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# Gödel, Wittgenstein and the Sensibility of Platonism

Abstract:
The paper presents an interpretation of Platonism, the seeds of which can be found in the writings of Gödel and Wittgenstein. Although it is widely accepted that Wittgenstein is an anti-Platonist the author points to some striking affinities between Gödel's and Wittgenstein's accounts of mathematical concepts and the role of feeling and intuition in mathematics. A version of Platonism emerging from these considerations combines realism with respect to concepts with a view of concepts as accessible to feeling and able to guide our behavior through feeling.

Keywords:
concepts, Gödel, intuition, mathematics, Platonism, realism, rule-following, Wittgenstein

*Our knowledge of the world of concepts may be as limited and incomplete as that of [the] world of things.*

K. Gödel[1]

*Auch Gott kann Mathematisches nur durch Mathematik entscheiden.*

L. Wittgenstein[2]

1) Gödel, *Some Basic Theorems on the Foundations of Mathematics and their Implications*, 321.

2) Wittgenstein, *Bemerkungen über die Grundlagen der Mathematik*, 408.

I would like to present a certain understanding of Platonism, different aspects of which can be found in the writings of Kurt Gödel and Ludwig Wittgenstein.[3] I am convinced that no other thinkers in the twentieth century have penetrated more deeply into the issues of Platonism, and no others contributed more importantly to the clarification of some of the systematic misunderstandings underlying both traditional interpretations of Platonism and standard lines of its critique. I shall proceed in four steps. First, I give a general characterization of Platonism as traditionally understood. Second, I outline the Gödelian version of Platonism. Third, I discuss Wittgenstein's attitude toward Platonism and show that however hostile it is with respect to traditional Platonism, it is nonetheless close to Gödel's non-standard version thereof. Fourth, I propose an extension of Platonism along the lines of both Gödel and Wittgenstein.

By putting both Gödel and Wittgenstein against the background of Platonism as traditionally understood it is possible to track the sources of the strong impression of antagonism one has when thinking about their respective views on concepts, mathematics, cognition, and the mind. The central role in producing this impression is played by the assumption – typically made only tacitly – of the object-character of the "Platonic" items, that is abstract entities postulated in order to account, among others, for some features of thought and cognition. It is what Wittgenstein (both early and mature) most certainly rejects, when he talks for example of the misleading picture of concepts and rules as "rails invisibly laid to infinity,"[4] being out there independently of whether we are aware of it and determining in advance the right and wrong with respect to our thinking and behavior. But, no less of a strong rejection of the objectual understanding of concepts and rules is to be found, although on slightly different grounds, in Gödel who, however, considered himself a Platonist. These observations are the starting point of an interpretation of Gödel and Wittgenstein as exemplifying a version of realism on which the problematic, "Platonic" entities are marked by a certain degree of indeterminacy precluding their treatment as objects. An important role in this interpretation is played by sensibility as a faculty of feeling through which concepts and rules make their way to consciousness in their role as guiding our behavior.

## 1. Standard Platonism

By standard forms of Platonism, I understand philosophical standpoints meeting the following criteria:

(a) They posit the existence of entities not reducible to physical objects or psychological phenomena.

(b) These entities are often labeled "abstract entities," but their nature, as construed by the standard view, is better captured by reference to the way they are individuated; namely, they are individuated through their essential properties (whereas physical and psychological entities are individuated through their contingent properties, their *accidentia*).[5] Sometimes this peculiar character of Platonic entities is expressed by saying that each of them is a species of its own – a very useful formulation since it combines abstract-

3) I am deeply grateful to two anonymous reviewers of this paper and to Dr. Mikołaj Sławkowski-Rode from the Editorial Board of *Eidos. A Journal for Philosophy of Culture* for their excellent, penetrating remarks that made me aware of many deficits and unclarities of my arguments. I tried to do my best to profit from their comments, but I am painfully conscious how far my paper still remains from the standards assumed by them. I would also like to sincerely thank Mr. Kamil Lemanek for excellent proofreading.

4) Wittgenstein, *Philosophical Investigations*, 85e.

5) Hegel's and Armstrong's "concrete universals" aside, the only spatiotemporally located objects are concrete particulars, and the spatiotemporal location and relations of such an object are clearly accidental. On the other hand, spatiotemporal locations and trajectories are according to Platonism among the central individuating mechanisms for particulars.

ness (a species is by definition not a concrete thing)[6] and essentiality (no contingent matters, such as the number of exemplars, enter the definition of a species).

(c) *Some* of these entities are related to spatiotemporal (physical or psychological) particulars as their essences. This point needs clarification. The relation *x is the essence of y* is fundamentally ambiguous, and only one of its meanings is of relevance for Platonism. First, it can refer to the role played by a property or a structural feature in the constitution of a particular thing, like "being haired" for a human or "being feathered" for a rooster. One and the same property, for example "being featherless," can be essential for one thing, for example a man, and not essential (accidental) for another, for example a rooster. This point was made for the first time by Aristotle, not least to solve the problem with Plato's anthropological insight culminating in the "featherless biped" formula. In the language of Aristotle, we can say a man is essentially featherless, whereas a plucked cock is featherless only accidentally, and this is why the definition "a featherless biped," rightly applied to Socrates, does not fit a plucked rooster (even if both a rooster and Socrates are essentially two-legged). Second, being the essence of something can mean a relation between two entities: a spatiotemporal particular and an essence (typically a composite one, e.g., "manhood" which is composed of "featherlessness" and "two-leggedness"). This relation is a kind of brute fact which obtains, for example, in the case of Socrates and manhood and does not in the case of a rooster and manhood. Certainly, only the second option does justice to Plato's original standpoint, whereas the first reflects rather the Aristotelian, largely anti-Platonic account of the matter. On the second reading, what belongs to a thing's essence is not a necessary property thereof (according to Plato, spatiotemporal particulars have *no* necessary properties); what is necessary pertains only to the content of an essence. Notwithstanding its drawbacks (clearly seen by Plato himself),[7] this account of essences has some important merits, perhaps even outweighing the drawbacks. First of all, it effectively blocks a whole range of modal speculations concerning putative necessary and contingent properties of sensible particulars. From the standpoint of Platonism thus understood, the thought that particulars possess necessary properties is a mixture of two thoughts, each innocent in itself but potentially harmful when combined: (1) each particular bears a contingent relation (termed by Plato "participation" – *μέθεξις*) to a range of essences whose constitution and relationships are in themselves necessary;[8] (2) depending on context and epistemic and other interests, some of these relations can *appear* necessary, and part of this appearance is the idea of necessity taken from what is truly necessarily constituted, that is, the essences themselves.

