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Which Option Pricing Model Is the Best? HF Data for Nikkei 225 Index Options[4]

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Abstract: In this study, we analyse the performance of option pricing models using 5-minutes transactional data for the Japanese Nikkei 225 index options. We compare 6 different option pricing models: the Black (1976) model with different assumptions about the volatility process (realized volatility with and without smoothing, historical volatility and implied volatility), the stochastic volatility model of Heston (1993) and the GARCH(1,1) model. To assess the model performance, we use median absolute percentage error based on differences between theoretical and transactional options prices. We present our results with respect to 5 classes of option moneyness, 5 classes of option time to maturity and 2 option types (calls and puts). The Black model with implied volatility (BIV) comes as the best and the GARCH(1,1) as the worst one. For both call and put options, we observe the clear relation between average pricing errors and option moneyness: high error values for deep OTM options and the best fit for deep ITM options. Pricing errors also depend on time to maturity, although this relationship depend on option moneyness. For low value options (deep OTM and OTM), we obtained lower errors for longer maturities. On the other hand, for high value options (ITM and deep ITM) pricing errors are lower for short times to maturity. We obtained similar average pricing errors for call and put options. Moreover, we do not see any advantage of much complex and time-consuming models. Additionally, we describe liquidity of the Nikkei225 option pricing market and try to compare the results we obtain here with a detailed study for Polish emerging option market (Kokoszczyński et al. 2010b).

Keywords: Option pricing models, high-frequency data, realized volatility, implied volatility, stochastic volatility, emerging markets

JEL Codes: O52, P33, R12

1 Introduction

The quest for the best option pricing model is at least 40 years old, but going back into the past, we could find its traces even few centuries earlier (e.g., the speculation during tulipomania or the South Sea bubble).

The futures option pricing model (Black 1976) began a new era of futures option valuation theory. The rapid growth of option markets in the 1970s[5] brought rapidly a lot of data and stimulated an impressive development of research in this area. Quite soon, numerous empirical studies put in doubt basic assumptions of the Black model: they strongly suggest that the geometric Brownian motion is not a realistic assumption. Many underlying return series display negative skewness and excess kurtosis (see Bates 1995, Bates 2003). Moreover, the implied volatility calculated from the Black-Scholes model often vary with the time to maturity of the option and the strike price (cf. Rubinstein 1985, Tsiaras 2009). These observations drove many researchers to propose new models that each relaxes some of those restrictive assumptions of the Black-Scholes model (Broadie and Detemple 2004, Garcia *et al.* 2010, Han 2008, Mitra 2009). Based on Han (2008), we can divide these researchers into a few groups. The first one engages into extending Black-Scholes-Merton framework by incorporating stochastic jumps or stochastic volatility (Amin and Jarrow 1992, Hull and White 1987), another one goes into estimating the stochastic density function of the underlying asset directly from the market option prices (Derman and Kani 1994, Dupire 1994) or using other distribution of the rate of return on the underlying asset rather than normal distributions (Jarrow and Rudd 1982, Corrado and Su 1996, Rubinstein 1998, Lim *et al.* 2005). On the other hand, the Black-Scholes model is still widely used not only as a benchmark in comparative studies testing various option pricing models, but also among the market participants. Christoffersen and Jacobs (2004) show that much of its appeal is related to the treatment of volatility – the only parameter of the Black-Scholes model, however, is not directly observed.

Detailed analysis of the literature (An and Suo 2009, Andersen *et al.* 2007, Bates 2003, Brandt and Wu 2002, Ferreira *et al.* 2005, Mixon 2009, Raj and Thurston 1998) seems to suggest that the BSM model with implied volatility calculated on the basis of the last observation performs quite well even when compared with many different and more sophisticated pricing models (standard BSM model, BSM with realized volatility, GARCH option pricing models or various stochastic volatility models).

Our motivation for this paper is to check the results of Kokoszczyński *et al.* (2010a), who conducted a similar study for emerging market HF data (WIG20 index options).[6] Their results show that the Black model with implied volatility (BIV) gives the best results, the Black model with historical volatility (BHV) is slightly worse and the Black model with realized volatility (BRV) gives clearly the worst results.

The complex comparison of Black model with different volatility assumptions presented only for an emerging market is definitely not enough to formulate conclusions of a more general nature. Therefore, we have decided to compare the results for the Polish emerging market with a similar research for the developed Japanese market. For this purpose, we choose the Nikkei 225 index option market (European style), which can be regarded as one of the most important option markets in the world, especially when we consider the level of its innovation and complexity. As a result, we hope we will be able to suggest some more general conclusions.

After a thorough analysis, we can say that the literature regarding the Japanese capital market and especially European style index options, is not so rich and this is our second motivation to write this paper. The reason for this can be that Nikkei 225 index is the basis instrument for many different derivatives that are quoted on many different exchanges and the literature is widely dispersed. We can easily find some papers focusing on pricing American style options or options quoted in different currency than Yen. On the other hand, the papers in English focusing on the European style Nikkei 255 index options are not so numerous.[7] The literature on American style options shows quite good results for the Black model (Raj and Thurston 1998), sometimes better than for various GARCH models (Iaquinta 2007). When we consider the second case (options in other currencies), we actually model not only option prices but the exchange rate fluctuations as well (Wei 1995); thus, the comparison of their results with ours could be regarded as not valid.

5 The Chicago Board of Options Exchange was founded in 1973 and it adopted the Black-Scholes model for option pricing in 1975.

6 The WIG20 is the index of twenty largest companies listed on the Warsaw Stock Exchange (further detailed information may be found at www.gpw.pl).

7 Unfortunately, because of language barrier we were not able to extend our literature review to papers written in Japanese.

Therefore, we are left with the very limited number of studies that focus on the European style options or otherwise touch this subject, sometimes only in an indirect way. Li (2006) shows that Nikkei 225 is rather an efficient market (in the sense of lack of arbitrage possibilities analysed through the existence of put-call parity). Yao *et al.* (2000) compare the BSM model with historical volatility with pricing done via neural networks and show that in some cases (mainly for ATM options) the BSM model gives better results. Kanoh and Takeuchi (2006) once again show that the BSM model is better (in terms of the RMSE statistics) from GARCH (1,1) and E-GARCH (1,1) model. On the other hand, Mitsui and Satoyoshi (2006) got better results for GARCH-T model for almost all moneyness classes, but their results are based on strong assumptions concerning the type of distribution of the basis instrument.

This review, covering those Nikkei 225 index options studies that are comparable with our approach, justifies quite strongly the positive assessment of the BSM model. We are going to check this by using the high-frequency data from 2008.

The structure of this paper has been planned in such a way as to answer the following detailed questions:

- Which model from among those we test can be treated as the best one?
- Can we observe any distinctive patterns in option pricing taking into account moneyness ratio (MR) and time to maturity (TTM)?
- Can we distinguish any patterns of liquidity behaviour in a developed market using transactional data?
- What is the proper measure of average pricing error? Is it robust to large errors that are likely to emerge when analysing HF data?
- Is there any substantial difference between the results for a developed (this paper) and an emerging market (Kokoszczyński *et al.* 2010a)?

The remainder of this paper is organized as follows. The second section describes some methodological issues. Next section presents data and the fluctuations of volatility processes derived from transactional data. The fourth section discusses the liquidity issues. Results are presented in section five and the last section concludes.

2 Option pricing methodology

2.1 The Black option pricing model with historical, realized and implied volatility

The basic pricing model we choose is the Black-Scholes model for futures prices, that is, the Black model (Black 1976). We call it further in the text the BHV model – the Black model with historical volatility. Below are formulas for this model:

$$c = e^{-r_f T}[FN(d_1) - KN(d_2)] \tag{1}$$

$$p = e^{-r_f T}[KN(-d_2) - FN(-d_1)] \tag{2}$$

where:

$$d_1 = \frac{\ln\left(\frac{F}{K}\right) + \sigma^2 T/2}{\sigma\sqrt{T}} \tag{3}$$

$$d_2 = \frac{\ln\left(\frac{F}{K}\right) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{4}$$

where c and p are respectively valuations of a call and a put option, T is time to maturity, r_f is the risk-free rate, F – the futures price, K – underlying strike, σ – volatility of underlying and $N(\cdot)$ is the cumulative standard normal distribution.

There are two reasons why we decided to use the Black model instead of the standard Black-Scholes model. First, we are able to relax the assumption about continuous dividend pay-outs.[8] Second, we can use additional data because usually derivatives (options, futures, etc.) are quoted much longer than the basis instruments (e.g., Nikkei 225 index).

To further justify such an approach, we assume that we can price a European style option on Nikkei 225 index applying the Black model for futures contract (with historical, realized and implied volatility), where Nikkei 225 index futures contract is the basis instrument. This is possible due to the following facts:

Nikkei 225 index futures expire exactly on the same day as Nikkei 225 index options do, the expiration prices

8 In this way, we are able to eliminate two possible source of pricing error: the necessity to estimate the dividend yield and the assumption about continuous payouts.

are set exactly in the same way, we study only European-style options; hence, early expiration – like in the case of American options – is impossible.[9]

One of the most important issues about option pricing is the nature of an assumption concerning the specific type of volatility process. Therefore, we check the properties of the Black model with three different types of volatility estimators: historical volatility, realized volatility and implied volatility, and additionally, we use the Heston model and the GARCH option pricing model. Below we provide a brief description of each of these volatility estimators and models.

