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Compatibility in tax reporting

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Purpose – to describe a compliance-monitoring equilibrium in presence of compatibility costs in a setting when managers and other parties have different attitude towards compliance.

Design/Method/Approach. Classical game theory – Nash equilibrium.

Findings. If compatibility costs are small, there exist a unique stable Nash equilibrium of the game between the tax authority and a population of heterogeneous firms. In this equilibrium, the relation between compatibility costs and compliance is non-monotonic and depends on the curvature of auditing function. However, compatibility costs reduce non-compliance in low cheating regimes and may enhance it when many firms are cheating.

Limitations. The model is at high level of abstraction and neglects many important detail that characterize each field where it could be potentially applied.

Theoretical implications. The results provide one rationale for developing countries to be cautious with employing refined auditing schemes and for developed countries to promote complicated accounting procedures.

Originality/value. Compatibility costs are not previously considered in economic analysis of compliance.

Paper type – conceptual.

***Keywords*:** tax evasion; compatibility; coordination; business partners; tax accounting.

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Сумісність у податковій звітності

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Мета дослідження – описати рівновагу у відносинах контролю за дотриманням встановлених вимог (compliance) при наявності витрат на сумісність (compatibility costs) в умовах, при яких менеджер і інші сторони відносин мають різні підходи до встановлених вимог.

Дизайн/Метод/Підхід дослідження. Класична теорія ігор – рівновага Неша (Nash equilibrium).

Результати дослідження. Виявлено, що якщо витрати на сумісність є невеликими, то існує унікальна рівновага Неша в грі між податковими органами і популяцією гетерогенних фірм. В даній рівновазі відношення між витратами на сумісність і відповідність встановленим вимогам є немонотонним і залежить від кривизни функції аудиту. Проте витрати на сумісність знижують невідповідність встановленим вимогам (non-compliance) в умовах низького рівня обману (low cheating regimes) і збільшують цю невідповідність при залученні великої кількості фірм в обманні дії.

Обмеження дослідження. Дана модель заснована на високому рівні абстракції і нехтує багатьма важливими деталями, які притаманні можливим сферам практичного застосування.

Практичне значення дослідження: Результати пропонують логічне обґрунтування для країн, що розвиваються, при прийнятті адаптованих схем аудиту, а для розвинених країн для впровадження більш складних процедур аудиту.

Оригінальність/Цінність/Наукова новизна дослідження. Витрати на сумісність раніше не використовувалися в економічному аналізі встановлених вимог.

Тип статті – теоретична.

***Ключові слова*:** ухилення від податків; сумісність; координація; бізнес-партенрство; оподаткування.

Совместимость в налоговой отчетности

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Цель исследования – описать равновесие в отношениях контроля за соблюдением установленных требований (compliance) при наличии затрат на совместимость (compatibility costs) в условиях, при которых менеджер и другие стороны отношений имеют различные подходы к установленным требованиям.

Дизайн/Метод/Подход исследования. Классическая теория игр – равновесие Нэша (Nash equilibrium).

Результаты исследования. Выявлено, что если затраты на совместимость невелики, то существует уникальное равновесие Нэша в игре между налоговыми органами и популяцией гетерогенных фирм. В данном равновесии отношение между затратами на совместимость и соответствием установленням требованием является немонотонным и зависит от кривизны функции аудита. Тем не менее, затраты на совместимость снижают несоответствие установленным требованиям (non-compliance) в условиях низкого уровня обмана (low cheating regimes) и увеличивают это несоответствие при вовлечении множества фирм в обманные действия.

Ограничения исследования. Данная модель основана на высоком уровне абстракции и пренебрегает многими важными деталями, которые свойственны возможным сферам практического применения.

Практическое значение исследования. Результаты предлагают логическое обоснование для развивающихся стран при принятии адаптированных схем аудита, а для развитых стран для внедрения более сложных процедур аудита.

Оригинальность/Ценность/Научная новизна исследования. Затраты на совместимость ранее не использовались в экономическом анализе установленных требований.

Тип статьи – теоретическая.

***Ключевые слова*:** уклонение от налогов; совместимость; координация; бизнес-партенрство; налогообложение.

Introduction

Managers of varying levels in hierarchy throughout the world face a dilemma whether to comply with various standards. Firms as represented by their managers add new dimensions to the problem over and above standard gambling and cat-and-mouse[1] approaches. Firstly, a firm is not a single decision maker and has its own agency problem, as stressed by Crocker and Slemrod (2005). Second, the interaction between firms can be important for the general outcome, as Bayer and Cowell (2009) and Sanchez (2006) point out, although Lipatov (2008) shows that the interaction matters in games with individual taxpayers as well.

Even in the simplest cases, successful hiding of information from a monitoring authority requires coordinated action whenever two or more parties involved in a transaction. For example, in sophisticated tax avoidance and evasion (tax evasion that requires certain expertise and involves intricate manipulation of accounts), there may be multiple parties as well as substantial costs of making accounts consistent and looking good at superficial checks of tax authorities. In the US, Sarbanes-Oxley act of 2002[2] has made these costs even higher[3].

The other aspect of costs to coordinate are differences in the reports about the monitored activity that should be similar a priory. In case of tax reports of transacting firms, the tax authority observes transactions and can audit both partners, having some idea of how correlated their incomes are. It is well known in the profession that the tax audits are not random. First, the taxpayers are divided in homogenous auditing classes. Second, within each class the tax authority may receive some signals that a given report is suspicious. One of such signals is a discrepancy in the reports of business partners. The importance of coordination in tax reporting is also confirmed experimentally by *Alm and McKee* (*2004*).

