, we also omit the fact that SOEs usually fulfill a broad set of goals, including social and political ones, and their presence might be supported by

(at least part of) society despite their economic inefficiencies (Shirley and Nellis 1991, Bai et al. 2000, Robinett 2006, Christiansen 2013, Matuszak and Kabaciński 2021). Finally, the conclusions derived from the analysis focusing on state-owned enterprises should be distinguished from and not extrapolated on those related to government spending, transfers, or other tools through which 'big government' influences economic growth.

State-owned enterprises still play an important role in the global economy. Nevertheless, both theoretical and empirical research on their economic growth effects is very limited. Future studies should expand the geographical and time coverage of the empirical analysis to provide a more solid basis for evaluating the consequences of keeping enterprises state-owned. Theoretical models in turn can benefit from investigating the political motivations to keep state ownership within the endogenous growth framework and considering more complex institutional features of the markets.

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A Appendix

A.1 List of countries included in the empirical analysis

Austria, Belgium, Bosnia and Hercegovina, Bulgaria, Croatia, Czech Rep., Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Italy, Latvia, Lithuania, North Macedonia, Montenegro, Netherlands, Norway, Poland, Portugal, Romania, Russia, Serbia, Slovakia, Slovenia, Spain, Sweden, United Kingdom.

Table A1: List of variables - macroeconomic analysis

Variable	**Description**	**N**	**Mean**	**Std. Dev.**	**Min**	**Max**
log GDP pc 2016	Log GDP per capita in PPP in 2016, in constant 2011 prices, source: World Bank	30	10.27	0.42	9.34	11.07
log GDP pc 2010	Log GDP per capita in PPP in 2010, in constant 2011 prices, source: World Bank	30	10.18	0.46	9.18	11.04
Growth rate 2010-16	Average growth rate of GDP pc in 2010-16, in PPP, constant 2011 prices, source: own calculations based on World Bank	30	1.45	1.56	-2.85	4.76
Growth rate 2007-16	Average growth rate of GDP pc in 2007-16, in PPP, constant 2011 prices, source: own calculations based on World Bank	30	1.11	1.39	-2.52	3.54
log Patents applications per m	Patents applications per 1 million people in 2010-16, source: the World Intellectual Property Organisation	30	5.53	1.52	2.57	7.78
SOE share	The share of state-owned enterprises in the group of large enterprises in terms of total assets, source: *TA25* from Szarzec et al. (2021)	30	27.66	17.13	1.43	69.90
Post-socialist	Dummy variable for post-socialist countries	30	0.53	0.51	0	1

Table A2: List of variables - microeconomic analysis

Variable	**Description**	**N**	**Mean**	**Std. Dev.**	**Min**	**Max**
Return on Assets	EBITDA to total assets, source: Amadeus database	853,402	4.76	7.22	-12.20	27.25
Return on Sales	EBITDA to operating revenues, source: Amadeus database	802,938	5.64	14.55	-52.51	74.28
State-owned	State-owned status of an enterprise (1 - state-owned, 0 - privately owned), according to the 25% ownership threshold, source: Amadeus database and ownership status classification from Szarzec et al. (2021)	1,272,300	0.06	0.23	0	1
log Operating Revenues	Log of operating revenues, source: Amadeus database	977,395	10.46	1.87	2.30	21.50
log Total Assets	Log of total assets, source: Amadeus database	1,071,332	10.87	1.70	2.30	19.92

B Appendix: Proofs

B.1 Proof of (15)

The right hand side of (15) is

$$\left(w_Y L_Y^{SOE} + p\left(x\right) x^{SOE}\right) - \left(\left(1-\sigma_{FG}\right)\gamma_{FG} A \left(L_Y^{SOE}\right)^{1-\alpha}\left(x^{SOE}\right)^{\alpha}\right), \tag{B1}$$

that is, the difference between production factors renumeration in SOEs in FG production and SOE FG production, which determines the aggregate value of subsidies granted to SOEs.