The historical volatility (HV) estimator is based on the formula:

$$VAR_{\Delta}^{n} = \frac{1}{(N_{\Delta} * n) - 1} \sum_{t=1}^{n} \sum_{i=1}^{N_{\Delta}} (r_{i,t} - \bar{r})^2 \quad (5)$$

where:

- VAR_{Δ}^{n} – variance of log returns calculated on high frequency data on the basis of last n days
- $r_{i,t}$ – log return for i-th interval on day t with sampling frequency equal to Δ, which is calculated in the following way:

$$r_{i,t} = \log C_{i,t} - \log C_{i-1,t} \quad (6)$$

- $C_{i,t}$ – close price for i-th interval on day t with sampling frequency equal to Δ
- N_{Δ} – number of Δ intervals during the stock market session
- n – memory of the process measured in days, used in the calculation of respective estimators and average measures
- $\bar{r}$ – average log return calculated for last n days with sampling frequency Δ, which is calculated in the following way:

$$\bar{r} = \frac{1}{(N_{\Delta} * n) - 1} \sum_{t=1}^{n} \sum_{i=1}^{N_{\Delta}} r_{i,t} \quad (7)$$

In this research, we use $N_{\Delta} = 1$, and hence, the HV estimator is simply standard deviation for log returns based on the daily interval. This approach is commonly used by the wide range of market practitioners.

The second approach is the **realized volatility (RV) estimator** proposed early by Black (1976) and Taylor (1986) and further popularised by Bollerslev (cf. Andersen *et al.* 2001). It is based on squared log returns summed over the time interval of N_{Δ}.

$$RV_{\Delta,t} = \sum_{i=1}^{N_{\Delta}} {r_{i,t}}^2 \quad (8)$$

The **implied volatility (IV) estimator** is based on the last observed market option price. It assumes that all parameters (with the exception of sigma) are also known. We calculate the implied volatility for the last market price for each option and then average them separately for each class of TTM and moneyness ratio, and for both call and put options.[10] Hence, for each observation, we have 50 different IV values (5 × 5 × 2). These values are then treated as an input variable for volatility parameter in calculations of the theoretical options price for the Black model with the implied volatility (BIV) for the next observation.

Before entering into the formula for the Black model, the HV and RV estimators have to be annualized and transformed into standard deviation. The formula for the annualization of the HV estimator is as follows:

$$HV = {}^{annual\_std}SD_{\Delta}^{n} = \sqrt{252 N_{\Delta} VAR_{\Delta}^{n}} \quad (9)$$

Contrary to the HV estimator, which is based on information from many periods ($n > 1$), RV estimator requires information only from a single period (time interval of Δ). Therefore, the procedure of averaging and annualizing realized volatility estimator is slightly different from that presented in formula (9):

$${}^{annual_{std}}[RV]_{\Delta}^{n} = \sqrt{252 \frac{1}{n} \sum_{t=1}^{n} [RV]_{\Delta,t}} \quad (10)$$

Having all these volatility estimators and additionally the Heston and GARCH (1,1) option pricing models we present below, we study several types of option pricing models, which will be described in details in section 2.5.

9 Early expiration of American-style option could result in the significant error in the case of such a pricing, because of the difference in prices of index futures and of Nikkei 225 index before the expiration date (the basis risk).

10 We divide 320 options (160 call and 160 put options) into 5 moneyness ratio classes and 5 time-to-maturity classes. The details of this classification are presented in Section 3.

2.2 The GARCH Model

Many classical option pricing models (e.g., the Black model) assume the constant level of volatility of log-returns of basis instruments. However, in reality, many financial time series are characterized by time varying volatility. GARCH models are one possible way to relax this initial assumption. They were proposed by Engle (1982) and Bollerslev (1986). GARCH model describes the dynamic of returns of the basis instruments with following equations:

$$r_t = \varepsilon_t \tag{11}$$

$$\varepsilon_t = z_t\sqrt{h_t}, z_t \sim IID\ N(0,1) \tag{12}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{13}$$

where $r_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$,

S_t is the price of the basis instrument in the moment t, and p and q define the order of GARCH (p,q) model. Currently, we have many extensions of standard GARCH(p,q) model, which mainly differ by the specification of the conditional variance equation and by various assumptions concerning the conditional distributions of residuals in the mean equation. Through the years, GARCH models have become the standard approach in volatility modelling, asset pricing, financial time series forecasting or risk management. Examples of this kind of research can be found in Bollerslev *et al.* (1988), Bollerslev *et al.* (1994), Campbell and Hentschel (1992), French *et al.* (1987), Glosten *et al.* (1993), Maheu and McCurdy (2004), Pagan and Schwert (1990), whereas the detailed description of GARCH models can be found in Bollerslev *et al.* (1992) or Campbell *et al.* (1997).

Finally, the GARCH models are also used in the option pricing models. Duan (1995) presents the methodology of European style call option pricing with the assumption that returns of the basis instrument can be described with the GARCH process. In order to become risk neutral in this approach, we differentiate between physical and martingale (risk free) probability measure. Garcia and Renault (1998) describe theoretical aspects of using GARCH models in risk hedging strategies, while Ritchken and Trevor (1999) use GARCH models in the American style option pricing applying trinomial trees. Duan *et al.* (2004) extend the methodology presented in his previous paper through inclusion of volatility jumps in prices of the basis instrument.

Option pricing based on GARCH model has been done here according to Duan (1995). This approach assumes that log returns undergo GARCH-M(p,q) process described by the following equations:

$$r_t = r_f + \delta\sqrt{h_t} - \frac{1}{2}h_t + \varepsilon_t \tag{14}$$

$$\varepsilon_t = z_t\sqrt{h_t}, z_t \sim N(0,1) \tag{15}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{16}$$

where parameters are denoted in the same way as in earlier formulas, and additionally, δ in equation (14) is interpreted as a unit risk premium.

The pricing of options are conducted assuming local risk-neutral valuation. It requires modification of log returns processes in such a way that the conditional variance one step ahead remains unchanged and simultaneously conditional expected return equals risk-free rate (Fiszeder 2008). Introduction of risk-neutral probabilistic measure Q enables us to price options through discounting expected option payoff.

The dynamic of basis instrument log returns with respect to measure Q can be described as follows:

$$r_t = r_f - \frac{1}{2}h_t + \xi_t \tag{17}$$

$$\xi_t = u_t\sqrt{h_t}, u \sim N(0,1) \tag{18}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left(\xi_t - \delta\sqrt{h_{t-1}}\right)^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{19}$$

The formula describing the dependence between the price of basis instrument on maturity day and its price in the time of pricing can be described:

$$S_T = S_t \exp\left[r_f(T-t) - \frac{1}{2}\sum_{i=t+1}^{T} h_i + \sum_{i=t+1}^{T} \xi_i\right] \tag{20}$$

while the price of European style call option is described by the discounted value of the option price on the maturity day:

$$call_t = \exp\left(-r_f(T-t)\right) E^Q[\max(S_T - X, 0)\,|\psi_t] \tag{21}$$

where E^Q is the operator of conditional expected value with respect to Q measure.

In practice, the pricing is done through Monte Carlo simulation. In the first stage, we estimate the parameters of the model (14), (15) and (16), and then on the basis of (17), (18), (19), and (20), we simulate N realization of basis instrument price. Call and put option prices are then calculated in the following way:

$$call_t = \exp\left(-r_f(T-t)\right)\frac{1}{N}\sum_{j=1}^{N}\max(S_{Tj}-X,0) \quad (22)$$

$$put_t = \exp\left(-r_f(T-t)\right)\frac{1}{N}\sum_{j=1}^{N}\max(X-S_{Tj},0) \quad (23)$$

We use GARCH-M(1,1) model in this study.[11] Many research papers show that this order of the model defines the dynamics of stock index returns in the most adequate way (Hansen and Lunde 2004 or Zivot 2008). Similarly, like in the case of the BSM model, we use index returns. We estimate the parameters of the equations (14), (15) and (16) on the basis of data from 1/1/2007 until the moment of option pricing. As a result, the size of sample used to estimate the GARCH parameters varies from one year (for pricing done on 2nd January, 2008) to 1.5 year (for pricing done on 30th June, 2008). In order to eliminate problems with instability of GARCH model parameters, we have decided to delete overnight returns from our data sample.

The number of replications in Monte Carlo simulations is another important choice to be made. Finance literature suggests strongly that N = 10,000 gives an adequate precision of estimates. However, due to the very large number of pricing (5-minute data) we need, we have to limit the number of replication to N = 1000. In order to minimize possible negative effects of that choice, we use two popular variance reduction techniques: antithetic variables sampling and control variates. The data we use in this study are described in detail in section 3.

2.3 The Heston Model

Log returns volatility in stochastic volatility models is represented by a given stochastic volatility process with dynamics set a priori. Hull and White (1987) are among the pioneers of applying stochastic volatility for option pricing. They assume that variance dynamics can be described with the following differential equation:

$$dV_t = a(b - V_t)dt + cV_t dZ_t \quad (24)$$

Under additional assumption – that volatility is not correlated with the basis instruments – Hull and White present the analytical formula for European style call option. One of the main conclusions of their research is that the BSM model systematically underestimates the prices of ITM and OTM options, and overestimates the prices of ATM options.[12]

The Heston model we used in our research is an extension of Stein and Stein (1991). Their option pricing formula assumes that volatility is described by the Ornstein-Uhlenbeck process and is not correlated with the basis instrument. On the other hand, Heston (1993) presents the call option pricing formula with no assumption on correlation of volatility with the basis instrument. His model assumes that the dynamics of underlying asset price S_t and its volatility V_t are given by the following set of differential equations:

$$dS_t = \mu S_t dt + \sqrt{V_t}S_t dW_t^1 \quad (25)$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dW_t^2 \quad (26)$$

$$dW_t^1 dW_t^2 = \rho dt \quad (27)$$

where $\{S_t\}_{t\geq 0}$ and $\{V_t\}_{t\geq 0}$ indicate the price and the variance of the basis instrument, and $dW^1_{t\ t\geq 0}$ and $dW^2_{t\ t\geq 0}$ are correlated Brownian motion processes (with parameter of correlation p). Additionally, it is assumed that $\{V_t\}_{t\geq 0}$ is a mean reverting process, with long memory expected value θ and mean reverting coefficient κ. The parameter σ is defined as volatility of volatility.