The counter-checking of reports is a standard procedure for some taxes. For VAT, this particularly makes sense, as a part of the tax that is paid by one party is then rebated by the other. Das-Gupta and Gang (2001) model the matching of purchase and sales invoices explicitly. They conclude that cross-matching can induce truthful reporting, but distorts purchase and sales decisions. In Russia the auditing of one firm involves checking accounts of the firms that are transacting with it, as described e. g. in *Sumina* (*2006*).

McIntyre (2005) writes that most of the modern sheltering schemes undermine the basic principle of tax law: a tax deductible item of one taxpayer is a part of taxable income of the other. The evasion opportunity arises when one firm deducts some payments made to the other firm, but this other firm is not taxable, e. g. it is an off-shore company. This kind of evasion looks simple in principle, but requires sophisticated organization and coordination not to be obvious. In turn, the detection of such evasion requires counter-checking of the reports provided by business partners. In Russia, the mechanism of evasion is similar, though the schemes are usually blunter: the accounting specialists register a lot of fictitious firms some of which just do not pay taxes and disappear.

We look here at a long run situation in an economy where firms exercise transactions with each other. Before entering the industry, a firm has to decide whether to adopt aggressive attitude towards tax reporting or to stay on the compliant side. This choice of accounting standard is analogous to the choice of a computer operating system in its compatibility aspect. That is, while operating together, the firms with different accounting machineries incur higher transaction costs than the firms with similar accounting procedures do.

If a firm decides to be aggressive, it hires a tax evasion specialist who arranges accounts for a certain fee[4]. A compliant firm manages accounts itself. After the accounting policy has been adopted, the firms start operating and transacting with other firms. Finally, the firms get profits and report them to a tax authority. The tax authority observes the transacting firms and decides on the auditing intensity.

Thus, in our economy the firms face two kinds of costs in addition to standard costs and benefits of evasion. The first type is compatibility costs, which have to be borne every time there is a transaction between firms with different accounting standards. These are related to the adjustment of accounts for different kinds of firms: e. g., the aggressive and complying firms often prefer transactions to be reflected in the books at different time points or at different locations[5]. The second type is endogenous costs, which arise every time the tax authority sets unequal probability of auditing for the cases of observing similar and different reports of the two firms whose income is known to be correlated.

The endogenous costs are also present in Sanchez (2006). The difference of his paper from our approach is not only in the lack of compatibility costs, but also that he considers tax authority with ability to commit. This is well explained by different ideas underlying the two papers: whereas we consider long-run equilibrium, Sanchez concentrates on the short-term with the aim of constructing auditing rule that minimizes mistakes of the tax authority (in sense of auditing the honest and not auditing cheaters). Furthermore, whereas Sanchez describes the situation in a homogenous auditing class, assuming perfect correlation of income and uncertainty about the auditing rule, we consider a pair of firms with imperfectly correlated income.

The paper by Bayer and Cowell (2009) stands even further from us: it looks at the effect of auditing on joint decision of competing firms to evade and to produce. Though their main result, the desirability of non-fixed auditing rule, survives in our setup, we consider firms that are partners rather than competitors, and we focus on the effect of compatibility costs rather than auditing rules. Crocker and Slemrod (2005) go inside a firm, whereas we treat it as a decision making unit.

In our model, the tax authority has no ability to commit. Firstly, this has a natural appeal for the long run modeling. Secondly, though the auditing rules are often announced by the tax authorities, there is no means to establish whether they are actually followed.

The main result of the paper is equilibrium characterization: We find out that equilibrium cheating and auditing differ substantially from the approach disregarding transactions among the firms, even if the compatibility the share of cheating firms as well as the auditing probability is costs are small. When evasion is small, likely to be overestimated, if the coordination of tax reports is not taken into account. In case of popular misreporting, both the share of non-compliers and the auditing probability may be underestimated. It is worth noting that the auditing probability in our setting varies with the reports combination, making comparison with uniform auditing probability of the representative case difficult in principle.

In general, we identify three effects that a change in compliance share has on attractiveness of aggressive accounting: positive “

[1] The term is borrowed from Cowell (2006) and refers to the modeling of evasion as a game between tax agency and a single taxpayer.

[2] The following information about the act is taken from http://www.fmsinc.org/cms/?pid=3253.

[3] The data availability requirements that are also part of costs can be checked at http://www.itcinstitute.com/display.aspx?id=2021.

[4] We treat the specialist as a passive player here. Her optimization problem is analysed in Lipatov (2008).

[5] A list of common tax shelters can be found at http://www.lowtax.net/lowtax/html/offon/usa_new/usashelt.html.

differential probability" and " saving" effects and negative " auditing change" effect. The positive effects reflect benefits from being compatible with more of the potential partners; the negative effect comes from strategic reaction of the monitoring authority. The total effect of any parameter on the endogenous variables is then influenced by the sum of the three effects identified.

For a large class of auditing technologies, we find that compatibility costs decrease cheating and auditing when only few firms are underreporting and increase them in case evasion is popular. The correlation of profits has a similar effect. In both instances, with coordination cost ascent the more popular strategy becomes more attractive; hence more firms choose it in equilibrium. Somewhat surprisingly, but following exactly the same logic, improvement in auditing technology and fines reduce cheating in low evasion regimes and enhance it in high evasion regimes.