As IGs are bought to the same extent, aggregate capital stock equals the amount of all IGs:

$$K = \int_0^A x_i di = Ax. \tag{B2}$$

Using (7) and (B2), (5) and (6) become:

$$Y^{POE} = \left(1-\gamma_{FG}\right) A {L_Y}^{1-\alpha} x^{\alpha}, \tag{B3}$$

$$Y^{SOE} = \left(1-\sigma_{FG}\right)\gamma_{FG} A {L_Y}^{1-\alpha} x^{\alpha}. \tag{B4}$$

Using (7), one can also transform (8) and (9):

$$\begin{aligned} w_Y &= (1-\alpha)\left(L_Y\left(1-\gamma\right)\right)^{-\alpha}\int_0^A \left(x\left(1-\gamma\right)\right)^{\alpha} di = \\ &= (1-\gamma)^{\alpha-\alpha}(1-\alpha)\,L_Y{}^{-\alpha}\int_0^A x^{\alpha} di = \\ &= (1-\alpha)\,L_Y{}^{-\alpha}\int_0^A x^{\alpha} di = \\ &= (1-\alpha)\,AL_Y{}^{-\alpha}x^{\alpha}, \end{aligned} \tag{B5}$$

$$\begin{aligned} p\left(x\right) &= \alpha\left(L_Y\left(1-\gamma\right)\right)^{1-\alpha}\left(x\left(1-\gamma\right)\right)^{\alpha-1} = \\ &= (1-\gamma)^{1-\alpha+\alpha-1}\,\alpha L_Y{}^{1-\alpha}x^{\alpha-1} = \\ &= \alpha L_Y{}^{1-\alpha}x^{\alpha-1}. \end{aligned} \tag{B6}$$

Given (B5) and (B6), production factors renumeration in SOEs in FG production is

$$\begin{aligned} &\gamma_{FG}\left(w_Y L_Y + p\left(x\right)Ax\right) = \\ &\qquad = \gamma_{FG}\left((1-\alpha)\,AL_Y{}^{1-\alpha}x^{\alpha} + \alpha AL_Y{}^{1-\alpha}x^{\alpha}\right) = \gamma_{FG}AL_Y{}^{1-\alpha}x^{\alpha}. \end{aligned} \tag{B7}$$

By subtracting (B4) from (B7) we obtain the aggregate subsidy to SOEs

$$\gamma_{FG}AL_Y{}^{1-\alpha}x^{\alpha} - (1-\sigma_{FG})\,\gamma_{FG}AL_Y{}^{1-\alpha}x^{\alpha} = \sigma_{FG}\gamma_{FG}AL_Y{}^{1-\alpha}x^{\alpha}. \tag{B8}$$

B.2 Proof of (23) and (24)

Under labour market clearing, $w_A = w_Y$, we can combine (B5) and (20):

$$p_A\eta A = (1-\alpha)\,AL_Y{}^{-\alpha}x^{\alpha}. \tag{B9}$$

Using (B3) and (B4), we can transform the aggregate FG production function (4) to

$$Y = (1-\gamma_{FG}\sigma_{FG})\,AL_Y{}^{1-\alpha}x^{\alpha}, \tag{B10}$$

and (B9) can be transformed to

$$p_A\eta A = (1-\alpha)\,AL_Y{}^{-\alpha}x^{\alpha} = (1-\alpha)\,\frac{Y}{(1-\gamma_{FG}\sigma_{FG})\,L_Y} \tag{B11}$$

Use (B10) in (13):

$$\pi_y = (1-\alpha)\,\alpha L_Y{}^{1-\alpha}x^{\alpha} = (1-\alpha)\,\alpha\,(1-\gamma_{FG}\sigma_{FG})^{-1}\,\frac{Y}{A}. \tag{B12}$$

As $\dot{Y}/Y = \dot{A}/A$ along the BGP, $\dot{\pi}_y = 0$ holds also along the BGP and implies $\dot{p}_A = 0$. Therefore, we can use (B12) in (22):

$$p_A = \frac{(1-\tau_{FG})\,\pi_y}{R-\delta} = \frac{(1-\tau_{FG})\,(1-\alpha)\,\alpha Y}{(R-\delta)\,(1-\gamma_{FG}\sigma_{FG})\,A}. \tag{B13}$$

Given that $L = L_Y + L_A$, we solve for L_Y and L_A using (B11), (B13) and (16):

$$L_Y = \frac{(R-\delta)}{(1-\tau_{FG})\,\alpha\eta} = \frac{(R-\delta)}{\left(1-\frac{\gamma_{FG}\sigma_{FG}}{(1-\alpha)\alpha}\right)\alpha\eta}, \tag{B14}$$

$$L_A = L - \frac{(R-\delta)}{(1-\tau_{FG})\,\alpha\eta} = L - \frac{(R-\delta)}{\left(1-\frac{\gamma_{FG}\sigma_{FG}}{(1-\alpha)\alpha}\right)\alpha\eta}. \tag{B15}$$