One of the main reasons, why the Heston (1993) model has become so popular is the fact that it is possible to obtain its closed-form solution for the European style call option pricing for an asset not paying dividend, which is given by:

$$C(S_t, V_t, t, T) = S_t P_1 - Ke^{-r_f(T-t)}P_2 \quad (28)$$

where

11 In the results section we will refer to this model as to GARCH(1,1).

12 The constant volatility assumption is responsible for this drawback of the BSM model.

$$P_j(x,V_t,T,K) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left\{\frac{e^{-i\phi\ln(K)} f_j(x,V_t,T,\phi)}{i\varphi}\right\} d\phi$$

$$f_j(x,V_t,T,\phi) = \exp\left\{r\phi i r_f + \frac{a}{\sigma^2}\left[\left(b_j - \rho\sigma\phi i\right)\tau - 2\ln\left(\frac{1-ge^{dr_f}}{1-g}\right)\right] + \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}\left(\frac{1-e^{dr_f}}{1-ge^{dr_f}}\right) + i\phi\ln(S_t)\right\} \quad (29)$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d}$$

$$d = \sqrt{\left(\rho\sigma\phi i - b_j\right)^2 - \sigma^2(2u_j\phi i - \phi^2)}$$

for $j = 1,2$ where:

$u_1 = \frac{1}{2}$, $u_2 = -\frac{1}{2}$, $a = \kappa\theta$, $b_1 = \kappa + \lambda - \rho\sigma$, $b_2 = \kappa + \lambda$

Formula 28 is not difficult to implement in practice. The only problem is to calculate the limit of the integral therein. This limit is often approximated by an adequate quadrature (Gauss-Legendre or Gauss-Lobatto), what can be done in many statistical software packages.

Practical implementation of the Heston model is done in two stages. First, we have to calibrate the model in order to find its parameters from equation (25), (26) and (27). Calibration can be done on the basis of call transactional prices observed in every one-hour interval. We choose parameter values in such a way as to minimize the difference between market and theoretical prices. Next, we use formulas (28) and (29) to calculate theoretical prices.

The calibration of the Heston model can be conducted in two ways – via global or local optimization. Global optimization guarantees that we find the true global minimum of our target function. The disadvantage of this method is that it is time-consuming and the parameters obtained here tend to be very unstable. On the other hand, local optimization gives only local minima but it is very fast and the parameters derived in this way are stable.

In our study, the global optimization is used for the first period and its results are the starting point for the local optimization in the second period. Then, the further iterations of the local optimization are being performed, for which the starting point is set to the local minimum from the previous stage.

In the second stage, the parameters found previously are used to calculate the theoretical prices in the next hourly interval. The prices of call options are calculated according to the formulas (28) and (29), while put option prices are found on the basis of call-put parity:

$$C_t + Ke^{-r_f(T-t)} = P_t + S_t \quad (30)$$

where C_t and P_t are European style call and put prices, S_t is the price of basis instrument, r_f is the risk-free rate, and finally, K is the strike price, and T is time to maturity for both call and put options.

The calibration of the Heston model in our study for the Japanese market has been done on the basis of an hourly interval. It means that in the time of calibration, we use transactional prices from the previous hourly interval, and then we use those results to price options for the current interval. The calibration of the Heston model was based on all the available transactional prices in one-hour interval.

Measuring option pricing error

To assess the accuracy option pricing models, we compare the option transactional prices with the theoretical prices obtained from each model. To measure the average pricing error, we use the median absolute percentage error (MdAPE). Since the distribution of errors is relatively strongly positively skewed, we claim that it is better to use the median rather than the mean value to express the average pricing error. The MdAPE statistic is defined as:

$$MdAPE = median\left\{\left|\frac{y_i - \hat{y}_i}{y_i}\right|\right\}_{i=1}^{N} \quad (31)$$

where y_i and $\hat{y}_i$ are, respectively, option transactional price and option theoretical price for the given time interval.

We also calculate the percentage of overprediction (OP) in order to see whether a given model on average over- or under predicts the transactional price of an option:

$$OP = \frac{1}{N}\sum_{i=1}^{N} OP_i \quad (32)$$

where $OP_i = 1$ if $\hat{y}_i > y_i$ and $OP_i = 0$ if $\hat{y}_i \leq y_i$.

Both statistics were calculated for all the models, for different TTM and MR classes, and for both call and put options.

2.4 Models' description

Thus, we study the properties of the following models:

- BHV – the Black model with historical volatility (σ as standard deviation, $n = 63$)
- BRV – the Black model with realized volatility (realized volatility as an estimate of σ parameter; RV calculated on the basis of observations with several different Δ intervals and different values for parameter n applied in the process of averaging)
- BIV – the Black model with implied volatility (implied volatility as an estimate of σ; IV calculated for the previous observation, separately for each TTM and MR class, and for both call and put options, hence for 50 different groups)
- Heston – the Heston option pricing model
- GARCH – GARCH (1,1) option pricing model based on the Duan's methodology.

Initially, we calculate the BRV models with four different Δ values: 5, 10, 15 and 30 minutes. Then, we check the properties of averaged RVs with different values of parameter n in pricing models. We find, like Kokoszczyński *et al.* (2010a), no significant differences between RVs with different Δ parameter. On the other hand, Sakowski (2011), after a detailed analysis of similar data for WIG20 index options, but for a longer data span shows that BRV, BHV and BIV models have better properties (basing on MdAPE statistics) for parameter Δ = 5 minutes compared with intervals of 10, 15 and 30 minutes. Additionally, it is a common approach in the literature to use observation intervals between 5 minutes and 15 minutes, since this constitutes the good trade-off between the nonsynchronous bias and other microstructure biases (cf. Ait-Sahalia *et al.* 2009). Therefore, we use the BRV model only for Δ = 5m interval with different values of averaging parameter (n = 1, 2, 3, 5, 10, 21 and 63). Having analysed their properties, we decided to present only the best and the worst model from the family of BRV models: BRV5m (non-averaged one), and BRV5m_63[13]. GARCH model has been estimated with the same Δ interval and the Heston model has been calibrated on hourly intervals but it still enables us to calculate the theoretical prices for Δ interval equal to 5 minutes.

13 Our choice is confirmed by results in Sakowski (2011) and Kokoszczyński *et al.* (2010a).

3 Data and the description of volatility processes

3.1 Data description

We use transactional data[14] for Nikkei 225 index options, Nikkei 225 index and Nikkei 225 index futures, which have been provided by the Reuters company[15]. The data cover the period from 2 January, 2008 to 30 June, 2008. Transactional prices for Nikkei 225 index options and Nikkei 225 index are in the form of 5-minutes data and we use such data for further calculations. However, in order to calculate different volatility estimators, we transform 5-minutes data into different frequencies. The risk-free interest rate is approximated by the LiborJPY3m interest rate, also converted into 5-minute intervals.

The market for Nikkei 225 index option started in this period at 1.00 CET and ended at 7.00 CET[16]. For that reason, we have 6745 observations (122 session days with 56 5-minutes intervals each[17]).

As a result, our data set for Nikkei 225 index options comprises transactional prices for 160 call options and 160 put options maturing in January, February, March, April, May, June and July 2008.[18]

14 Some papers that test alternative option pricing models and include the Black-Scholes model among models tested therein use instead of transactional data bid-ask quotes (midquotes), as they allow to avoid microstructural noise effects (Dennis and Mayhew 2009). Ait-Sahalia and Mykland (2009) state explicitly that quotes 'contain substantially more information regarding the strategic behaviour of market makers' and they 'should be probably used at least for comparison purposes whenever possible' (p. 592). On the other hand, Beygelman (2005) and Fung and Mok (2001) argue that midquote is not always a good proxy for the true value of an option.

15 Thanks to the financial support of the government, we were able to buy all the necessary data (5 minutes intervals) from Reuters Datascope company.

16 In practice, the market session lasted from 1.00 CET to 3.00 CET, then there was a pause, and later session lasted from 4.30 CET to 7.00 CET. Therefore, we get 56 5-minutes intraday returns.

17 Some days, close to the most important national holidays, the market session finished before 7.00 CET.

18 Maturity days of these options and their symbols for each call and put series are as follows: 11.01.2008 (*call*-A8, *put*-M8), 08.02.2008 (*call*-B8, *put*-N8), 14.03.2008 (*call*-C8, *put*-O8), 11.04.2008 (*call*-D8, put-P8), 09.05.2008 (*call*-E8, *put*-Q8), 13.06.2008 (*call*-F8, *put*-R8) i 11.07.2008 (*call*-G8, *put*-S8).

The results of our analysis will be presented with respect to 2 types of options, 5 classes of MR and 5 classes of TTM:

- 2 types of options (call and put)
- 5 classes of moneyness ratio, for call options: deep OTM (0–0.85), OTM (0.85–0.95), ATM (0.95–1.05), ITM (1.05–1.15) and deep ITM (1.15+), and for put options in the opposite order[19]
- 5 classes for time to maturity: [0–15 days], [16–30 days], [31–60 days], [61–90 days], [91+ days).