The auditing probability in our model can be positively affected by the amount of fines, unlike in representative case. This becomes possible because the direct effect of larger fines to make auditing more attractive may overplay the indirect effect coming through the reduced cheating. Finally, the effectiveness of fine always decreases as a result of an increase in compatibility costs.

We also shed some light on the mechanism of evasion game when compatibility matters: we show that correlation of profits solely generates the difference in auditing probabilities. The compatibility costs alone change equilibrium cheating and auditing, but leave the latter independent from the report configuration.

The rest of the paper is structured as follows. The model setup is presented in the next section, followed by the description of equilibrium structure. Section four is devoted to the discussion of the results for the mixed equilibrium. Conclusion is followed by appendix with derivations of equilibria and results.

Intended contribution

The research aims at characterizing equilibrium outcomes when firms' management follows an optimal strategy of compliance with respect to some regulated activity (e.g. tax compliance, labor protection compliance, environmental compliance). In particular, the optimal strategy takes into account the effect of the actions of the other firms' managers actions on the profits of the firm. This research question is answered using the tools of classical game theory, i.e Nash equilibrium in mixed strategies.

Compliance game

Single firm benchmark

Let us start with the case when there are no transacting pairs and no compatibility costs. A single firm decides whether to evade its profit, facing the tax authority that can perform auditing. We use the approach of Graez, Reinganum and Wilde (1986) in this benchmark, with a convex rather than linear cost function for auditing.

First, the nature moves, assigning a type to the firms: high profit $h = \pi$ or low profit $l = 0$. The types are drawn from a distribution characterized by a density function

$$f(x) = \begin{cases} \gamma & \text{if } x = \pi \\ 1-\gamma & \text{if } x = 0 \end{cases}.$$

Second, the high profit firms decide whether to submit a high report $H = \pi$ (be honest) or a low report $L = 0$ (cheat).

The tax authority does not audit high reports and exerts effort a in auditing low reports. A continuous function $a:[0,1) \to R_+$ is a mapping from detection probability defined on the unit interval to the auditing effort defined for non-negative real numbers. The inverse function determines detection probability from the effort $p: R_+ \to [0,1)$. We assume that the firms can never be detected with certainty, and zero effort results in zero detection probability $p(0) = 0$. The low report is honest with probability $\frac{1-\gamma}{1-\gamma+q\gamma}$ and not with the complementary probability, where q is the probability that high profit firm is cheating.

The authority is maximizing its expected revenue $\frac{q\gamma}{1-\gamma+q\gamma} p(a)(1+s)t\pi - a$, the high income firm - its expected profit $\pi - p(a)(1+s)t\pi$. Here S is a surcharge rate for being caught, t is a tax rate. In equilibrium with positive detection probability FOC for the tax authority $p'(a^*)(1+s)t\pi = \frac{1-\gamma}{q^*\gamma} + 1$, and indifferent condition for the firm is . Hence equilibrium effort is

$$p(a^*) = \frac{1}{1+s}$$

and equilibrium evasion probability is

$$q^* = \frac{1-\gamma}{\gamma} \frac{1}{p'\left(p^{-1}\left(\frac{1}{1+s}\right)\right)(1+s)t\pi - 1}.$$

Sufficient conditions for the existence of such an equilibrium: p is strictly increasing and strictly concave, $p'\left(p^{-1}\left(\frac{1}{1+s}\right)\right) > \frac{1}{\gamma(1+s)t\pi}$. The latter actually ensures mixed equilibrium. If, to the contrary, detection probability does not increase fast enough or the fine is too small, the equilibrium is all cheating. The equilibrium, either in mixed or pure strategies, is unique with strictly increasing and strictly convex p. We retain this assumption for the rest of the paper. The mixed equilibrium is of most interest to us, since we do not observe full cheating and the fines are usually high enough to cover auditing costs in reality. Moreover, this mixed equilibrium is evolutionary stable (Weibull 1995), as even if a small part of taxpayers gives honest reports, the reduction in detection probability is not enough to off-set a loss from lower evasion.

Two transacting firms

Recall the story behind our model, presented in the introduction. Firstly, the firms choose their accounting standards. Second, the firms are matched according to some rule. Third, the firms draw pre-tax incomes from participating in a match. The second and third stages may repeat a number of times. Fourth, the firms summarize the realized income and submit a tax report. Finally, the tax authority audits the tax reports of some firms (and all its partners).

To make the analysis as simple as possible while preserving the coordination aspect, we make the following simplifying assumptions: (i) each firm meets only one transacting partner; (ii) each firm makes only one transaction; (iii) the aggressive firm does not report truthfully. Under these assumptions the game above is equivalent to the following 3 player game.

The setup

Consider a simultaneous game between two risk neutral firms and a tax authority.

The first move is made by the firms. They decide whether to adopt aggressive accounting policy and pay a price b per evaded euro for it, $0 \le b < t$, or to use compliant accounting that comes at a cost normalized to zero.

The second move is made by the nature that assigns a type to each of the two firms: high profit $h = \pi$ or low profit $l = 0$. We assume now that the profits are correlated with the correlation coefficient $r, 0 \le r < 1$[6]. We do not consider negative correlation, as our firms are cooperating rather than competing. The joint distribution of two types in a match is given by the following density function:

$$f(x,y) = \begin{cases} \delta, & \text{if } x = y = \pi, \\ \gamma - \delta, & \text{if } \{x,y\} = \{0,\pi\}, \\ 1 - 2\gamma + \delta, & \text{if } x = y = 0. \end{cases}$$

where $\delta := \gamma^2 + \gamma(1-\gamma)r, \delta \in [\gamma^2, \gamma)$.