(d) The central epistemological tenet of standard Platonism is a strict distinction between essences and concepts. Simplifying, we can say that essences make up an objective order, a world of their own, whereas

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6) Whereas it is open to debate whether universality metaphysically implies abstractness (see important arguments to the contrary, due among others to Hegel and Armstrong) it is rather uncontroversial that species are abstract entities since they are sets. Whether the same is true of the concept "population" as it is used in biology is a different question.

7) See for example the dialogues *Philebus* and *Parmenides*.

8) The idea that a particular might stand in a contingent relation to its own essence can at first glance seem strange, but this is precisely how it *must* seem to a philosopher reading Plato through Aristotelian glasses. On the Aristotelian account of essence it can be illustrated by such familiar contemporary examples as $H_2O$'s being the essence of water. It is the essence of water since it accounts for its causal powers, and, on the other hand, H2O's being the essence of water is itself a necessary fact about water. Formally speaking, a thing's standing in a necessary relation to its own essence is captured by the axiom $\Box A \rightarrow \Box\Box A$ of the systems K4 and S4. On the other hand, if one thinks of an essence as a separate entity, then it is quite natural to assume that, even if due to having such and such an essence a thing has a set of necessary properties, it is nevertheless contingent that it has the necessary properties it has. On this, standard-Platonic account of essence, the above axiom does not hold.

concepts are a psychological affair, something belonging to *ψυχή* (soul), so both must form disjoint sets. This distinction was expressly denied by Aristotle, but one has to bear in mind that this denial was accompanied by a fundamentally different account of essence: for Aristotle, the essence of a thing was its *form*, existing only as instantiated in it. Forms can be multiply instantiated, and concepts are but a special subclass of instantiations of forms – their instantiations in the *ψυχή*. So in a sense we can say that, according to Aristotle, concepts *are* the essences (forms), as instantiated in the order of cognition. The point concerning the relationship of concepts and essences is of special importance in the context of Gödel's Platonism.

For our purposes the most important features of standard Platonism are (a) its twofold objectivism, that is its treatment of essences as mind-independent (and as such intersubjective), and at the same time as having the character of objects, in the sense of self-subsisting, "saturated" (to borrow the term from Frege) entities, as opposed to conceptual and other operations, as essentially depending on an input to yield their values; (b) its strict distinction between essences (ideas), as belonging to the order of being, and concepts, as belonging to the order of thought.[9]

## 2. Gödel: Mathematics as a Physics of Concepts

Gödel consistently referred to his philosophy as Platonism. However, his version of Platonism is certainly a non-standard one. The following are the most obvious differences:

(a) The essences referred to by Gödel as "concepts" are not essences *of* things – a thing's possessing of a certain property (be it essential or contingent) does not entail its standing in a *μέθεξις*-like relation to any concept. For that matter, they are not essences of anything; calling them "essences" is a typical example of the Pickwickian usage of the term, which is so common among philosophers.

(b) Concepts exist objectively, therefore the distinction between essences and concepts, characteristic of standard Platonism (on which essences are distinctively objective and concepts subjective), is not legitimate. The relationship between objectively existing concepts and concepts as items in the order of thought is among the central issues in Gödel's philosophy and, as such, will be discussed at length in what follows.

(c) Objectively existing concepts are in an important sense non-objective: they do not have the character of objects. This view of concepts as non-objective, even if objectively existing, has its sources in Leibniz and Kant, and the Fregean interpretation of the Leibnizian-Kantian heritage might have been especially important for Gödel.

Gödel insisted upon the analytic character of mathematical propositions, but at the same time he expressly refrained from regarding them as tautologies.[10] They can be informative in a strong sense, not merely in the standard-logicist sense in which an analytic proposition can indeed be informative with respect to our limited

9) This feature of Platonism as traditionally understood was especially hard to digest for its modern interpreters who, like the neo-Kantians, were at pains to show that the Platonic entities are at bottom the first attempt to capture the transcendental nature of concepts. However, for all its ingenuity, this interpretation is certainly non-standard. See for example Natorp, *Plato's Theory of Ideas: An Introduction to Idealism*.

10) Gödel, *Some Basic Theorems,* 319–21.

awareness of what follows from our axioms (for that matter, they can be even surprising). They can extend and change our concepts (taken in the subjective sense), and this process is possible precisely because of the relationship between concepts understood subjectively, as forms of thought, and concepts understood as objectively existing. This possibility is a joint effect of two facts: the non-objectivity of concepts and the way objectively existing concepts are represented in the order of thought.

Let us begin with the better-known aspect of Gödel's view, which is related to the Leibnizian-Kantian--Fregean tradition – the non-objectivity of concepts. Here are the important facts:

(1) For Leibniz, concepts were in a sense the fundamental building blocks of all of reality. Reality is at bottom composed of spiritual beings ("monads," which can be translated as "singletons") identified through their perceptions, whose nature is conceptual – each perception being a representation of some manifold *as* a unity. Perceptions are not static, object-like structures representing something outside the substance to which they belong (nothing like a painting depicting a scene), but rather a kind of *doing*: they are operations unifying a manifold, where the unification proceeds through imposing an order. Leibniz himself proposed the term "function" to refer to this sort of conceptual operation and suggested its use in mathematics as a formal discipline dealing with transformations (arithmetic, algebraic and others) operating on different manifolds.[11] We can say, simplifying a bit, that each conceptual operation is an ordering of a certain manifold, possibly leading to assigning a number to it.[12] This last observation is of some importance with respect to Gödel's contribution to the foundations of mathematics.

(2) In an important sense, the concept of function, both with respect to mathematics *and* to conceptual cognition, was a byproduct of the nominalism still reigning in eighteenth century philosophy. The rejection of abstract entities led Leibniz, Euler, and Kant to postulate something, itself not an object (no concrete particular), responsible for the order discovered by mathematics and other sciences. Functions seemed almost perfectly suited for this task.

(3) It was in accord with the nominalist tendency in the early history of the concept of function that Kant extended its use to the domain of subjective thinking, defining concepts as "functions of unity" in judgements,[13] and – even more importantly – reducing strictly mathematical functions to conceptual operations performed on a sensory manifold of "pure intuition."[14]

(4) During the realist turn of the nineteenth century (Bolzano, Frege, Peirce, Cantor and others) the nominalistically motivated concept of function proved, quite surprisingly, very useful for realist purposes. The

11) This happened for the first time in a 1673 manuscript entitled *Methodus tangentium inversa, seu de functionibus* and then, more importantly, in correspondence with Johann Bernoulli in the 1690s. It is due to this exchange with Bernoulli that the Leibnizian concept of function started to exert its immense impact on mathematics, first in the works of the Bernoullis, then of Euler and others. See Youschkevitch, "The Concept of Function up to the Middle of the 19th Century," 55–60.