This categorization allows us to compare the different pricing models along several dimensions.

The number of transactional prices, theoretical prices and pricing errors are presented in Tab. 2 and Fig. 5.

3.2 The descriptive statistics for Nikkei 225 futures time-series.

We begin our study with the basic analysis of the time series of returns of the basis instrument. Tab. 1 presents the descriptive statistics for 5-minute interval data. They are calculated for two samples: with and without opening jump effects.[20]

Both samples have high kurtosis and are asymmetric. The distribution for the full sample has negative skewness, while removing jump effects makes the distribution right skewed. Overall, both Jarque-Bera and Kolmogorov-Smirnov statistics indicate that returns in both samples are far from normal. Nevertheless, we observe interesting feature that – contrary to the data from the Polish market (Kokoszczyński *et al.*, 2010b) – for adjusted sample skewness and kurtosis are larger when we consider their absolute values. In case of the Japanese market, it is not the jump effect that is responsible for the non-normality of returns, but returns' general features.

Fig. 1 and Fig. 2 additionally confirm this observation showing high negative and positive returns in both time series with and without jump effects. Formally, the lack of normality of the basis instrument means that the standard BSM model should not be applied for option pricing with these data. Accordingly, we transform this model varying its assumption about the nature of the volatility process. Moreover, we also

Tab. 1. The descriptive statistics for Nikkei 225 index returns for samples with and without opening jump effects

		sample with opening jump effects	sample without opening and mid-session jump effects
N		6745	6504
Mean		-0,000025394	-0,000014111
Median		0,000032644	0,000036116
Standard Deviation		0,0030907	0,0028429
Minimum		-0,0319108	-0,0319108
Maximum		0,0216127	0,0216127
Kurtosis		10,4364219	12,7560437
Skewness		-0,6227228	0,7206586
	Normality tests		
Kolmogorov-Smirnov	Statistic	0,093349	0,086497
Jarque-Berra	Statistic	30995,9195	44584,7971

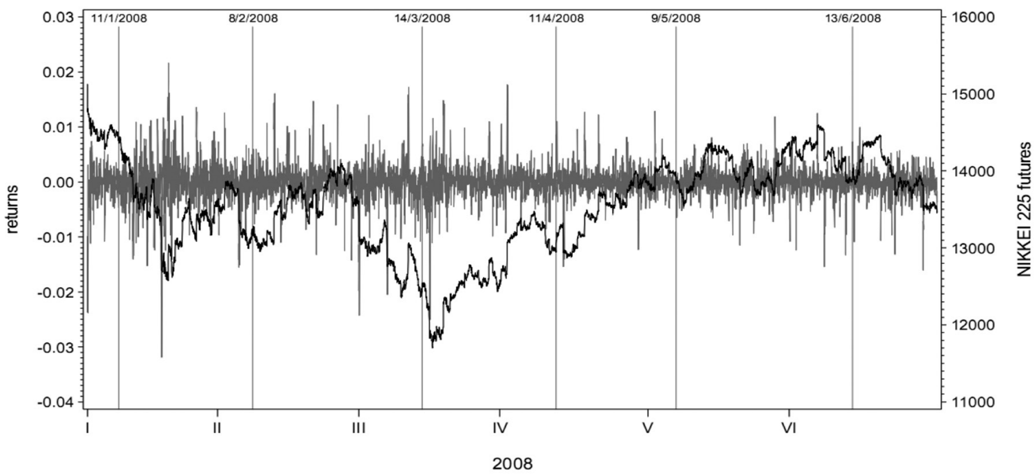
19 Moneyness ratio is usually calculated according to the following formula:

$$moneyness\ ratio = \frac{S}{K/e^{r_f T}} = \frac{F}{K}$$

where K is the option strike price, S is the price of underying, F is the futures price of underlying, r_f is the risk-free rate and T is time to maturity.

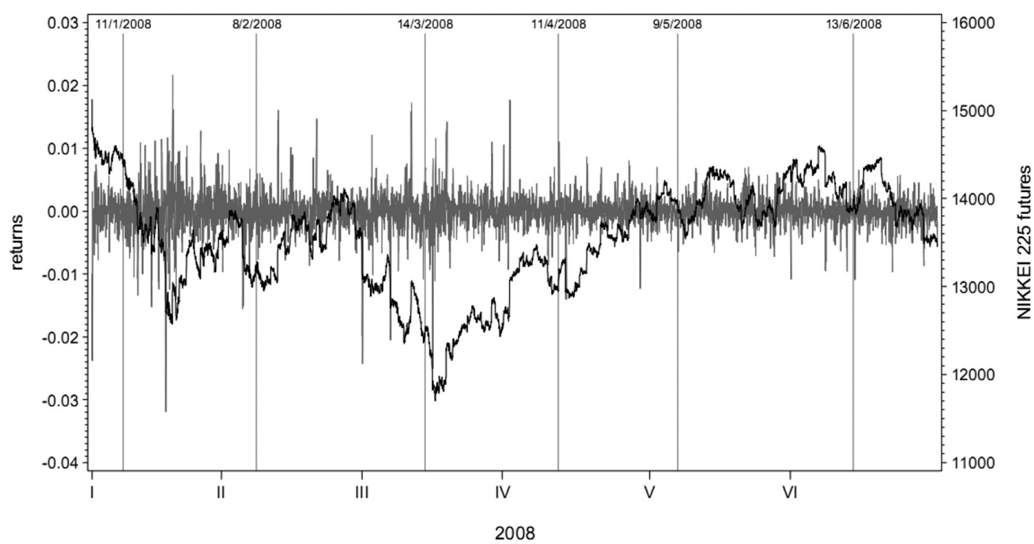
20 By *opening jump effects* we mean returns between 7.00 CET and 1.00 CET on the next day. Thus, sample without opening jump effects does not include observations with these returns. In case of the Japanese market two returns were excluded: one overnight return and second one including the return from the mid-session break.

Fig. 1. Index returns with the opening jump effect[a]



[a] The returns and index prices cover the data span between 2 January, 2008 to 30 June, 2008.

Fig. 2. Index returns without the opening and mid-session jump effect[a]



[a] The 10-second returns between the closing price from each day and the opening price from the next day have been excluded. The same was done with the mid-session jump. The returns cover the data span from 2 January, 2008 to 30 June, 2008.

apply the Heston and GARCH option pricing models to the same data.

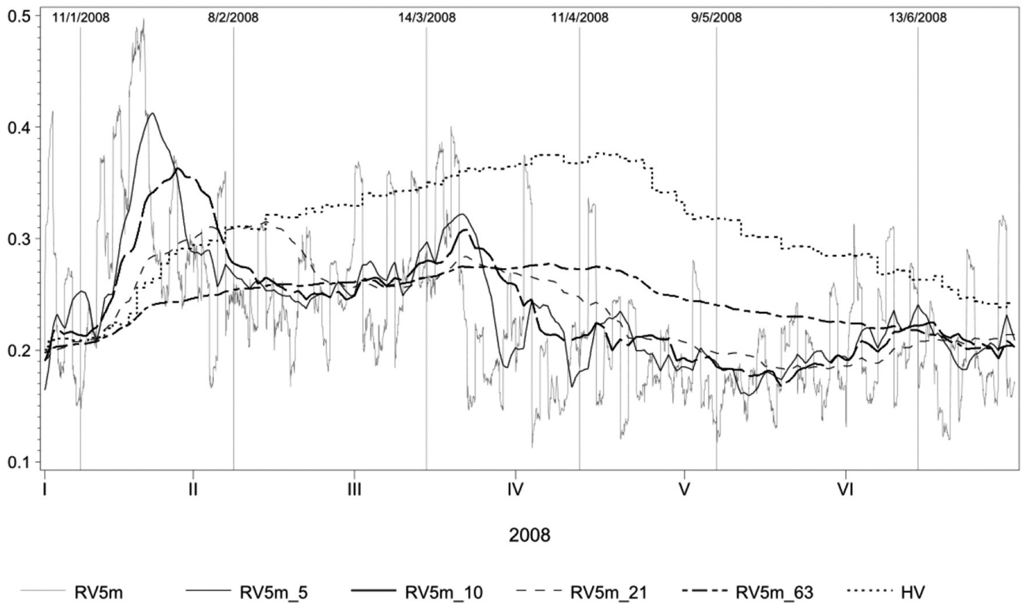
3.3 The description of volatility processes; historical, realized and implied.

We consider three different volatility measures: historical, realized and implied volatility for the Black option pricing model and – in addition to that – stochastic volatility and GARCH model. Obviously, the volatility process assumed in pricing is one of important reasons for differences among theoretical option prices we compare.

In the case of the historical volatility estimator $N_\Delta = 1$ for every ri,t (daily log returns) and Ci,t in formulas (5), (6) and (7). Moreover, we use the constant value of parameter n being equal to 63, because we want to reflect historical volatility from the last three trading months.