After the pre-tax profit is realized, the firms submit their reports according to the procedure they chose in the first stage. Namely, the low income firm submits a low report and gets a payoff normalized to 0, if its partner has the same accounting standard, and $-c$, if it has a different standard. The high income firm submits a high report $H = \pi$ (be honest) if chose compliant policy or a low report $L = 0$ (cheat) if chose aggressive policy. Each firm of type h (high profit) gets ex interim expected payoff (before the coordination costs c) of $u(i,j)$, where i is its own report and j is a report of its partner:

$$u(L,L) = \pi - p(a^{LL})(1+s)t\pi - b\pi,$$
$$u(L,H) = \pi - p(a^{HL})(1+s)t\pi - b\pi,$$
$$u(H,H) = u(H,L) = \pi(1-t).$$

Ex ante expected profit is then the following. If a firm decides to use aggressive accounting,

$$u(A) = \delta(qu(L,L) + (1-q)u(L,H)) + (\gamma - \delta)u(L,L) + (1-\gamma) * 0 - (1-q)c.$$

Here the event when both the firm and its partner get high profit defines the first term, the event when the firm gets high profit and its partner gets a low one defines the second term. The third term contains the payoff in the event of our firm getting low profit, normalized to zero. In any event we have to subtract coordination cost c in case our aggressive firm is matched with the compliant one, and that is what the last term takes care of.

If a firm decides to use compliant accounting, it is

$$u(C) = \delta(qu(H,L) + (1-q)u(H,H)) + (\gamma - \delta)u(H,L) + (1-\gamma) * 0 - qc.$$

The terms are similar: both firms getting high profit, only the compliant firm getting high profit, and the compliant firm getting low profit.

The third move is by the tax authority, which chooses an auditing effort $a \in R_+$ conditional on the reports observed: $a(LL)$ (two low reports), $a(HL)$ (a low and a high report in any order), $a(HH)$ (two high reports). The tax authority gets expected revenue of $p(a)(1+s)t\pi - a$ from each cheater it audits and the revenue $t\pi - a$ from each honest report it audits.

The game takes into account both types of costs outlined in the introduction. Compatibility costs are fixed to c per transaction. The endogenous coordination cost reflects the difference in detection probabilities the tax authority might want to generate. Namely, the authority can exert different efforts in auditing low profit report depending on whether it comes with another low report or with a high report. Compared to the case of two low reports, it needs a half of resources to provide the same auditing probability if one of the reports is high. Thus, we do not consider the case in which coordinated evasion requires more effort to discover than uncoordinated does.

We choose the simultaneous formulation rather than a sequential one, because we do not want to consider a particular industry structure or a relation between an entrant and an incumbent. Our goal is to characterize the economy where two firms from different populations (again, think of buyers and sellers) meet to play a coordination game. Even more, since the decisions are long-term, they become a property of the firms, so that they can be characterized as evaders or honest. In this way, the Nash equilibria of the simultaneous game show us where these populations could converge to.

Optimization problem of the tax authority

The tax authority observes the match. Recall that we denote with lower-case letters the profits, and with upper-case the reports. We have then the following profit - report table which represents the measures (or shares) of taxpayer pairs reporting incomes given by the column entries, while actually receiving incomes given by row entries.

Table 1

Profit – report table

	total	*HH*	*HL*	*LL*
hh	δ	$\delta(1-q)^2$	$2\delta q(1-q)$	δq^2
hl	$2(\gamma - \delta)$	0	$2(\gamma - \delta)(1-q)$	$2q(\gamma - \delta)$
ll	$1 - 2\gamma + \delta$	o	o	$1 - 2\gamma + \delta$

[6] We have also analyzed the case when r = 1, but since this is not likely to happen in reality, we do not present the results here. It turns out that the equilibrium structure in this case is distinctly different from correlation arbitrary close to perfect, so we also cannot use it as a benchmark. The derivation of equilibrium is available upon request.

Table 2

A breakdown of the compatibility costs propagation mechanism

	$c=0, r=0$	$c>0, r=0$	$c=0, r>0$
$p^*(LL)$	$\frac{t-b}{(1+s)t}$	$p\left(p^{'-1}\left(\frac{\gamma q^*+1-\gamma}{\gamma q^*(1+s)t\pi}\right)\right)$	$p\left(p^{'-1}\left(\frac{\delta q^{*2}+2q^*(\gamma-\delta)+1-2\gamma+\delta}{(\delta q^{*2}+q^*(\gamma-\delta))(1+s)t\pi}\right)\right)$
$p^*(HL)$	$\frac{t-b}{(1+s)t}$	$p\left(p^{'-1}\left(\frac{\gamma q^*+1-\gamma}{\gamma q^*(1+s)t\pi}\right)\right)$	$\frac{t-b}{(1+s)t}$

The following lemma characterizes the best response of the tax authority in this case.