12) Of course, as a matter of fact in the vast majority of cases, conceptual ordering does not yield an explicit assignment of a number to the manifold ordered. But the only graspable difference between ordered and unordered manifolds consists in the possibility of estimating the number of the elements of the first kind of manifolds.

13) I. Kant, *Critique of Pure Reason*, A 69–70 (B 94–95), A 79 (B 104–105). A and B refer, respectively, to the first (1781) and second (1787) original edition of *Critique of Pure Reason*.

14) Ibid., A 46–49, B 64–74. One should be careful, however, not to interpret Kant's account of mathematics in constructivist terms. That mathematical concepts receive their meanings through intuitive constructions by no means implies logically that they also *refer* to them; on the contrary, it rather suggests that their reference lies elsewhere. Constructions in pure intuition are scarcely more than drawings on a blackboard used to prove general theorems. And just as no one would seriously claim that the Pythagorean theorem is about a certain triangular drawing, it is almost as implausible that Kant should regard intuitive constructions as the subject-matter of mathematics.

essence of this new role of the concept of function was famously expressed by Frege in the distinction between functions and objects, where functions were characterized by "unsaturatedness" (*Ungesättigtheit*) consisting in their being operations needing some input (the role typically played by individual objects, either concrete or abstract) in order to yield an output (the value of a function for an argument).[15] However, unlike Kant, who had a similar idea of functions and concepts as operations, Frege insisted upon their being fully independent from the order of thought, be it understood psychologically as in psychologism or transcendentally as in Kant. Functions (and concepts as a special subclass of functions yielding true and false as their sole values) make up a reality of their own for Frege, irreducible both to physical and psychological objects and processes.

Gödel's account of mathematics as a science of an objectively existing realm of concepts is a clear variation on this Kantian-Fregean theme. Interestingly, in philosophical contexts, Gödel prefers the talk of concepts to that of sets as the subject matter of mathematics, even if he admits that taking mathematics to be reducible to abstract set theory is "the simplest and most natural standpoint."[16] It is with respect to this set-theoretical standpoint that Gödel gives his basic formulation of the inexhaustibility thesis: mathematics, understood as set theory, is inexhaustible in Gödel's sense because of two facts: (1) the impossibility of capturing all the infinitely many axioms of set theory in a finite rule of production;[17] (2) the dependence of even relatively simple parts of mathematics (like the Diophantine problems) on truths concerning arbitrarily high levels in the hierarchy of sets.[18] In other words, the phenomenon of inexhaustibility presents itself not merely on the edges of mathematics, through the possibility of introducing more and more powerful axioms describing exotic, immensely complicated structures, but also, so to speak, at home, in our closest vicinity, in the domain of relatively simple problems definable by means of elementary arithmetical notions. So, if Gödel – at least for philosophical purposes – remains ultimately unsatisfied with this account of mathematics as abstract set theory and switches to another – in terms of concepts, it must be for some deeper reasons than the insufficiency of the set-theoretical approach to capture the infinite complexity of the mathematical world.

I think that the decisive reason behind this move from sets to concepts was epistemological in nature. Gödel considered mathematical cognition to be conceptual in character, relying on a kind of conceptual intuition remotely connected with the Kantian pure intuition (whose essence is also at bottom conceptual.[19]) Gödel's conceptual intuition, issuing in analytic but nontrivial claims making up the substance of mathematics, is a very special case of the employment of concepts. It consists in using concepts in a way most suitable for revealing their own properties, not the properties of things normally apprehended with their help. Gödel gives the example of simple analytic statements like "it will rain or it will not rain."[20] The purpose of the statement just quoted is not to say something about the weather (since it is true whatever the conditions) but to reveal certain

15) See, for example, Frege, *Function and Concept*, 24–25, 31, 38.

16) Gödel, *Some Basic Theorems*, 305.

17) Ibid., 305–306.

18) Ibid. 306–307.

19) Kantian pure intuition, even if made of the non-conceptual stuff of sensory impressions (as *sensory*, human intuition – be it empirical or pure – is always dependent on affection producing impressions), is a *cognitive* achievement insofar as what is intuited is a concept, not a sensed particular. A white triangular drawing on a blackboard can be both a piece of empirical (e.g. when we measure one of its sides and find it to be 60 cm long) and of pure intuition (when we attend to the rule of its production, identical with the concept "triangle.")

20) Gödel, *Is Mathematics Syntax of Language?*, 336, 362.

properties of the concepts "or" and "not."[21] These properties are normally not spoken of and remain hidden in ordinary discourse, even if they make it possible. It requires special effort and attention to understand their presence, and it is very reasonable to call this attention *intuition*, since it proceeds immediately, with the help of no other concepts (if we do not have this intuitive grasp of "or" and "not" in the first place, it will be of no use to introduce them with the help of other concepts like "sentential connective.")

But the concepts studied in mathematics are of course not identifiable with the concepts whose intuitive grasp is the mode of mathematical cognition. The latter belong – if not exclusively – to the order of thought, in a sense they are thoughts, more precisely: they are formal structures of thought, not their psychological ingredients. Meanwhile, large parts of mathematics are about items principally unable to thus enter our thoughts, either due to their sheer vastness (like the natural number series as a completed whole) or their complexity (like spaces of many – even if finitely many – dimensions). At this point, ontology enters the Gödelian picture of concepts. It appears that at least some of them are capable of a twofold mode of existence: an objective (as "concepts in themselves") and a subjective one (as forms of thought). It is actually the case with some finitary concepts, with many such concepts it is possibly the case, and whether it is also a possibility with infinitistic concepts depends on some obscure metaphysical considerations pertaining to the existence of infinite minds, into which it is not necessary to penetrate now. Before I make some conjectures as to what the existence of a concept outside the mind might consist in, let me briefly outline the general scheme of how mathematical cognition as conceptual cognition of objectively existing concepts works according to Gödel.

Gödel famously saw a strict analogy between mathematics as a cognition of objectively existing concepts and physics as a cognition of an objectively existing material world.[22] The conceptual intuition briefly described above is the counterpart of sensory perception for the outside physical world. Both are in a sense "in the mind," both serve as starting points for a cognition of a reality outside it (physical and conceptual). They are both in the mind primarily in that what they directly operate on, or better what they are made of, are mental items, impressions, feelings, considered either as representing things (in perception) or as representing concepts – better yet, as incorporating concepts. But the "in the mind" idiom can be slightly misleading in this context due to its narrowness and dependence on Cartesian, consciousness-oriented epistemology. Equally well one can say that both sensory perception and conceptual intuition operate on a special subclass of spatiotemporal objects *outside* our bodies, like all kinds of diagrams for mathematical concepts or the everyday, middle-sized objects in our immediate vicinity, which we can think of as percepts representing, for example, the broader spatiotemporal context (ultimately the observable universe)[23] or some basic realities theorized about in physics, and so on.