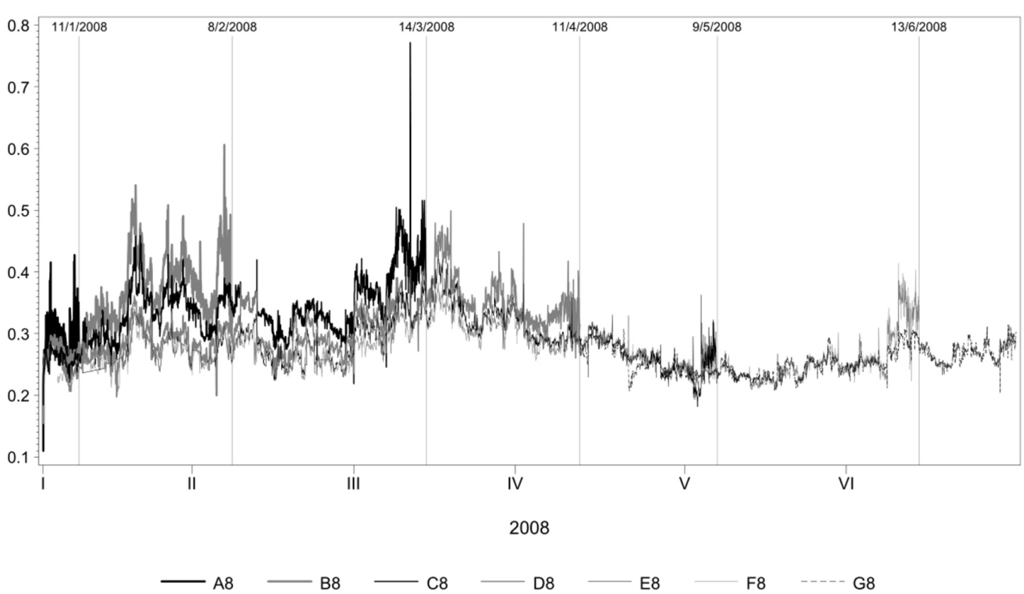
On the basis of similar studies for the Polish market (Kokoszczyński *et al.* 2010a, Kokoszczyński *et al.* 2010b), the realized volatility has finally been calculated on

Fig. 3. Historical and realized volatility (5m, 5m_5, 5m_10, 5m_21, 5m_63)[a]



[a] The volatility time series cover the data period between 2nd January, 2008 and 30th June, 2008. Vertical lines represent end of month and additionally the day of 11th January, 8th February, 14th March, 11th April, 9th May and 13th June, when the option series expired.

Fig. 4. Implied volatility for ATM call options[a]



[a] The volatility time series cover the data period between 2nd January, 2008 and 30th June, 2008. IV are presented for 7 series of options. Vertical lines represent end of month and additionally the day of 11th January, 8th February, 14th March, 11th April, 9th May and 13th June, when the option series expired.

the basis of Δ equal to 5 minutes. Therefore, at this stage, we limit our selection of volatility time series only to the RV calculated for Δ = 5 minutes, with averaging parameter n = 5, 10, 21 and 63 days. Fig. 3 presents the realized volatility compared to historical volatility. The distinguishing fact is that the non-averaged RV time series (RV_5m) is much more volatile than the averaged RV or HV time series. Obviously, such a high volatility of volatility can strongly influence the theoretical prices from the BRV model and their stability over time. One can thus expect that in periods of high volatility, the BRV model with the non-averaged RV estimator may produce high pricing errors.

On the other hand, the implied volatility time series exhibits substantially different trajectories than RV or HV time series. Fig. 4 presents how IV estimates (for ATM call options) evolve in time.[21] Similar to Kokoszczyński *et al.* (2010), we observe that for the short TTM (5–10 days), the IV tends to increase with shortening of the TTM. Contrary to the Polish market for the TTM lower than 5 days, we do not observe explosion of the IV and it does not reach the level of over 200% (annualized). This happens mostly for the call and put (deep) OTM and ATM options. However, jump of the IV to 70% can be the reason of big mispricing of options with the low TTM. For that reason, some researchers often exclude from comparison options with the short TTM and market prices lower than 5–10. However, we have consciously decided to conduct this research on the full sample, believing that such an approach would allow us to better answer the question what kind of observation should be treated as outliers.

Tab. 2. Number of theoretical premiums for different classes of MR and TTM for BRV model*

option	moneyness	0–15 days	16–30 days	31–60 days	61–90 days	91+ days	Total
CALL	deep OTM	372	4327	27089	23799	10494	66081
	OTM	6501	11635	22572	19567	8959	69234
	ATM	8199	9681	17385	12141	5368	52774
	ITM	3880	4510	5373	1484	761	16008
	deep ITM	1205	1935	3032	1044	1335	8551
	total CALL	20157	32088	75451	58035	26917	212648
PUT	deep OTM	6964	20580	44831	31225	7768	111368
	OTM	6109	8142	15466	12674	5631	48022
	ATM	8028	9669	17014	12001	6413	53125
	ITM	4278	4826	7427	1790	1096	19417
	deep ITM	2411	3002	3098	1161	1962	11634
	total PUT	27790	46219	87836	58851	22870	243566
total CALL and PUT		47947	78307	163287	116886	49787	456214

*456 thousand for BIV, Heston and GARCH(1,1) model and 445 thousand for BHV

4 Market liquidity

Liquidity constraints are typical features of an emerging derivatives market and they put severe limits for conducting such a study as we have done for the Polish market. To make our comparisons of both markets (WIG20 and Nikkei 225) as comprehensive as possible, we have also decided to present a detailed discussion of developed market liquidity on the example of the Nikkei 225 index option market with respect to (1) the number of transactional prices available in the sample, (2) the volume and (3) the turnover of option transactions.

The number of transactional prices is shown in Tab. 2 and Fig. 5. Their distribution suggests that the activity of market participants (measured by the number of single trades and not by their volume) concentrates on call ATM, OTM and deep OTM and put deep OTM option with the TTM between 16 and 90 days.

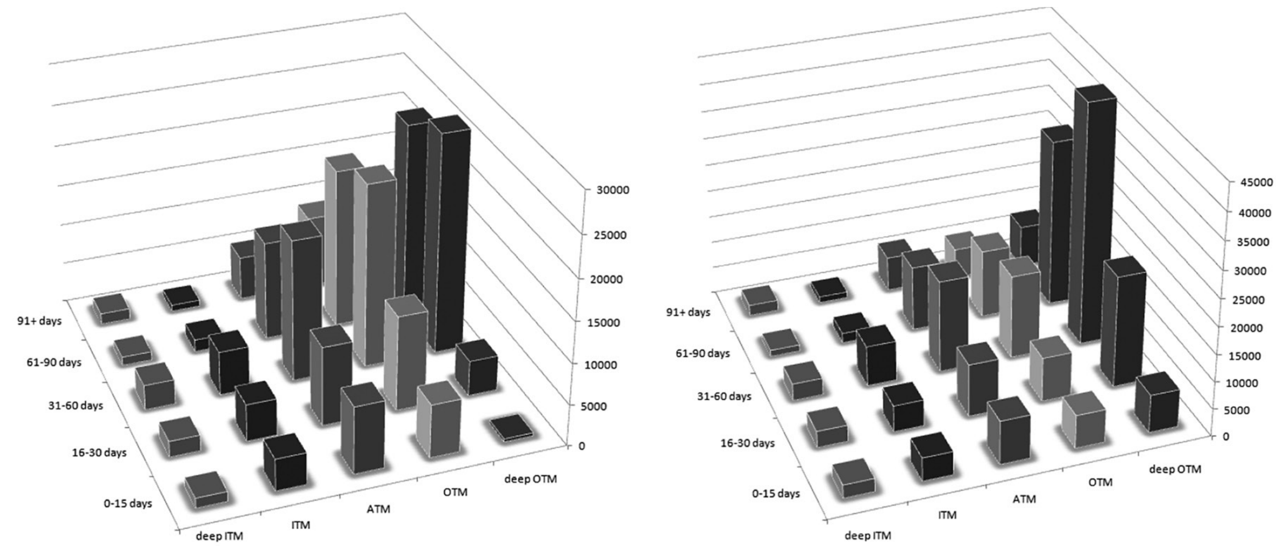
On the other hand, observing the distribution of volume for call options, presented in Fig. 6 (left panel), we notice that the highest volume is observed for the MR equal to ATM and OTM for the TTM up to 60 days. The lowest volume we see for the low TTM and the MR equal to ITM, deep ITM, and deep OTM. This suggests that investors rarely trade highly valued options (deep ITM and ITM) or options with the long TTM.

The distribution of volume for put options (Fig. 6, right panel) is very similar. The only difference is that the volume is also high for deep OTM options with the TTM less than 60 days. However, this is mostly due to the fact that put options are used as an insurance against sharp downward movement of the basis instrument.[22] Generally, we could say that the volume distribution for call and put options is very similar and that investors focus their trades on low-valued options with the short TTM.