Using (18) and (B15), the rate of technological progress in an economy with SOEs in FG production is

$$g^{SOE,\ FG} = \frac{\dot{A}}{A} = L\eta - \frac{(R-\delta)}{\left(1-\frac{\gamma_{FG}\sigma_{FG}}{(1-\alpha)\alpha}\right)\alpha}. \tag{B16}$$

Using (3) and (B16), in terms of the fundamentals of the model, the economy with SOEs in FG production grows along the BGP at rate

$$g^{SOE,\ FG} = \frac{L\eta\left(\alpha - \frac{\gamma_{FG}\sigma_{FG}}{(1-\alpha)}\right) - \rho}{\alpha - \frac{\gamma_{FG}\sigma_{FG}}{(1-\alpha)} + 1}. \tag{B17}$$

B.3 Proof of (38)

The aggregate subsidy to SOEs in R&D is the difference between the total renumeration paid to labor in SOEs in the R&D sector and SOE production in R&D:

$$\begin{aligned} L_A^{SOE} w_A - p_A\,(1-\sigma_{R\&D})\,\eta A L_A^{SOE} &= \gamma_{R\&D} L_A p_A \eta A - p_A\,(1-\sigma_{R\&D})\,\eta A \gamma_{R\&D} L_A = \\ &= \gamma_{R\&D} L_A p_A \eta A\,(1-1+\sigma_{R\&D}) = \\ &= \gamma_{R\&D}\sigma_{R\&D} L_A p_A \eta A. \end{aligned} \tag{B18}$$

B.4 Proof of (43) and (44)

As $\dot{Y}/Y = \dot{A}/A$ along the BGP, $\dot{\pi}_y = 0$ holds also along the BGP and implies $\dot{p}_A = 0$. Using (40) in (42):

$$p_A = \frac{(1-\tau_{R\&D})\,\pi_y}{R-\delta} = \frac{\left(1-\gamma_{R\&D}\sigma_{R\&D}\frac{L_A}{\alpha L_Y}\right)\pi_y}{R-\delta}. \tag{B19}$$

Given labor market clearing $L = L_Y + L_A$, $w_Y = (1-\alpha)\,Y/L_Y = p_A\eta A = w_A$, (32) and (B19) yield:

$$L_Y = \frac{(R-\delta) + \gamma_{R\&D}\sigma_{R\&D}\eta L_A}{\alpha\eta}, \tag{B20}$$

$$L_A = \frac{L\alpha\eta - (R-\delta)}{\eta\,(\alpha + \gamma_{R\&D}\sigma_{R\&D})}. \tag{B21}$$

Using (36) and (B21), the rate of technological progress in an economy with SOEs in R&D is

$$g^{SOE,R\&D} = \frac{\dot{A}_t}{A_t} = (1-\gamma_{R\&D}\sigma_{R\&D})\,\frac{L\eta\alpha - (R-\delta)}{\alpha + \gamma_{R\&D}\sigma_{R\&D}}. \tag{B22}$$

Using (3) and (B22), in terms of the fundamentals of the model, the economy with SOEs in R&D grows along the BGP at rate

$$g^{SOE,\ R\&D} = \frac{L\eta\alpha - \rho}{(\alpha + \gamma_{R\&D}\sigma_{R\&D})/(1-\gamma_{R\&D}\sigma_{R\&D}) + 1}. \tag{B23}$$

B.5 Proof of (48)

The growth rate along the BGP that would emerge from the solution to a social planning problem with the SOE R&D sector can be found by solving the problem:

$$\max_{\{c_t, L_{A,t}^{SOE}\}} \int_0^\infty e^{-\rho t}\ \ln c_t dt, \tag{B24}$$

subject to

$$\dot{K}_t = \left(A_t\left(L - L_{A,t}^{SOE}\right)\right)^{1-\alpha} K_t{}^{\alpha} - Lc_t, \tag{B25}$$

$$\dot{A}_t = \eta A_t\,(1-\sigma_{R\&D})\,L_{A,t}^{SOE}, \tag{B26}$$

$$L_{A,t}^{SOE} + L_{Y,t} = L. \tag{B27}$$

The current value Hamiltonian for the problem is:

$$H(\cdot) = e^{-pt}u(c_t) + \lambda_t\left[A_t{}^{1-\alpha}\left(L - L_{A,t}^{SOE}\right)^{1-\alpha} K_t{}^{\alpha} - Lc_t\right] + \\ + \mu_t\left[\eta A_t\,(1-\sigma_{R\&D})\,L_{A,t}^{SOE}\right]. \tag{B28}$$