Lemma 1 In the tax evasion game above the best response of the tax authority $a(q)$ to the firms cheating with probability $q\in(0,1]$ is implicitly defined by:

$$a^{HH}=0, \tag{1}$$

$$p'\left(a^{HL}(q)\right)=\frac{\delta q+\gamma-\delta}{\delta q(1+s)t\pi}, \text{if } q\geq q^0_{HL}; \tag{2}$$

$$p'\left(a^{LL}(q)\right)=\frac{\delta q^2+2q(\gamma-\delta)+1-2\gamma+\delta}{\left(\delta q^2+q(\gamma-\delta)\right)(1+s)t\pi}, \text{if } q\geq q^0_{LL}; \tag{3}$$

$$a^{HL}(q)=0, \text{if } q<q^0_{HL}; a^{LL}(q)=0, \text{if } q<q^0_{LL}. \tag{4}$$

The proof is left to the appendix A, q^0_{HL} and q^0_{LL} are also defined there. Obviously, observing two high reports the tax authority does not audit them. Observing different reports in a match, the authority audits the low one with probability determined by the effort $a^{HL}(q)$. When two low reports are observed, the optimal auditing effort is given by $a^{HL}(q)$.

Note that the two efforts (and corresponding probabilities) are only equal, when $r=0$, that is the report of one firm does not contain any information about the profit of the other firm. With $r>0$ we have $a^{HL}(q)\geq a^{LL}(q)$, which is quite intuitive: different reports indicate possible cheating, so it makes sense to audit them more.

Equilibria

Before stating the result it is useful to introduce the following terminology:

We call an equilibrium of our game full cheating, if all the firms are submitting low (zero) reports in this equilibrium $q^*=1$; we call an equilibrium full honesty, if all the high income firms submit high reports $q^*=0$.

The proposition 1 characterizes the equilibria arising in case of correlated draws. We denote the equilibrium values of cheating probability with q^* and of auditing effort with a^*.

In the tax evasion game with two transacting firms

(i) there exists a symmetric evolutionary stable equilibrium with q^* implicitly defined by

$$\gamma(t-b)\pi-(1-2q^*)c=\left(\left(\gamma-\delta(1-q^*)\right)p\left(a^{LL}(q^*)\right)+ \tag{5}$$
$$+\delta(1-q)p\left(a^{HL}(q^*)\right)\Big)(1+s)t\pi,$$

$a^{HH*}=0$, $a^{HL*}=a^{HL}(q^*)$, $a^{LL*}=a^{LL}(q^*)$ as given by (1), if the compatibility costs are small and

$$\gamma(t-b)\pi-(1-2q^0_{LL})c>\delta(1-q^0_{LL})p\left(a^{HL}(q^0_{LL})\right)(1+s)t\pi, \tag{6}$$

where q^0_{LL} reflects auditing technology and is defined in the appendix.

(ii) There exists a symmetric evolutionary stable equilibrium with q^* implicitly defined by

$$\gamma(t-b)\pi-(1-2q^*)c=\delta(1-q)p\left(a^{HL}(q^*)\right)(1+s)t\pi, \tag{7}$$

$a^{HH*}=0$, $a^{HL*}=a^{HL}(q^*)$, $a^{LL*}=0$, if the compatibility costs are small and (6) does not hold[7].

(iii) If $\gamma(t-b)\pi-c\leq 0$, there exist a full honesty equilibrium with $q^*=0, a^*\equiv 0$.

(iv) If $\gamma(t-b)\pi+c\geq\gamma p\left(a^{LL}(1)\right)(1+s)t\pi$, there exist a full cheating equilibrium, and $q^*=1, a^{HH*}=0$, $a^{HL*}=a^{HL}(1)$, $a^{LL*}=a^{LL}(1)$.

The proof of the proposition is left to appendix B. The structure of equilibria is very intuitive: for small compatibility costs (how small they should be depends on the auditing technology) there is a unique stable mixed equilibrium, as in a standard game without coordination issues. A small qualification here is that it takes a different form depending on whether consistent low reports are audited (i) or not (ii).

With larger compatibility costs, multiple equilibria may arise. More importantly, full honesty and full cheating may become equilibrium, as with everybody around being honest it is too costly in terms of compatibility to use aggressive accounting and visa versa. Whereas only the magnitude of the compatibility costs (relative to the evasion benefits) decides whether there exist full honesty equilibrium (iii), the auditing technology also plays a role in determining the existence of full cheating equilibrium (iv).

Discussion of results

Summary

Since we believe that the exogenous coordination costs are relatively small, we can concentrate on the regions of parameter values where a mixed equilibrium exists. As it has been already noted, the probability of auditing for dissonant reports is higher than that for the similar reports as long as $r>0$. A further breakdown of the compatibility costs propagation mechanism is represented in the table below:

[7] The equilibria characterized in (i) and (ii) are also unique under the conditions specified in the appendix B.

From this table we see clearly that the differential auditing probability is generated from some correlation even in the absence of exogenous compatibility costs. On the other hand, only exogenous costs c shift equilibrium cheating probability even in the absence of auditing intensity differential: The following expression determines the share of aggressive firms with independent draws.

$$\gamma(t-b)\pi-(1-2q^*)c=\gamma p\left(p^{'-1}\left(\frac{\gamma q^*+1-\gamma}{\gamma q^*(1+s)t\pi}\right)\right)(1+s)t\pi. \quad (8)$$

Thus, the two channels of the compatibility costs can be clearly separated.

The following remark shows how the expected payoff of the firms I depend on the compatibility costs. The payoff is easy to compute because in the mixed equilibrium the firms a ex ante indifferent between aggressive and compliant accounting.