21 IV estimates for ATM put options show similar pattern.

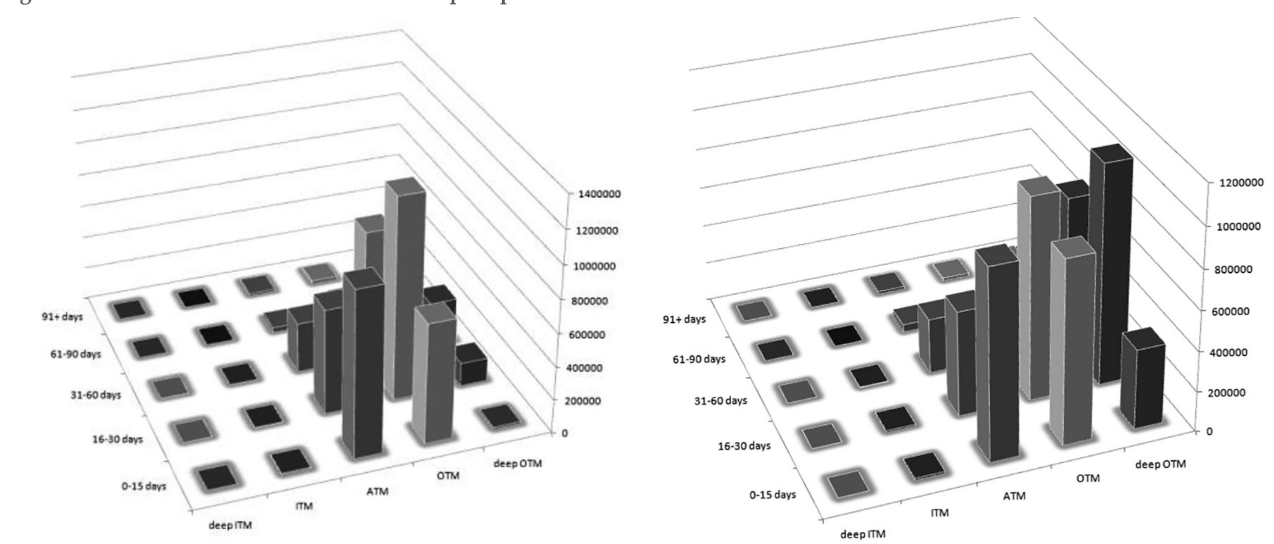
22 One buys the right to sell the basis instrument in the case of an extreme financial catastrophe, e.g., financial crash, for a relatively low cost (put option premium).

Fig. 5. The number of theoretical values for call and put options with respect to TTM i MR ratio[a,b]



[a] call options [a] put options

Fig. 6. The distribution of volume for call and put options[a,b]



[a] call options [a] put options

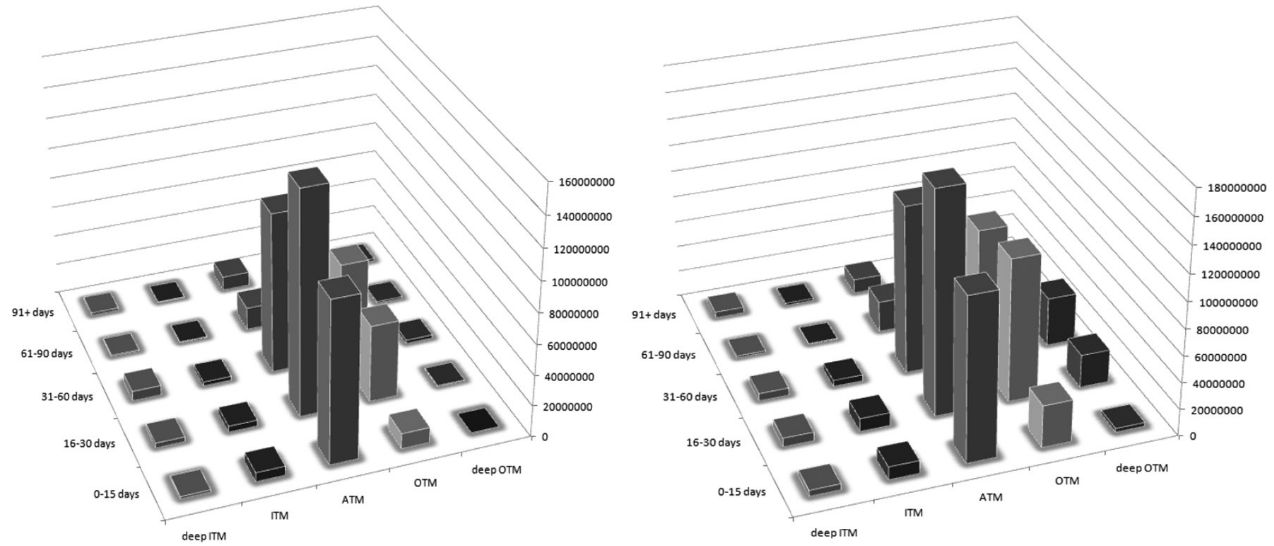
[b] the volume for both call and put options quoted in the period between 2nd January, 2008 and 30th June, 2008.

Fig. 7 addresses the liquidity issue from another perspective by focusing on the turnover volume that increases the importance of traded options' value. We observe significant shift from deep OTM to ATM and then to OTM options. It obviously means that most investors involved in the option trades concentrate in the ATM-OTM range. The same results are observed for call and put options with only slightly higher turnover volume for put options. However, this latter feature can be tied to the behaviour of the basis instrument in the period we study.[23]

The most important outcome from the liquidity analysis is that we can indicate where the volume of options concentrates. We notice that after dividing the set of options into different MR and TTM classes, we can distinguish options with the low TTM, which are ATM,

23 We observe sharp downward movement of Nikkei 225 index in the time of research.

Fig. 7. The volume of turnover for call and put options[a]



[a] call options

[a] put options

OTM or ITM that cumulate more than 90% of the total volume, both in case of call and put options. Similar situation has been observed in the case of the Polish market. Finally, it is worth noticing that the accumulation of volume in the given class of the MR and the TTM is partly conditional on the availability of options with the specified MR or TTM ratio.

5 Results

We divide this section into two subsections containing results presented separately for call and put options (section 5.1), and the comparison of joint results with respect to different dimension (section 5.2). This enables us to present a multidimensional comparative analysis of option pricing models.

5.1 Error statistics distribution

For all the available transactional and theoretical prices, we calculate two pricing error measures (MdAPE and OP), separately for six different pricing models (1. Heston, 2. GARCH(1,1), 3. BRV5m, 4. BRV5m_63, 5. BHV, and 6. BIV). The discussion of our results is based on two-dimensional charts containing five panels (Fig. 8 to 11). Each panel presents results for the separate MR class. Values of statistics have been joined with dashed or solid lines for a given TTM class.

Fig. 8 presents MdAPE statistics for call options. We can observe that the Black model with the implied volatility estimator (BIV) has the smallest average pricing errors for the majority of option classes. Slightly higher errors we got for the Heston model, but on the other hand, it is the best model for ATM options with the TTM less than 60 days. In the next place, depending on the given option class, we can rank BRV5m_63, BRV5m and BHV model (despite relatively large errors for deep OTM for the latter one). The worst results are observed for the GARCH(1,1) model with very large pricing errors for OTM options with time to maturity below 15 days.

Moreover, for all the models, we can observe distinctive relationship between average pricing errors and MR and TTM classes. Values of MdAPE statistics decline when we go from deep OTM through deep ITM options. The influence of TTM classes looks a little bit different. For deep OTM and OTM options, pricing errors are higher for short times to maturity, whereas for ITM and deep ITM options, we observe higher errors for longer times to maturity. Smallest differences between all the models, and simultaneously, most precise theoretical premiums we obtain for deep ITM options with time to maturity below 15 days.

Fig. 9 – with OP values for call options – indicates that the BIV model (number 6) is the best one. It is characterised by almost the same level of over- and underprediction (the value of OP is approximately equal to 0.5). Results for other models differ. The second best models according to this metric are the Heston model (number 1)

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Fig. 8. MdAPE statistics for call options with respect to MR and TTM classes