Necessary first-order conditions for interior maxima:

$$\frac{\partial H(\cdot)}{\partial c_t} = 0 \implies e^{-\rho t} u'(c_t) = \lambda_t, \tag{B29}$$

$$\frac{\partial H(\cdot)}{\partial L_{A,t}^{SOE}} = 0 \implies -\lambda_t \left[(1-\alpha) A_t{}^{1-\alpha} \left(L - L_{A,t}^{SOE} \right)^{-\alpha} K_t{}^{\alpha} \right] + \\ + \mu_t \left[\eta A_t \left(1 - \sigma_{R\&D} \right) \right] = 0, \tag{B30}$$

$$\frac{\partial H(\cdot)}{\partial \lambda_t} = \dot{K}_t \implies A_t{}^{1-\alpha} \left(L - L_{A,t}^{SOE} \right)^{1-\alpha} K_t{}^{\alpha} - L c_t = \dot{K}_t, \tag{B31}$$

$$\frac{\partial H(\cdot)}{\partial \mu_t} = \dot{A}_t \implies \eta A_t \left(1 - \sigma_{R\&D} \right) L_{A,t}^{SOE} = \dot{A}_t, \tag{B32}$$

$$\frac{\partial H(\cdot)}{\partial K_t} = -\dot{\lambda}_t \implies \alpha \lambda_t A_t{}^{1-\alpha} \left(L - L_{A,t}^{SOE} \right)^{1-\alpha} K_t{}^{\alpha-1} = -\dot{\lambda}_t, \tag{B33}$$

$$\frac{\partial H(\cdot)}{\partial A_t} = -\dot{\mu}_t \implies (1-\alpha) \lambda_t A_t{}^{-\alpha} \left(L - L_{A,t}^{SOE} \right)^{1-\alpha} K_t{}^{\alpha} + \\ + \mu_t \eta (1-\sigma) L_{A,t}^{SOE} = -\dot{\mu}_t. \tag{B34}$$

From log-differentiation of (B29):

$$\frac{u''(c_t)\dot{c}_t}{u'(c_t)} - \rho = -\frac{\dot{c}_t}{c_t} - \rho = \frac{\dot{\lambda}_t}{\lambda_t}. \tag{B35}$$

From (B30):

$$\frac{\mu_t}{\lambda_t} = \frac{(1-\alpha) A_t{}^{-\alpha} \left(L - L_{A,t}^{SOE} \right)^{-\alpha} K_t{}^{\alpha}}{\eta \left(1 - \sigma_{R\&D} \right)}. \tag{B36}$$

From (B34):

$$(1-\alpha) \frac{\lambda_t}{\mu_t} A_t{}^{-\alpha} \left(L - L_{A,t}^{SOE} \right)^{1-\alpha} K_t{}^{\alpha} + \eta \left(1 - \sigma_{R\&D} \right) L_{A,t}^{SOE} = -\frac{\dot{\mu}_t}{\mu_t}. \tag{B37}$$

Using (B36) in (B37):

$$-L\eta \left(1 - \sigma_{R\&D} \right) = \frac{\dot{\mu}_t}{\mu_t}. \tag{B38}$$

For the BGP, is holds that $\dot{\lambda}(t)/\lambda(t) = \dot{\mu}(t)/\mu(t)$, then using (B35) and (B38):

$$\frac{\dot{c}_t}{c_t} = L\eta \left(1 - \sigma_{R\&D} \right) - \rho. \tag{B39}$$

For the BGP it also holds that $\dot{c}_t/c_t = \dot{A}(t)/A(t)$ and the growth rate from this social planning problem is

$$g^{SOE^*,R\&D} = L\eta \left(1 - \sigma_{R\&D} \right) - \rho. \tag{B40}$$

B.6 Social planning optimum with SOEs in FG production

The growth rate along the BGP that would emerge from the solution to a social planning problem with SOEs in FG production can be found by solving the problem:

$$\max_{\{c_t, L_{Y,t}^{SOE}\}} \int_0^{\infty} e^{-\rho t} \ \ln c_t dt, \tag{B41}$$

subject to

$$\dot{K}_t = (1-\sigma_{FG}) \left(A_t L_{Y,t}^{SOE}\right)^{1-\alpha} K_t{}^{\alpha} - Lc_t, \tag{B42}$$