Compatibility costs put a burden on the firms unless there is a full honesty: $I=\gamma(1-t)\pi-q^*c$

Comparative statics

Firstly, we are interested in how the equilibrium value of cheating depends on the compatibility costs. For $q^*>q^0_{LL}$, from (5) we have

$$(1-2q^*)dc=Qdq, \quad (9)$$

$$Q:=\begin{pmatrix}\delta\left(p\left(a^{HL}\right)-p\left(a^{LL}\right)\right)-\left(\gamma-\delta\left(1-q^*\right)\right)p'\left(a^{LL}\right)a^{LL}_q\\ -\delta\left(1-q^*\right)p'\left(a^{HL}\right)a^{HL}_q\end{pmatrix}(1+s)t\pi+2c. \quad (10)$$

On the lhs we see the direct effect of c on the costs of evasion: When there are more compliant firms ($q^*<1/2$), the effect is positive, as there is a higher chance to meet a compliant firm and incur the compatibility costs. Otherwise ($q^*>1/2$), the effect is negative, as there is a higher chance to meet a firm with aggressive accounting.

On the rhs we see the indirect effect of c on the benefits of evasion through changing q. The effect is more intricate and can be divided into three terms. The first term is a positive differential in auditing probability for similar and different reports. Indeed, with higher q there is a lower chance to submit different reports, so the evading firms can enjoy lower auditing probability (" differential probability" effect). The second term reflects the negative effect of q on the attractiveness of evasion through raising auditing probability for both types of the reports (" auditing change" effect). The third term is positive and reflects the increase in benefits from evasion through saving on compatibility costs (" saving" effect).

Thus, the total indirect effect is ambiguous. Note that this is true not only for compatibility costs, but for any parameter affecting q, since it is actually change in q itself that either increases or decreases attractiveness of evasion depending on how responsive the auditing probability is. This in turn depends on the curvature of the auditing function (we see $p'(a)$ directly in (9), in appendix we show that a_q depends on $p''(a)$). As $p'(a)$ is decreasing in q with $\partial^2 p'(a(q))/(\partial a\partial q)<0$, the effect of the auditing change is most likely to outweigh other effects for small q, and visa versa. Because of strict monotonicity and $p(+\infty)\le 1$, auditing functions satisfy $p'(+\infty)=0$. So, the auditing change effect evaporates for large q, and the total effect becomes positive.

For the class of functions with $p'(0)=+\infty$, $q^0_{LL}=q^0_{HL}=0$ and the auditing change effect grows unboundedly large at zero, whereas the lhs is bounded, so the total effect is certainly negative. Thus, for such functions there is a threshold value of equilibrium share of cheaters q^c, below which the total indirect effect is negative (and hence $dq^*/dc<0$ for $q^*<\min\{1/2, q^c\}$), and above which the total indirect effect is positive (and hence $dq^*/dc<0$ for $q^*>\max\{1/2, q^c\}$).

We also note that the differential probability effect is reinforced by profit correlation more than the auditing change effect, so the total is more likely to be negative with lower correlation. At the extreme of independent draws we shall have

$$Q=-\gamma p'(a)a_q(1+s)t\pi+2c,$$

which is negative, if compatibility costs are small.[8]

If the equilibrium is at the intersection when only inconsistent reports are audited, that is $q^0_{HL}<q^*<q^0_{LL}$, we have

$$(1-2q)dc=Q^0dq \quad (11)$$

$$Q^0:=\left(\left(p\left(a^{HL}\right)-\delta(1-q)p'\left(a^{HL}\right)a^{HL}_q\right)\delta(1+s)t\pi+2c\right)$$

We can see the play of all effects described above also here. The differential probability is represented by $p\left(a^{HL}\right)$ and the auditing change effect is weakened, because the similar reports are not audited. We know that close to q^0_{HL} the total effect is negative, as $p\left(a^{HL}\left(q^0_{HL}\right)\right)=0$. With higher q the differential probability effect kicks in and the auditing change effect is less pronounced, so that the total may even change its sign.

Second, we are interested in the effect of correlation on the equilibrium share of firms that use aggressive accounting. For an equilibrium with non-zero auditing of both report combinations[9], we have

$$Dd\delta=Qdq, \quad (12)$$

$$D:=\begin{pmatrix}\left(\left(\gamma-\delta\left(1-q^*\right)\right)p'\left(a^{LL}\right)a^{LL}_\delta+\delta\left(1-q^*\right)p'\left(a^{HL}\right)a^{HL}_\delta\right)\\ +\left(p\left(a^{HL}\right)-p\left(a^{LL}\right)\right)\left(1-q^*\right)\end{pmatrix}(1+s)t\pi \quad (13)$$

The direct effect of the correlation on the costs of evasion is always positive (the last term in D). It increases in the probability differential and the share of compliant firms. Intuitively, with higher correlation different reports are more likely, other things being equal. And since different reports are more likely to be detected and punished than similar, expected fine increases in profit correlation. The indirect effect of the correlation through auditing probability is represented by the first two terms in the expression for D. The first term is a negative effect through the decrease in auditing of similar reports ($a^{LL}_\delta<0$), the second term is a positive effect through the increase in auditing of different reports ($a^{HL}_\delta>0$).

Here we can observe that for small q the negative effect becomes small, whereas for large q the positive effects vanish. The total effect of correlation on q depends then on the indirect effect discussed at length above. For example, for auditing functions satisfying Inada conditions $dq^*/dr<0$ for both very small and very large q^*.

[8] For sufficiently large compatibility costs the total effect is positive, but this is most likely to be irrelevant, as we are not sure about existence and uniqueness of the equilibrium under consideration.

[9] The sign of the expression does not change if only high-low report combinations are audited.