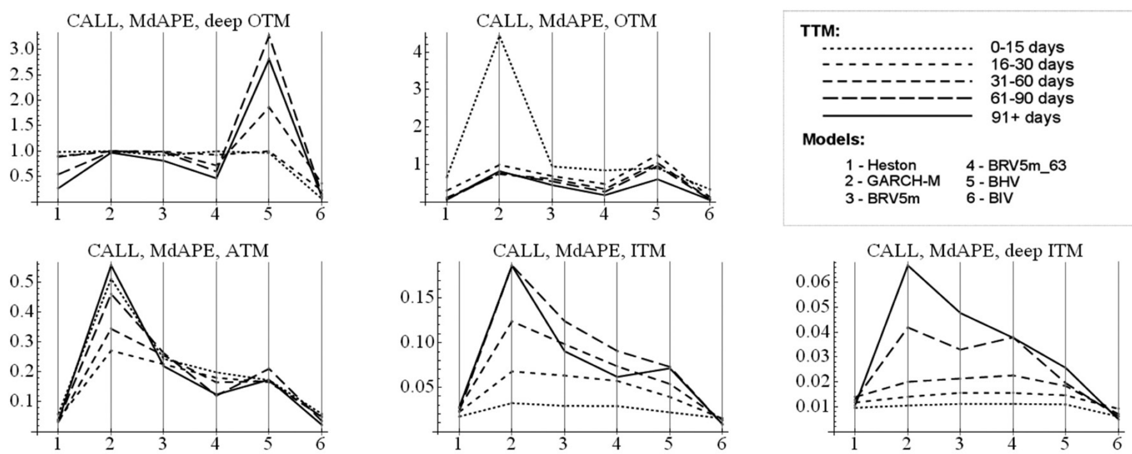


Fig. 9. OP statistics for call options with respect to MR and TTM classes

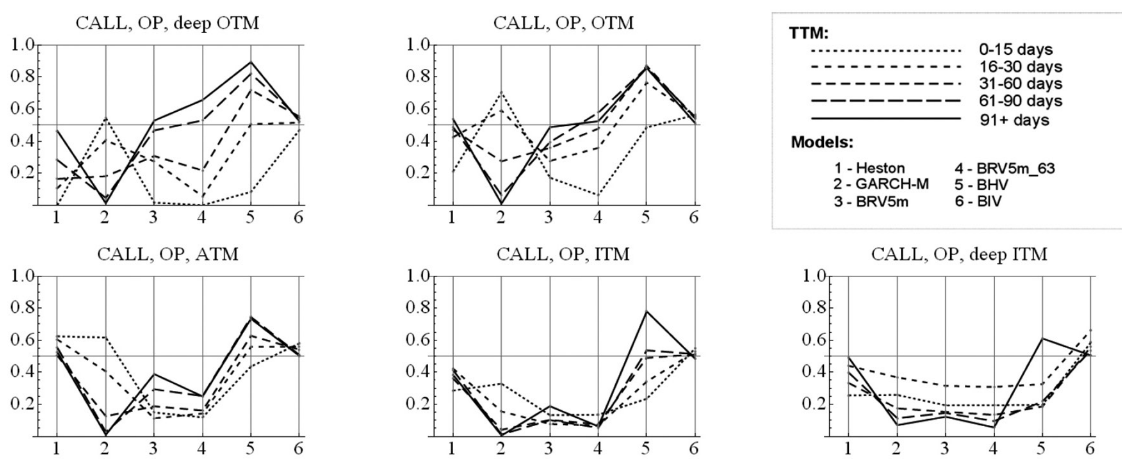


Fig. 10. MdAPE statistics for put options with respect to MR and TTM classes

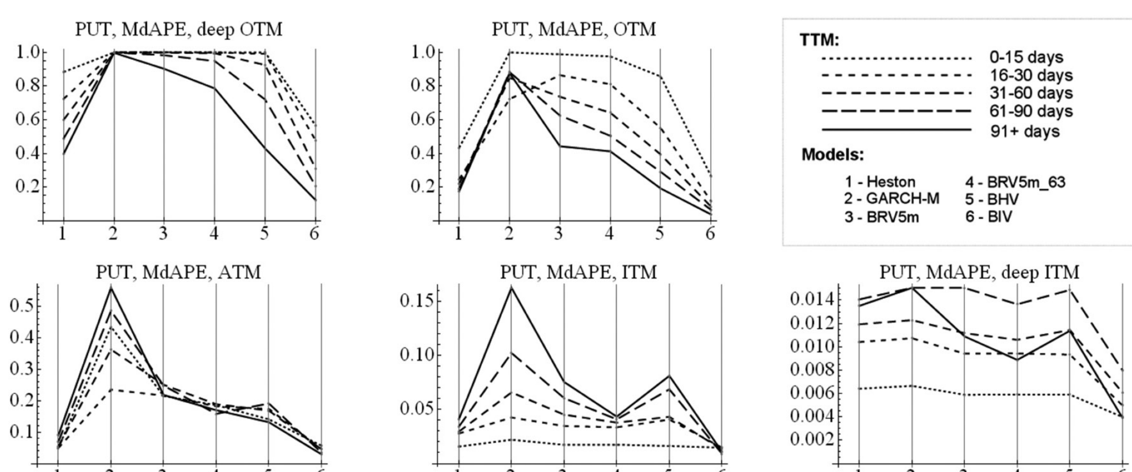
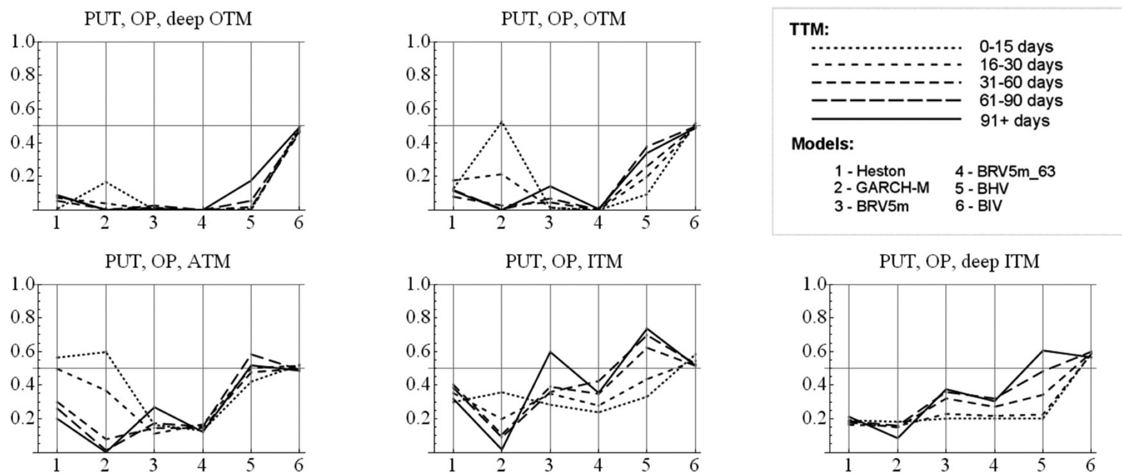


Fig. 11. OP statistics for put options with respect to MR and TTM classes



and the BHV model. On average, GARCH, BRV5m and BRV5m_63 models underpredict the market values of options, especially for ATM, ITM and deep ITM options. Nevertheless, we can see that results – with the exception of the BIV model – vary strongly with changes in the TTM.

MdAPE statistics for put options are shown in Fig. 10. The results are similar to those for call options. Once again we see that the best model is BIV. The Heston model performs only slightly worse. On the next places, we can rank BHV, BRV5m_63 and BRV5m. The highest average pricing errors are produced by the GARCH model.

Moreover, we can observe the same relationship between average pricing errors and MR and TTM classes as previously described for call options. Errors are significantly smaller for the higher-valued options (ITM and deep ITM), while the effect of TTM classes depends on the MR class. For deep OTM and OTM options, we get smaller pricing errors for longer times to maturity, whereas for ITM and deep ITM options, the errors are smaller for shorter times to maturity.

Fig. 11 with OP statistics for put options confirms the ranking of models derived from results for call options. The BIV model is the best one, then the Heston model is the second one and as the third one, we have the BHV model. The results for models other than the BIV depend strongly on TTM classes. We also observe strong underestimation of market prices for all the models with the only exception of the BIV model.

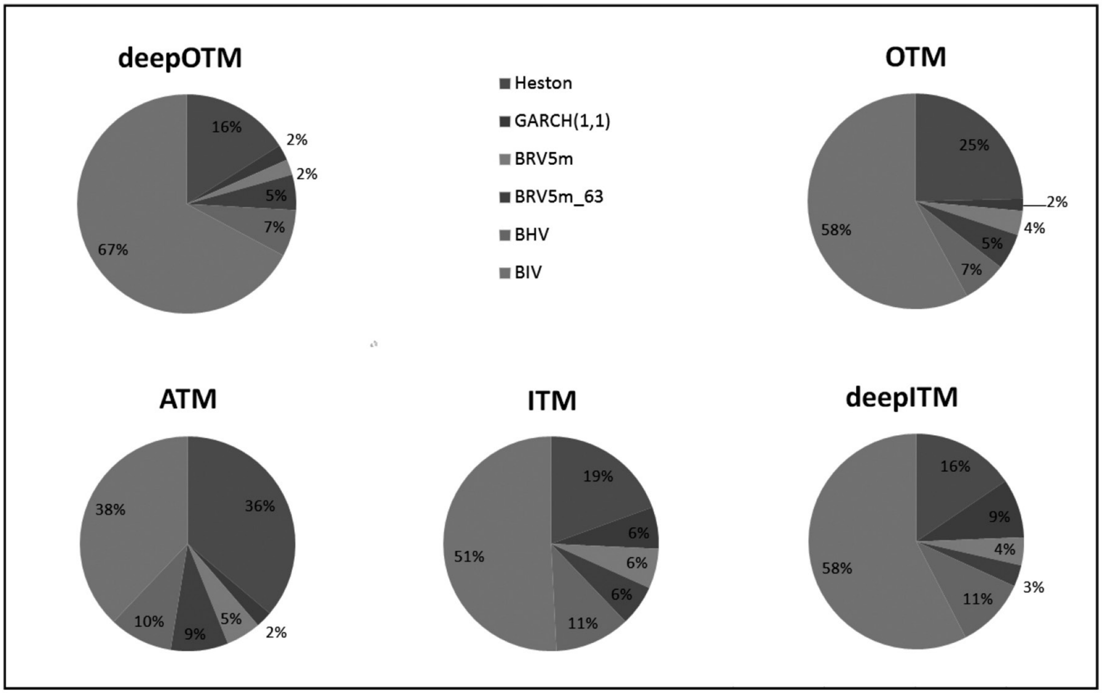
Finally, it has to be emphasized that our results are based on all the available theoretical prices of the analysed options. We did not remove any observations from the original sample. In order to omit the problem of possible outliers among the pricing errors, we used median (absolute percentage) error, which is a robust measure to the possible large deviations of errors from their average value. We argue that this approach, as opposed to removing from the sample problematic observations (low-valued options or options with few days to maturity), is a better solution of the problem of outliers, since it allows analysing models' properties in all the option classes.

5.2 Multidimensional comparisons of results.

In this closing subsection, we present our conclusions in a more formal way. Fig. 12 presents the frequency of best pricing for all the tested models in 5 diagrams for each moneyness ratio for call and put options together. Our initial conclusions from section 5.1 are confirmed here by this aggregated approach. BIV is clearly the best model, the Heston model is the next one, and the third one is BHV. Additionally, we see that the Heston model seems to behave much better for OTM, and especially for ATM options, while the BIV model is the worst model for ATM and then for ITM options. Finally, we noticed that the BRV and the GARCH models are the worst models for every MR.