What is important is that conceptual intuition, pretty much like sensory perception, although a fundament of cognition, is far from infallible. It is prone to illusions (like for example the illusions surrounding the concept of a set)[24] and thus requires control and support from theory, to which at the same time it gives

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21) To be sure, there is still some role to be played by the context. Certainly, the utterances of the kind "it is raining or it is not raining," if not strictly informative about the weather, can nevertheless sometimes convey an information which is not strictly logical, for example about a speaker's uncertainty about the weather.

22) Gödel, *Some Basic Theorems*, 313, 320. Cf. Gödel, *Is Mathematics Syntax of Language?*, 360.

23) In some respects, even its unobservable parts (beyond the so-called particle horizon) may be represented, namely insofar as we can theorize about them using analogy or some general considerations pertaining for example to the isotropy of the universe.

24) Like the seemingly obvious idea that whenever we have an intuitively understood concept (e.g. the concept of a set), we can form a set of all and only those items that fall under that concept. Or – to name a more sophisticated example – the belief of for example Cohen that the Continuum Hypothesis is "obviously false," since the set of all countable ordinals ($\aleph_1$) is generated by a far less powerful principle (the replacement axiom) than C, which is generated by the power set axiom. Well, supposing that CH might after all be true, the seeming obviousness of its falsity for Cohen and many others would be an illusion, perhaps to be accounted for by some prop-

foundation. Accordingly, a mathematical theory behaves in several respects like a physical theory: (1) it has an empirical basis (conceptual intuition) which, like sensory perception, is indispensable even if not infallible; (2) theories concerning non-intuited concepts and structures do not logically follow from intuitions, they (to be precise, their axioms) rely partly on induction; (3) what is stated in (2) finds ultimate support in Gödel's realistic faith in an independently existing realm of concepts we discover, not create.[25]

Notwithstanding some striking analogies between sensory perception and conceptual intuition, there is also at least one fundamental difference: sense perception and its data (from immediate feelings to the perceived features of the environment) derive their representational capacities from their causal embeddedness in the order of things and processes represented, whereas conceptual intuition stands in no causal relation to concepts, neither the immediately intuited ones (like, e.g., the concept of addition of integers) nor the non-intuitable but mathematically cognizable concepts like real or complex numbers or many-dimensional spaces. The presence of, for example, the concept "triangle" in what I am doing when drawing a triangular figure on a blackboard explains what is happening, but not in a causal way. It rather resembles the way arithmetical truths explain the results of physical calculations. When I put together two apples and three pears and find that I have five fruits, we are tempted to say "It is so *because* two plus three equals five." While it is certainly an explanation, it is almost certainly not a causal one (after all, what would it mean to say, e.g., "If two plus three *hadn't* been equal to five, then upon counting two apples together with three pears I *wouldn't* have obtained five"?). The case of concepts is slightly more complicated, since one *can* in a sense causally explain my drawing a triangular shape by my being in possession of the concept "triangle" (as a fact about, say, my brain). But the way the concept "triangle" features in this explanation is utterly different than when I say, for example "I am drawing three lines enclosing a surface, *because* a triangle is by definition such and such a figure." In the latter case it is hard to imagine what kind of counterfactual conditionals would support this claim if it were meant as a causal one. Tentatively, this second kind of explanation could be considered as a species of interpretation: by describing a piece of behavior as governed by a concept, I am making sense of it – not explaining it causally, but rather explaining its meaning.

To be sure, the above suggestion that concepts behave like meanings with respect to linguistic behavior contains a lot of question-begging. I am going to address this problem more systematically when I come to Wittgenstein, for the time being suffice it to say that conceptual intuition, as a species of immediate awareness of a concept through its manifestations in consciousness and behavior, differs from sensory perception in that concepts do not "affect" us in any sense, but is at the same time like perception in that it plays the role of a fundament – if a fallible one – of our knowledge of the world of concepts.

Let us now turn to the status of mathematical theories according to Gödel. Sometimes they make a huge departure from what might be sensibly regarded as intuitable, so the question arises: what serves as the bridge between the relatively simple concepts we are able to access intuitively[26] and the rather less intuitive concepts studied in more advanced branches of mathematics? Gödel's answer to this question can be read from the main line of his argumentation in favor of realism with respect to the mathematical world. Namely, the higher-order facts to which only theory gives us access prove to be essential for understanding and solving problems belonging

erties of set-theoretical concepts and axioms, as giving us only a partial grasp of the universe of sets. See Cohen, *Set Theory and the Continuum Hypothesis*, 151.

25) "If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics;" see Gödel, *Some Basic Theorems*, 313.

26) Like for example addition of natural numbers. To be sure, their intuitive character is partly a relative matter depending on training we receive in childhood.

to lower levels, the intuitive levels, for example that of arithmetic, included. And this essentiality, or fruitfulness, is what matters when it comes to assessing ontological claims concerning mathematical objects proper, that is concepts considered as items in principle capable of existing independently of language and thought.[27] The situation here is analogous to the way theoretical objects feature in scientific explanations of observable (perceptible) things, properties and events, where explanatory success supports our faith in there being *something* like the postulated entities. The analogy seems to me all the more profound, as the relations underlying the explanatory service done by entities postulated in fundamental physical theories are typically non-causal,[28] unlike the relations underlying the perceptual givenness of the objects of our everyday experience, where causal dependence plays a decisive role.[29]

One more aspect of Gödel's view deserves closer inspection. Consider a typical concept in the mind whose intuitive understanding is rather unquestionable (at least for those who admit of intuitive understanding in the first place): the concept of addition. The kind of knowledge Gödel has in mind can be read from remarks like the following one: "it is by exactly the same kind of evidence that we judge that 2 + 2 = 4 and that $a + b = b + a$ for any two integers $a$, $b$."[30] The evidence in question seems to be an intuitive understanding of addition as an operation, and thus it seems to be something in the order of thought. But the intuited concept "addition" itself (or "… + … = …") is – according to Gödel's radically anti-psychologistic *and* anti-formalistic attitude – nothing psychological or purely conventional, but an objectively existing item belonging to an order of its own, and only partly and imperfectly perceived or intuited in thought. Analogously, a realist (i.e., an anti-phenomenalist and anti-conventionalist) will say of a physical object that it exists independently of the mind and is only imperfectly represented in perception. Also – still within the same analogy – one can say that in both cases this independence consists, among others, in the fact that there is always more to the item perceived than what is really represented (and that is so no matter how far our interpretations of what we perceive might go).