Third, we would like to see how an improvement in auditing technology affects the equilibrium. Consider a new auditing technology $p^1(a) = kp(a), k > 0$. Then

$$Kdk = Q^1 dq, \tag{14}$$

$$K := \begin{pmatrix} (\gamma - \delta(1-q^*))(p(a^{LL}(q^*)) + kp'(a^{LL})a_k^{LL}) \\ +(1-q)p(a^{HL}(q^*) + kp'(a^{HL})a_k^{HL}) \end{pmatrix}(1+s)t\pi. \tag{15}$$

where Q^1 is a correspondingly adjusted version of Q that takes into account k. As expected, the direct effect of an improvement in auditing on the costs of evasion is positive: the same effort of the tax authority results in higher expected fine for a firm. The indirect effect is negative: with more effective auditing the optimal auditing effort is reduced, so the expected fine goes down as well. The total effect K depends on the size of $p''(a)$: If $p(k)$ is concave, the direct effect is higher than the indirect one, so the total effect of an improvement in auditing on the evasion costs is positive; the opposite is true for convex $p(k)$.

The effect through equilibrium cheating Q^1 is not affected much, as both differential probability and auditing change effects are amplified to the same extent, only the compatibility effect becomes relatively less important. Then with Inada conditions and positive K, $dq^*/dk < 0$ for $q^* < q^c$, $dq^*/dk > 0$ for $q^* > q^c$, that is improvement in auditing technology reduces cheating in low evasion regimes and enhances it in high evasion regimes.

Fourth, we look at the fine. In our model the cheating is not necessarily decreasing in the surcharge rate S. The deterrence effect depends again on whether an increase in q curbs or boosts benefits of evasion, i.e. on the sign of Q:

$$((\gamma - \delta(1-q^*))p(a^{LL}(q^*)) + \delta(1-q)p(a^{HL}(q^*)))t\pi ds = Qdq. \tag{16}$$

With Inada conditions that means $dq^*/ds < 0$ for $q^* < q^c$, $dq^*/ds > 0$ for $q^* > q^c$.

We define the measure of effectiveness of the fine as the absolute value of the derivative of the equilibrium cheating $\left|\frac{dq^*}{ds}\right|$.

We immediately see that this measure is decreasing in compatibility costs, so the fines loosen their grip with higher costs in our equilibrium. This is important to have in mind while formulating a tax/enforcement/accounting policy.

Conclusion

The compliance game with costs of accounting compatibility between transacting firms or monitored parties is considered in this paper. We show that when compatibility costs are small, there is a unique stable equilibrium[10] with a positive share of non-compliant firms and a positive share of audited reports. When the costs are large, there may be multiple equilibria, in some of which either everybody or nobody complies.

The game yields the insights that are impossible to obtain within the representative agent framework. Firstly, the monitoring authority should put more effort in auditing parties that did not coordinate their compliance decision, if it maximizes its expected revenue. Second, the compatibility costs may affect the extent of compliance in the opposite directions depending on what the auditing technology and the equilibrium share of noncompliance are. If there are many non-compliant agentss, the compatibility costs are more likely to increase noncompliance, and visa versa. In the application to tax avoidance, the correlation of taxpayer profit affects equilibrium in a similar way. Third, the effect of the fines and auditing technology on equilibrium values crucially depends on the prevailing reporting standard. When most of the firms use an aggressive standard, an increase in fines or auditing effectiveness may have an adverse effect on compliance.

There is a number of policy recommendations arising from our analysis. Firstly, compatibility costs reduction efforts are only justified for economies (or industries) with substantial shadow sector. Such efforts include simplified accounting (exogenous costs) and little interest in the business links (endogenous costs through auditing probability differential). Second, the marginal increases in fines may be dangerous in economies or industries with low levels of compliance. Third, compatibility costs enhancement may be a sensible strategy for high compliance countries, and it may even be financed by eventual reduction in enforcement costs.

We hope that our paper opens up a whole pile of issues that could not be addressed by the literature before. How do the links between managers of transacting firms affect their decision to comply with regulations, e.g. to pay taxes? How are these links taken into account by the monitoring authority? Could the government change the structure of these links for the benefit of the whole society? We cannot answer these questions in a far too simplified setting of transacting pairs we have here. However, what we can do is to say that the equilibrium behavior of the agents is affected significantly by the links between them, that it is affected through the costs of behaving differently, and it is affected in the direction of harmonization of this behavior.

[10] The uniqueness is guaranteed under mild technical assumption presented in the appendix.

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Appendices

A - Proof of Lemma 1

The expected revenue of the auditor is

$$\delta(1-q)^2 t\pi + (\gamma-\delta)(1-q)t\pi + \delta q(1-q)t\pi \quad (17)$$

$$+(1+s)p(a^{HL})\delta q(1-q)t\pi \quad (18)$$

$$-(\delta q(1-q)+(\gamma-\delta)(1-q))a^{HL}$$

$$+(1+s)p(a^{LL})(\delta q^2+q(\gamma-\delta))t\pi$$

$$-(\delta q^2+2q(\gamma-\delta)+1-2\gamma+\delta)a^{LL}$$

Here the first term is the revenue from the firms that have high profits and do not evade (they are of measure $\delta(1-q)^2$). The second group of 3 term is the revenue from the mixed reports: the high reports bringing $t\pi$ are of measure $\delta q(1-q)+(\gamma-\delta)(1-q)$, and low reports bringing in the fine are $\delta q(1-q)$. Correspondingly, the costs of auditing must be subtracted for these cases. Finally, the last terms are the revenue from low reports and costs of auditing them. The same fine is levied in the cases of two firms or only one firm misreporting.