$$\dot{A}_t = \eta A_t \left(L - L_{Y,t}^{SOE}\right), \tag{B43}$$

$$L_{Y,t}^{SOE} + L_{A,t} = L. \tag{B44}$$

The current value Hamiltonian for the problem is:

$$\begin{aligned} H(\cdot) = e^{-pt} u(c_t) + \lambda_t \left[(1-\sigma_{FG}) \left(A_t L_{Y,t}^{SOE}\right)^{1-\alpha} K_t{}^{\alpha} - Lc_t\right] + \\ + \mu_t \left[\eta A_t \left(L - L_{Y,t}^{SOE}\right)\right]. \end{aligned} \tag{B45}$$

Necessary first-order conditions for interior maxima:

$$\frac{\partial H(\cdot)}{\partial c_t} = 0 \implies e^{-\rho t} u'(c_t) = \lambda_t, \tag{B46}$$

$$\begin{aligned} \frac{\partial H(\cdot)}{\partial L_{Y,t}^{SOE}} = 0 \implies \lambda_t \left[(1-\alpha)(1-\sigma_{FG}) A_t{}^{1-\alpha} \left(L_{Y,t}^{SOE}\right)^{-\alpha} K_t{}^{\alpha}\right] + \\ - \mu_t [\eta A_t] = 0, \end{aligned} \tag{B47}$$

$$\frac{\partial H(\cdot)}{\partial \lambda_t} = \dot{K}_t \implies (1-\sigma_{FG}) \left(A_t L_{Y,t}^{SOE}\right)^{1-\alpha} K_t{}^{\alpha} - Lc_t = \dot{K}_t, \tag{B48}$$

$$\frac{\partial H(\cdot)}{\partial \mu_t} = \dot{A}_t \implies \eta A_t \left(L - L_{Y,t}^{SOE}\right) = \dot{A}_t, \tag{B49}$$

$$\frac{\partial H(\cdot)}{\partial K_t} = -\dot{\lambda}_t \implies \lambda_t \left[\alpha(1-\sigma_{FG}) A_t{}^{1-\alpha} \left(L_{Y,t}^{SOE}\right)^{1-\alpha} K_t{}^{\alpha-1}\right] = -\dot{\lambda}_t, \tag{B50}$$

$$\begin{aligned} \frac{\partial H(\cdot)}{\partial A_t} = -\dot{\mu}_t \implies \lambda_t \left[(1-\alpha)(1-\sigma_{FG}) A_t{}^{-\alpha} \left(L_{Y,t}^{SOE}\right)^{1-\alpha} K_t{}^{\alpha}\right] + \\ + \mu_t \left[\eta \left(L - L_{Y,t}^{SOE}\right)\right] = -\dot{\mu}_t. \end{aligned} \tag{B51}$$

From log-differentiation of (B41):

$$\frac{u''(c_t)\dot{c}_t}{u'(c_t)} - \rho = -\frac{\dot{c}_t}{c_t} - \rho = \frac{\dot{\lambda}_t}{\lambda_t}. \tag{B52}$$

From (B47):

$$\frac{\lambda_t}{\mu_t} = \frac{\eta A_t}{(1-\alpha)(1-\sigma_{FG}) {A_t}^{1-\alpha} \left(L_{Y,t}^{SOE}\right)^{-\alpha} {K_t}^{\alpha}} \tag{B53}$$

From (B51):

$$\frac{\lambda_t}{\mu_t} \left[(1-\alpha)(1-\sigma_{FG}) {A_t}^{-\alpha} \left(L_{Y,t}^{SOE}\right)^{1-\alpha} {K_t}^{\alpha}\right] + \left[\eta \left(L - L_{Y,t}^{SOE}\right)\right] = -\frac{\dot{\mu}_t}{\mu_t}. \tag{B54}$$

Using (B53) in (B54):

$$\eta L = -\frac{\dot{\mu}_t}{\mu_t}. \tag{B55}$$

For the BGP, is holds that $\dot{\lambda}_t/\lambda_t = \dot{\mu}_t/\mu_t$, then using (B52) and (B55):

$$\frac{\dot{c}_t}{c_t} = L\eta - \rho. \tag{B56}$$

For the BGP it also holds that $\dot{c}_t/c_t = \dot{A}_t/A_t$ and the growth rate from this social planning problem is

$$g^{SOE^*,FG} = L\eta - \rho. \tag{B57}$$