In the case of concepts, this independence has at least two dimensions. The first is quite obvious: even such "simple" concepts as addition conceal an immense complexity, not even sensed on the level of their intuitive grasp. However, this complexity typically reveals itself in connection with other concepts, for example multiplication, in itself almost equally simple and intuitively graspable. But the moment we consider both concepts in combination we immediately face problems of unimaginable complexity, like Goldbach's conjecture (that every even number greater than two is representable as a sum of two prime numbers), where all intuition for how to prove it fails us. This complexity together with the apparent simplicity of concepts as intuited items resembles the unimaginable complexity of physical systems presenting themselves in perception as the familiar middle-sized objects of everyday experience. The second is that intuition, however partial and indistinct with respect to concepts, is at the same time systematically ahead of all physical, psychological and syntactic facts pertaining to the actual execution of operations with which concepts can be identified. This was the chief

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27) See, for example, Gödel, *Some Basic Theorems,* 307, 314, 318; cf. Gödel, *What is Cantor's Continuum Problem?*, 182, 261.

28) It would be extremely strange to say, for example, that what happens at the quantum level is *causally* responsible for the objects and events making up the world of everyday experience (the notorious "tables and chairs" of all the classrooms where philosophy of physics is lectured about). The true story (if there is any truth to physics in the first place) is rather that what "really" happens is quantum processes and there is nothing non-quantum to be accounted for in causal terms, since even measurement is at the bottom a quantum process.

29) There is a seemingly obvious disanalogy between the inter-level relation of grounding between for example basic and non-basic physical properties on the one hand and the relation obtaining, say, between higher levels in the hierarchy of sets and the intuitively graspable facts pertaining for example to finitary arithmetic on the other. Whether this disanalogy can be shown to be after all not so fundamental as it seems to be, is a topic deserving a separate study.

30) Gödel, *Some Basic Theorems*, 311.

reason behind Gödel's rejection of finitism and its attempts at a reduction of mathematics to, for example, the syntax of language.[31]

The main purpose of this section was to outline Gödel's conception of mathematics as a science of an objectively existing realm of concepts. The non-standard character of Gödel's Platonism lies chiefly in (a) the non-objectual nature of concepts making up the subject-matter of mathematics and (b) pointing to an analogy between mathematics as a study on the realm of concepts and physics as a study of the material world. The last point issues in Gödel's insistence upon the role of incomplete induction in mathematics and the unavoidable margin of uncertainty of mathematical cognition, as well as its proneness to illusions. An equally important feature of Gödel's account is the role it ascribes to intuition. I propose to interpret it as an extension of the Kantian conception of mathematics as based on pure sensory intuition: whereas the sensory intuition gives us access to the more elementary concepts of arithmetic and set theory, the concepts making up the subject-matter of more advanced branches of mathematics are introduced as hypotheses needed to solve some problems couched in the more elementary, intuitively accessible terms, but not readily solvable on the intuitive level.

## 3. Wittgenstein's Secular Platonism

Wittgenstein's own standpoint in the *Tractatus* was, against the suggestion of Gödel,[32] very far from the conventionalism[33] of Hahn, Carnap and Schlick (conventionalism probably remained the standpoint Wittgenstein most firmly rejected throughout his life). But he seemed to share with the finitists the view that mathematics does not aim at describing an independently existing reality, even less at registering or codifying the psychological processes of mathematical reasoning. However, the reasons behind what might appear as Wittgenstein's finitism were fundamentally different from those of Carnap and others, and, however strange it might seem, closer to Gödel's than commonly believed.

According to Wittgenstein's early doctrine, mathematics, as a "logical method,"[34] consists of equations, that is pseudo-propositions (*Scheinsätze*) whose role consists in revealing some aspects of the logical form of the world.[35] Other aspects of the same form are revealed by the tautologies of logic, and both kinds of pseudo-propositions make up the body of senseless (*sinnlos*), not nonsensical (*unsinnig*), propositional facts. Two important lessons can be drawn from the teachings of the *Tractatus*:

(1) As to tautologies, and logic in general, their task is not definable without reference to the truth-values of elementary propositions and therefore to semantics. Logical constants are – whatever they might look like in the context of different notations – at the bottom, different distributions of truth-values over a set of points (elementary propositions) for which the corresponding compound proposition is true (alternatively, for which it is false). The role of tautologies is different: they display some "global" properties of the world, in that they are true for all distributions of truth-values over the set of *all* elementary propositions. That

31) Gödel, *Is Mathematics Syntax of Language?*, 340–43.

32) See for example ibid., 334.

33) What I mean is a version of conventionalism in some respects strongly diverging from the conventionalism of Poincaré and Duhem which saw it as a matter of convention which mathematical theories (e.g. which geometry) we use to describe physical reality. As opposed to this, Carnap and others extended the conventional treatment to mathematics itself, regarding it as a description and codification of syntactical conventions embodied in the language of science and in ordinary language.

34) Wittgenstein, *Tractatus Logico-Philosophicus*, 6.2, cf. 6.234, 169, 171.

35) Ibid., 6.2, 6.21, 6.22., 169.

is why Wittgenstein insists upon their being pseudo-propositions: they exclude nothing, and precisely for this reason they carry no information – they are devoid of content. This aspect of Wittgenstein's theory is rather easily reconciled with Gödel's doctrine of analytic (conceptual) truths: the difference almost completely boils down to different ways of understanding content. For early Wittgenstein, content (sense) always means *factual* (and, as such, contingent) content, whereas for Gödel there are (at least) two kinds of content: factual (empirical), akin to Wittgenstein's sense, and logical (conceptual), displayed by the analytic propositions of logic and mathematics.

(2) With equations the story is different and, in some respects, more complicated. They penetrate more deeply into the structure of compound propositions (non-tautological propositions included), and their use lies primarily with non-tautological propositions. Although equations look like identities between results of different operations on numbers, what they truly convey in this form is equivalences between results of different operations on propositions, no matter what kind of propositions they are.[36] The policy behind this approach is, among other things, to account for the astonishing fruitfulness of mathematics in empirical science. The propositions with which we describe the world (like that the bottle is on the table or that every two bodies attract each other with a force directly proportional to the product of their masses) are – without exception – immensely complex truth-functions of elementary propositions. Mathematics, through its equations, allows us to describe this functional dependence – and through it the structure of compound propositions – in an extremely efficient manner. Leaving aside the question of the adequacy of this account of the role of mathematics in empirical science,[37] let me point to one problem which in the context of later Wittgenstein is of critical importance. I term it the problem of invariance of operations with respect to complexity. In the simplest terms, on what grounds do we suppose that, by applying a logical operation several times successively to its own product, we are performing one and the same operation? Putting it in terms of numbers, on what grounds do we believe that "addition," applied, say, to 1 and 2, is the same operation as applied to 2 and 3, 1001 and 1002, 5 and 7, and so on. On Frege's account, there is no such worry: addition is the set of ordered triples <1, 2, 3>, <2, 3, 5>, <1001, 1002, 2003>, <5, 7, 12>, and so forth, and its identity is guaranteed by its extension. On Wittgenstein's intensional approach,[38] such abstract definitions are unavailable; we have to rely upon our grasp of what we are doing when, for example, adding two numbers. But that is precisely what the problem of invariance is about. We intuitively feel that we are doing the same thing (e.g., adding numbers). But on what grounds?