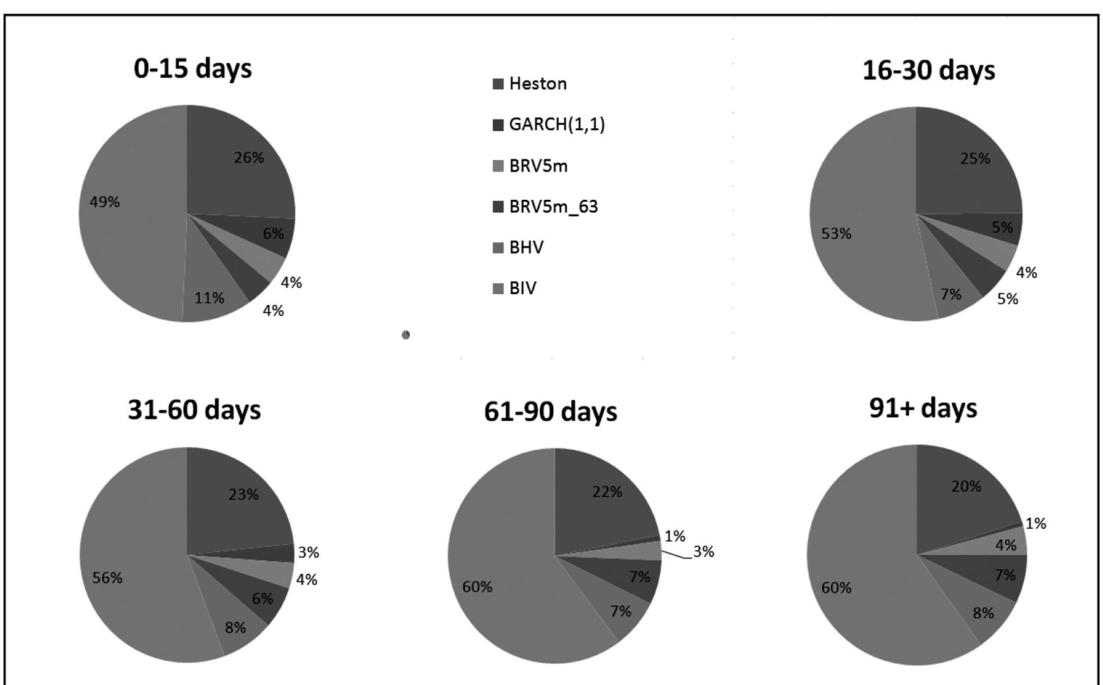
Fig. 13 shows next the frequency of best pricing for all the tested models, but for each TTM class for call and put together. The BIV model is – as expected – the best one, and the Heston model is ranked as the second one, the BHV model follows. Additionally, we see that

Fig. 12. The frequency of the best option pricing for Nikkei 225 index options with respect to MR based on MdAPE error statistic[a]



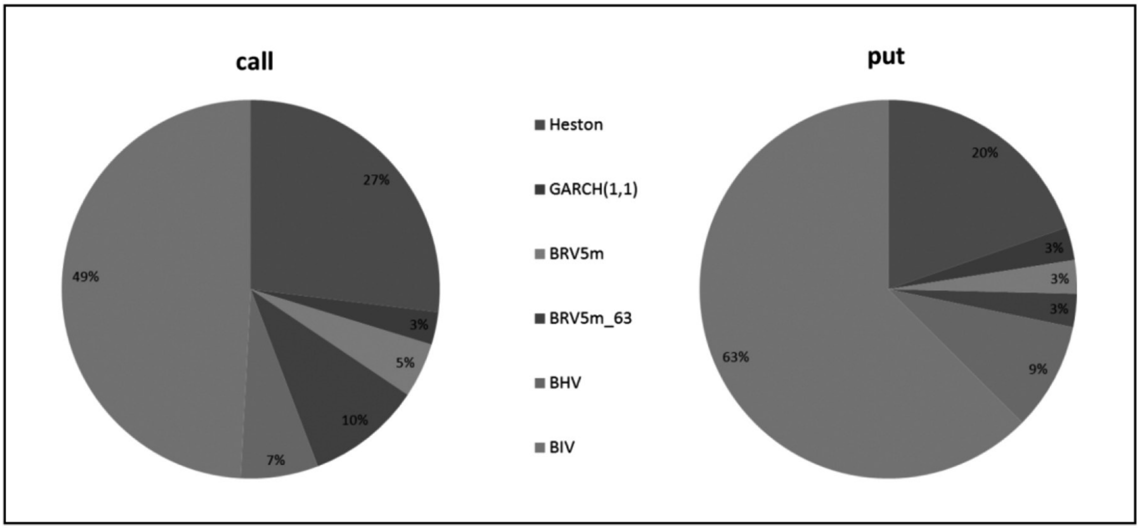
[a] The charts present the data for call and put options together. On each panel, the order of the models is the same: we start with the Heston model at the top and going clockwise end up with the BIV model.

Fig. 13. The frequency of the best option pricing for Nikkei 225 index options with respect to TTM on MdAPE error statistic[a]



[a] The charts present the data for call and put options together. On each panel, the order of the models is the same: we start with the Heston model at the top and going clockwise end up with the BIV model.

Fig. 14. The frequency of the best option pricing for Nikkei 225 index options with respect to the type of option on MdAPE error statistic[a]



[a] On each panel, the order of the models is the same: we start with the Heston model at the top and going clockwise end up with the BIV model.

the BIV model gains on efficiency, while the Heston model worsens its performance when we go from the lowest TTM to the highest one. Other models do not change their performance with respect to the TTM class.

The final Fig. 14 that presents the frequency of best pricing for all the tested models with respect to the type of options obviously does not change the model ranking. However, we see a very interesting pattern concerning the two best models. The BIV model performs much better for put options, while the Heston model is better for call options. Here again we do not observe any significant differences for other models.

6 Conclusions and further research.

In this study, we present a detailed analysis of option pricing models' performance using 5-minutes transactional data for the Japanese Nikkei 225 index options. We compare 6 different types of option pricing models: the Black model with different assumptions about the volatility process (BRV – two cases, BHV, BIV), the Heston model and the GARCH model. Then, we present detailed error statistics describing how efficient in option pricing are the models we test. Furthermore, we focus on the analysis of liquidity for option market in order to better understand different behaviours of options within various classes of the TTM and the MR. Here, we try to summarize our conclusions from this study and we formulate some thoughts concerning further research.

First of all, when we consider the performance of models we have tested, the model ranking, from the most efficient to the least efficient one, is as follows: BIV, Heston, BHV, BRV5m_63, BRV5, and GARCH(1,1). The BIV model comes out as the best in majority of option classes. The model of Heston occurred to be only slightly worse. Next places, with similar results, belong to the Black model with historical volatility and the Black model with realized volatility averaged across last 63 trading days. Average pricing errors for the Black model with realized volatility (not averaged) were higher, due to the more volatile estimates of RV compared to HV estimates. We obtained the worst results for the GARCH(1,1) model. Generally, these results confirm the previous findings for the Polish and Brazilian emerging option markets (Kokoszczyński 2010a, Sakowski 2011). The only exception is the Heston model, which performed significantly worse for the less developed markets. The probable reason is that calibration of the Heston model is strongly dependent on the number of options with different maturities. Nevertheless, to some extent, we can claim that this model ranking is not only a feature of an individual market, but can also be regarded as robust to the level of development, liquidity or various other market characteristics.

Secondly, for both call and put options, we observe the clear relation between average pricing errors and option moneyness: high error values for deep OTM options and the best fit for deep ITM options. We can explain this pattern by noting that highly valued options (ITM or deep ITM) are relatively better priced because of the more active participation of market makers and institutional investors in this market segment, where we do not observe strong under- or overreaction to new information as it happens with individual investors. The concentration of liquidity for low-valued options with short maturities may mean high error for options that are traded more frequently. Such error distribution can explain higher interest of speculative investors for deep OTM and OTM options, where information noise, responsible for larger departure of transactional prices from the theoretical ones, is of greater importance. All these outcomes confirm our previous results for the Polish WIG20 index option market (Kokoszczyński 2010a).

Thirdly, focusing on parameter n (RV averaging parameter) for BRV models, we observe that much lower error values are obtained for n = 63 than in the case of the non-averaged RV, what confirms our initial hypothesis that the non-averaged RV estimator (Fig. 3) is rather a poor choice considering the efficiency of option pricing model. This is the confirmation of results presented in the literature on the efficiency and accuracy of various volatility estimators (Ślepaczuk and Zakrzewski 2009).

Fourthly, we would like to focus on two models with the most time-consuming estimation process (the Heston model and GARCH models). Results we have presented earlier make us doubt whether there is any gain from using them, especially in the case of the GARCH model, which comes out as the worst one, when better models are formally much less complicated, and additionally, less time-consuming in the process of estimation.

Analysing the liquidity issues, we observe several interesting features of the Japanese index option market data. First of all, the volume of calls and puts concentrates in ATM, OTM and deep OTM options, with hardly any volume noticed for deep ITM and ITM options. Secondly, the turnover volume peaks around ATM and ITM options, indicating that the highest (in terms of transaction value) liquidity is observed for ATM options, and then for ITM options. Thirdly, the liquidity – however measured – is significantly higher for put options. Nevertheless, we are aware of the fact that the latter conclusion could result from the sharp downward movement of the market in the time we study and the high demand for put options for hedging purposes.

This final observation shows clearly how important are the liquidity issues for patterns we get while comparing performance of various option pricing models. They should be certainly the subject of further studies. Our intention is thus to conduct a similar study for other markets.

There are suggestions in the literature that notwithstanding unrealistic assumptions of the BSM or the Black model, they can produce results of the same quality than much more sophisticated models do. Our paper constitutes an argument supporting this opinion, because superiority of this model is shown for majority of option classes.

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