That even the simplest and clearest concepts like addition rely on a feeling of certainty concerning what to do at each step of the corresponding procedure, together with the observation that this certainty cannot be justified by any kind of facts concerning behavior, material brain, immaterial soul or whatever else, belong famously to the fundamental tenets of Wittgenstein's later philosophy. They are, one is tempted to say, its pillars, if only such a predominantly dialectical and aporetic philosophy had any "pillars" in the first place.[39] Both insights

36) Ibid., 6.23, 6.231, 169.

37) After all, mathematics seems to work in science through such factors as (1) representation of different aspects of physical, economic and other processes in the form of numbers (mostly real and complex) and (2) model-building using the tools of analysis, geometry and topology. Neither of these seems to derive its fruitfulness from the fact that "numbers are exponents of [logical] operations," ibid., 6.02,1, 155.

38) The intensional character of Wittgenstein's treatment of numbers is best expressed in the proposition 6.031 of the *Tractatus*: "The theory of classes is altogether superfluous in mathematics." In other words, neither in order to do nor to understand mathematics do we need to take account of the extensions of concepts.

39) Save, perhaps, the rather poorly justifying "pillars" of the language-game described in §2 of *Philosophical Investigations*.

are famously elaborated within the so-called rule-following argument, Wittgenstein's master argument in *Philosophical Investigations*, even if it is not fully clear what this argument is supposed to prove. In the following, I give an interpretation of the argument in the spirit of realism and along lines similar to Gödel's theory of concepts and conceptual intuition. This will be a rather unorthodox interpretation of Wittgenstein; it will also shed some unexpected light upon Gödel's philosophy of mathematics and mind. The standpoint I am going to attribute to Wittgenstein deserves to be called "secular Platonism," because unlike standard Platonism it dispenses with any kind of immediate access to concepts "in themselves" (supra-sensory vision) and relies fully on blind rule-following which, once reflected upon, bears a striking analogy to Gödel's conceptual intuition.

One of the results of the rule-following argument is that no course of action can be justified as being the correct interpretation of a rule, since for every imaginable justification we can imagine a different (i.e. nonequivalent) course of action justifiable in exactly the same way. But one should beware of the frequent error of interpreting this as implying that there is no right and wrong with respect to rule-following and that total relativism is the only remaining option. That we have no justification for doing something by no means implies – surely not for Wittgenstein – that it is wrong or incorrect to do so, nor that it is equally correct (or incorrect) as doing anything else. Wittgenstein's point is not to show that, say, every continuation of an addition is correct, but that it is impossible to objectively justify any such continuation as the intended one. The problem is not so much how to justify my intention as the correct interpretation of, for example, addition, as it is to communicate my intention to others (or, for that matter, to myself). Whatever I try to communicate as my intention, I have to reckon with the answer "Oh, I thought you meant *this*…," followed by an utterly odd interpretation of my meaning. But if this is so, then what makes me so certain as to what I ought to do myself? According to Wittgenstein, it has nothing to do with my saying or thinking to myself what I should do. "I follow the rule *blindly*"[40] – this is probably the most condensed single expression of Wittgenstein's view on rule-following. But if, after all, there is right and wrong with respect to our following a rule,[41] then it must be due to something. It must have some ground, and a rather deep one, judging from the fundamental character of the rule-following phenomenon. So, *the* question seems to be as follows: what kind of ground is it, and where does it lie? In many respects this question resembles the one asked by Kant: what is the ground of our ability to immediately orient ourselves in space, insofar as it depends on our ability to distinguish "right" and "left" as directions in space.

At bottom it is the same problem that once opened the gate for Platonism: if there is right and wrong with respect to our behavior, then it must have some ground. And the Platonic answer is the most straightforward you can imagine: the right-wrong distinction is grounded in the nature of reality; reality is, so to speak, "orientable" with respect to right and wrong.[42] It means not merely that there are some standards for how to correctly proceed, but that these standards are readily, intuitively recognizable as instantiated in particular

40) Wittgenstein, *Philosophical Investigations*,1:219, 85, 85[e]. I translate "*folgen*" in German as "follow" instead of "obey," as G.E.M. Anscombe does. Anscombe's translation is misleading because it suggests that Wittgenstein is analyzing a different concept ("obeying a rule," whatever that might mean), whereas what he has in mind is precisely the same concept of rule following that lies at the center of his discussion. Even if I were happy to obey Anscombe's translation, I could not follow it.

41) In one of the concluding paragraphs of his rule-following discussion, Wittgenstein says: "Disputes do not break out, among mathematicians, say, over the question whether a rule has been [followed] or not [*ob der Regel gemäß vorgegangen wurde, oder nicht*]. People don't come to blows over it, for example. That is part of the framework on which the working of our language is based" (Wittgenstein, *Philosophical Investigations*, 240). I read it as implying that in matters of rule-following there not only *is* right and wrong, but it is so unambiguous that, save for rare and limiting cases, there is no place for doubt or controversy.

42) Precisely as in the Kantian doctrine of space and time – a distant echo of Plato's *Timaeus* – the fundamental feature of both forms, crying out for a deep explanation, is their orientability – of space with respect to right and left and of time with respect to past and future.

pieces of behavior. This is precisely where Wittgenstein's critique applies to standard Platonism. The essence of this critique lies not in questioning the existence of rules, but rather the way they find expression in behavior. Wittgenstein's main point is that, however unhesitatingly we follow the rules, it by no means implies that the contents of the rules are clear to us. We intuitively feel the rules, but once we try to put this intuition into words or to capture it in symbolic formulae of whatever kind, all clarity and distinctness of intuition disappear, since any formula can be brought into accord with any kind of behavior.

## 4. Toward Sensible Realism

Gödel's conceptual intuition and Wittgenstein's blind rule-following seem to be two different but complementary perspectives on one and the same problem. It might be called the problem of incommensurability between intuitions and concepts – a distant cousin of Kant's problem of the heterogeneity of concepts and intuitions, with the power of judgement as a mediating faculty between them. Gödel's intuition is severely restricted in its scope and can be easily misleading – that is why it needs support from theory. For Wittgenstein it is rather the abstract formulations of rules that are in need of support because they are notoriously ambiguous in themselves. They receive support from our unhesitatingly following the rule, say, of addition of integers, whereas the formula "x + y = z" can be interpreted in many different ways. So, it seems, Gödel's and Wittgenstein's accounts go in opposite but fully complementary directions: the first points to the need of supporting intuition with theory, the second to the dependence of theory on "blind" intuition.[43] But the view that opens up from both vantage points is one: it is the view of concepts and rules as objective, if not of the character of objects. It is a view that deserves to be seen as a form of Platonism, even if a rather nonstandard species of it. In the following, I explore a further aspect of this view, concerning the relationship between concepts and sensibility, as well as the status of *sensibilia* in general.

Immediately following the rule-following discussion (beginning with § 243 of the *Investigations*) is Wittgenstein's exploration of the problem of privacy, of sensations, and private language. It bears striking resemblance to the problem of rules: just as the phenomenon of being guided by a rule is not accountable for in causal-functional terms and not derivable from any formula, the private episodes of "feelings, moods, and the rest"[44] resist linguistic description: they are largely inaccessible for public language, and a private language adequate for their description is in principle out of the question, due to the inconsistency of the very idea of such a language. In effect, we have two seemingly heterogeneous phenomena: being guided by rules on the one hand and private episodes of feeling on the other that nevertheless both behave very similarly with respect to the possibility of their linguistic expression. Is this a mere coincidence or a manifestation of some deep-lying feature?

Let us begin with Wittgenstein's observations concerning two radically different modes of expression. This well-known theme culminates in § 317: "Misleading parallel: the expression of pain is a cry – the expression of thought, a proposition." As we remember from the *Tractatus*, a proposition expresses a thought due to a morphism between the structures of both: a thought is, so to speak, a mental proposition, and that's why it is readily expressible in a linguistic proposition. That the same does not work with feelings might suggest (and has indeed suggested to many a thinker) that they are structureless "atoms" admitting of no formal relations with anything like propositions. But equally well – and I think this was the case for Wittgenstein – it can be interpreted as a symptom of their immense complexity, drastically surpassing the characteristic "manifold" of propositions. Feelings cannot be expressed like thoughts; that is why they are sometimes regarded as "inex-

43) Once again, we see a clear analogy with Kant: although "intuitions without concepts are blind" (Kant, *Critique of Pure Reason*, A 51, B 75), they are nonetheless necessary to disambiguate pure and reflexive concepts.

44) Wittgenstein, *Philosophical Investigations*, 243.

pressible." But of course, they are expressible, only by means of a different relation between what is expressed and the means of expression.

A similar ambiguity holds with respect to the expression of rules. What makes the topic of rule-following so mysterious is the seeming parallel between the expression of rules in the form of different instructions and formulae and their expression in behavior. The parallel is not exactly the same but equally misleading as the one between a cry and a proposition: we normally consider appropriate behavior as the necessary and (typically) sufficient criterion of mastering a rule, and since we also use linguistic representations of rules (e.g., arithmetical formulae), we tend to believe that both modes of expression work the same way. It is not exactly the same since, *on the face of it*, thoughts and feelings are generically different mental items differently expressed, whereas, in the second parallel, one and the same rule can be expressed either in behavior or in a formula or prescription. The tendency to think that both modes of the expression of rules are equivalent receives additional support from the fact that philosophers typically work with the model of *linguistic* rules, where the dominant form of rule-following behavior is linguistic (speaking, writing, etc.), so that sometimes expression in a formula and in a piece of behavior are outwardly indiscernible. However, Wittgenstein's project is to show that in fact these are two completely different expressions of a rule: the one in behavior is "blind" (unhesitating, instinctive, unreflective), but after suitable training practically unerring. The one in a formula is proposition-like, typically reflective and systematically ambiguous, since every formula allows for a multiplicity of interpretations.

So it seems that in the context of its expression in behavior (linguistic behavior included) a rule appears as an unstructured atom, whereas in the context of its linguistic expression (e.g., in a formula) it reveals a structure, if rather ambiguously. In other words, in contexts of the first kind, a rule acts like a feeling with respect to its manifestations in behavior; in those of the second kind, a rule acts like a thought with respect to its expression in a proposition. It suggests a two-aspect view of rules, as akin both to feelings and concepts but, depending on the context, revealing only one side of their nature at a time. Rules issue in behaviors in a feeling-like way, resembling a cry issuing from pain – instinctively, unhesitatingly, blindly. Rules appear in this context as mere dispositions to such and such behavior, like black boxes showing only a reading on an external display. But when it comes to expressing (saying, illustrating with an example, drawing, etc.) what a rule prescribes, its content, all blindness comes to an end; the rule appears as a structured thing, and its structure as immensely complex. An important dimension of its complexity pertains to the fact that virtually every rule covers a potential infinity of cases and *in a sense* can be identified with the totality of them.[45] But for Wittgenstein the decisive point is the relationship between two structures: the structure of a rule (as an operation applied to an array – be it finite or infinite – of arguments) and of a proposition, meant to reflect the first. In his youth, Wittgenstein believed in the possibility of finding homomorphisms between such structures (propositions are logical pictures of facts due to sharing aspects of their logical form). The mature Wittgenstein of *Philosophical Investigations* – for reasons lying beyond the scope of this article – does not have this faith any more. To be sure, structures are still important but in a less straightforward way: a structure (say, a proposition) reflects another (say, a state of affairs) only relative to an interpretation, and interpretation is never determined by the syntactic structure alone. Applied to the relationships between rules, their expression in behavior and their linguistic expression in formulae and other proposition-like structures, the account in the *Investigations* yields the following picture. Observing a rule is not a matter of interpretation, otherwise it would lack the character of "blindness" (unhesitating, unreflective, mostly unerring reproduction of a pattern of behavior). From the standpoint of a follower, the rule she follows need not reveal its structure; therefore, it may appear as simple (better, under the aspect of simplicity). The mode of our being conscious of rules thus expressing themselves in behavior is

45) Much like Frege's *Wertverlauf einer Funktion*, defined as set of ordered pairs ‹argument, value for this argument›.

feeling, referred to by Wittgenstein as the experience or feeling of "being guided."[46] In contrast, when looking for a formula in order to isolate the structural features of a rule, which are responsible for the way it guides my behavior, I am not guided by this very same rule any more (perhaps I am guided by other rules I might try to interpret on another occasion), and consider it as, in a sense, a static object. But this static object – a rule as captured in a formula, prescription, instruction, and so forth, – is indeterminate precisely with respect to what behavior is in accord with the rule.

However strongly that might suggest an affinity between rules/concepts and quantum systems (finding an expression of a rule being a counterpart of measurement and "blind" rule-following of a spontaneous evolution of a quantum system), for the time being it is at most an analogy requiring further elaboration. Perhaps, after all, it might turn out that rules, concepts and, together with them, mathematical entities have an indeterminacy of their own. Such indeterminacy would by no means entail that they are not real, that they are mere constructions or inessential accompaniments of purely formal transformations of signs, just as no such unreality is entailed by the indeterminacy of a quantum system (on the contrary, they seem to be far more real than the seemingly determinate systems of classical physics and everyday experience). This line of reasoning would lead to a form of Platonism that is much more extravagant than standard Platonism. However, for the time being I must leave this possibility unexplored and focus on what is in a sense half-way between standard Platonism and a hypothetical (and highly problematic) "quantum" Platonism, namely on what might be called a "sensible Platonism" I believe to be derivable from Gödel and Wittgenstein. Instead of providing an elaborate narrative, let me define this form of Platonism with a few tenets.

(a) Neither Gödel's concepts nor Wittgenstein's rules[47] have the character of objects; they are rather kinds of doings, conceived under the aspect of their unchanging (or better, recurring) form. In other words, they are operations or functions intensionally understood – a concept stemming indirectly from Kant and directly from Frege.

(b) They in a sense accidentally become instantiated in our thought and behavior. On the face of it, it looks very different for both thinkers: Gödel seems to focus on "private" thoughts as realizations of concepts, whereas Wittgenstein focuses on "public" behaviors as realizations of rules.

(c) Gödel stays within the broadly Kantian-Fregean paradigm in which thinking – insofar as it interests logic and epistemology – is not identifiable with psychological phenomena which, at least in the case of human psychology, happen to be "private" (not directly perceptible to anyone except their possessors). Even if some psychological processes are indispensable for thinking, logic and epistemology investigate their intersubjective, publicly communicable content, and it is precisely this publicly available aspect of thinking that concepts are instantiated in. So, on closer inspection, Gödel's thinking is not very far from Wittgenstein's rule-following behavior. Surely the difference between their conceptions cannot be identified with the private-public distinction.

(d) Both concepts and rules, insofar as they are instantiated in linguistic and other behavior, can be experienced, and this experience has a necessarily sensory character. The sensing has of course a private aspect (feeling), but what it makes cognitively available is the felt presence of concepts and rules in what we do – and this by no means reduces to the private, rather, on the contrary, it is all public.

46) Wittgenstein, *Philosophical Investigations,* 1:169–177, 68$^{e}$–72$^{e}$.

47) The rules of the *Investigations* are a fairly clear counterpart of the formal concepts of the *Tractatus*. In both cases, complete ineffability in propositional form contrasts with the ease and immediacy with which concepts and rules can be demonstrated (however, the modes of demonstrating are very different in both works).

(e) What we sense is always aspects, parts of concepts and rules. For example, what we sense is never the concept of addition taken absolutely, but only insofar as it is present in the actual instances of addition performed so far.

(f) This readily accounts for one salient common feature of both conceptions: the necessary incompleteness, uncertainty and indeterminacy of our knowledge of concepts and rules, insofar as that knowledge is meant to be contained in definitions, formulae, axioms or written prescriptions. They always go beyond what is intuitable but, on the other hand, are unable to determine further cases in a unique way. That is why the formulae always have to be verified by actual procedures of concept application and rule-following.

(g) On the other hand, both conceptual intuition and the closely related experience (feeling) of being guided by a rule require support from all kinds of guidelines (definitions, axioms, formulae, prescriptions, drawings, diagrams, etc.), because both can be misleading and, left to themselves, in the long run, inevitably lead us astray. All of this looks pretty much like the relationship between sensory intuition and theory in natural science.[48]

To sum up, let me point to two features of this version of Platonism which make it fundamentally different from the standard doctrine. First, it does not introduce a special kind of *object*, different both from physical and psychological phenomena, but rather a class of operations – objective but lacking the character of objects. Second, the primary mode of cognitive access to concepts and rules thus understood is through a kind of sensory experience, feeling, and not, as in standard Platonism, through pure reason detached from sensibility.

However, one fundamental problem is left open by this account. According to standard Platonism, the mark of ideas was their immutability and obedience to the laws of non-contradiction and excluded middle, as opposed to sensorily experienced reality, conceived in a Heraclitean fashion, as in many respects underdetermined or even inconsistent. The "new way of ideas," here attributed to Gödel and Wittgenstein, is marked *inter alia* by a primarily sensory access to concepts and rules (conceptual intuition and the feeling of being guided). But precisely this shift in epistemology, especially in the context of contemporary science, puts into question the anti-Heraclitean character of Platonistic ontology. To be sure, this new physics-oriented epistemology of mathematics, envisaged by Gödel and tacitly assumed by Wittgenstein, is compatible with the view of Platonic reality as a world of immutable, fully determined, supra-sensible entities (even if conceived as functions, not objects), and such was probably the working assumption of both thinkers. On the other hand, the overall scientific context of our times invites us to reconsider this last stronghold of traditional realism with respect to the mathematical world and to replace it with an equally realistic view of abstract entities as characterized by a degree of indeterminacy that only partially gets resolved when we try to observe and capture the way they find expression in our thinking and doing. But further pursuit of this line of thought is a project for another time.

What I tried to outline as a non-standard version of Platonism to be found in the writings of Gödel and Wittgenstein is certainly far from being just another interpretation of Plato. Readers might wonder whether what I claim does not after all boil down to an instance of the observation made by A.N. Whitehead that all history of Philosophy after Plato is just footnotes to his works. Though I completely agree with Whitehead on this point, I nevertheless believe that there is a strand in the history of thought that not only stems from Plato

48) It would be unfair to unrestrictedly attribute this view to Wittgenstein, who, in the epoch of the *Investigations*, frequently stressed the difference between mathematics and science. On the other hand, it should be noted that the picture of science in the *Investigations* is pretty much like the one in the *Tractatus*, whereas that of mathematics is fundamentally different. Taking into account that natural science was not the focus of Wittgenstein's philosophy from the 1930s onward, it seems not utterly implausible that if he had reflected upon it more, his picture of science would have changed equally profoundly.

but tries to be faithful to his realistic commitments, to his belief in there being a reality beyond the physical world, which somehow finds expression in the sensorily perceived reality. And it is precisely this expression that poses the greatest problem for philosophy. In this respect, I believe, Gödel and Wittgenstein made some extremely important points. And since their points can be regarded as elaborations on a problem lying at the heart of Platonic philosophy, they can be considered contributions to Platonism, not just footnotes to Plato.

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