Rearranging and taking first order conditions with respect to a^{LL} and a^{HL} gives

$$a^{HL}: -(\delta q(1-q)+(\gamma-\delta)(1-q))+\delta q(1-q)(1+s)p'(a^{HL})t\pi = 0,$$

$$a^{LL}: (\delta q^2+q(\gamma-\delta))(1+s)t\pi p'(a^{LL})-(\delta q^2+2q(\gamma-\delta)+1-2\gamma+\delta)=0.$$

Working this out, we arrive at

$$g^{HL}(q) := \frac{\delta q+\gamma-\delta}{\delta q(1+s)t\pi} = p'(a^{HL}(q)),$$

$$g^{LL}(q) := \frac{\delta q^2+2q(\gamma-\delta)+1-2\gamma+\delta}{(\delta q^2+q(\gamma-\delta))(1+s)t\pi} = p'(a^{LL}(q)),$$

where the equality holds for $a>0$. In this case we can show that $p'(a^{HL}(q))< p'(a^{LL}(q))$, as

$$g^{LL}(q)-g^{HL}(q) = \frac{\delta q^2+2q(\gamma-\delta)+1-2\gamma+\delta}{q(\delta q+\gamma-\delta)(1+s)t\pi} - \frac{\delta q+\gamma-\delta}{\delta q(1+s)t\pi} = \frac{\delta-\gamma^2}{\delta q(\delta q+\gamma-\delta)(1+s)t\pi},$$

which is positive for any positive correlation and zero for independent draws. Note also that the difference is decreasing and convex in q, so $\partial^2 p'(a(q))/(\partial a\partial q)<0$, $\partial^3 p'(a(q))/(\partial a\partial q^2)>0$.

Under concavity assumption second order conditions are trivially satisfied and $a^{HL}(q)>a^{LL}(q)$. Our intuition is confirmed: low reports paired with high reports are audited more intensively than those paired with low reports.

Note though that because $\lim_{q\to 0} g^{HL}(q)=+\infty$, there may also be a corner solution. Indeed, for any auditing function $p(a)$: $\lim_{a\to 0} p'(a)<+\infty$ there will be a corner solution. Formally, for all such functions $\exists q^0_{HL}>0: a^{HL}(q)=a^{LL}(q)=0\forall q\le q^0_{HL}$. By construction it is also true that $\exists q^0_{LL}>q^0_{HL}: a^{HL}(q)>a^{LL}(q)=0\forall q\in[q^0_{HL},q^0_{LL}]$. These threshold values can be found from the auditing function. For the different reports we have

$$q^0_{HL} = \frac{\gamma/\delta-1}{p'(0)(1+s)t\pi-1}.$$

For the similar reports the threshold value f the share of firms with aggressive accounting is implicitly defined by

$$1-2\gamma+\delta = (p'(0)(1+s)t\pi-1)\delta(q^0_{LL})^2+q^0_{LL}(\gamma-\delta)((1+s)t\pi-2).$$

Since $0\le q^0_{HL}\le 1$, if $p'(0)<1/((1+s)t\pi)$, tax authority will never audit, as the marginal revenue from audit is negative. Furthermore, if $p'(0)<\gamma/(\delta(1+s)t\pi)$, the best response function is degenerate with $a(q)\equiv 0$; if $p'(0)<1+1/(\delta(1+s)t\pi)-\gamma/\delta$, the similar reports are never audited: $a^{LL}(q)\equiv 0$.

Thus, both best responses (for mixed and similar reports) of the tax autority are weakly increasing continuous functions of q.

B - Proof of proposition 2

To show that p^*, q^* is indeed a Bayesian Nash equilibrium, we need 1) p^* is a best response of tax authority given the belief about q; 2) each firm plays best response to p^* and the share of cheating firms q^*; 3) the belief of the authority is consistent with equilibrium play of the firms.

For 1) we need (??) and (1); for 2) in a mixed equilibrium it is sufficient that each firm is indifferent between cheating and honesty given that the partner is cheating with probability q:

$$u(A) = u(C)$$

or

$$\delta(qu(L,L)+(1-q)u(L,H))+(\gamma-\delta)u(L,L)-(1-q)c =$$
$$\delta(qu(H,L)+(1-q)u(H,H))+(\gamma-\delta)u(H,L)-qc$$

Rearranging, we get

$$\gamma(t-b)\pi-(1-2q)c=((\gamma-\delta(1-q))p(a^{LL})+\delta(1-q)p(a^{HL}))(1+s)t\pi. \quad (19)$$

Note that this expression depends on q unlike in the benchmark case, so we cannot present the resulting equilibrium explicitly. However, the two sides of the equation admit quite a straightforward intuitive explanation. The lhs is the benefit from evasion net of accounting costs b and coordination costs c. The rhs is the expected cost of fines in two types of matches: two low reports and high-low reports. Both costs and benefits of evasion increase with q. The higher population share of evaders relieves the coordination problem for a firm that chose aggressive accounting. At the same time, higher share of wrong reports calls for more auditing thus increasing expected fine.

Formally, from the properties of best response functions $a^{LL}(q), a^{HL}(q)$ we can see that rhs of (19) is weakly monotonically increasing in q. Namely, it is zero for $q \le q^0_{HL}$, it is $\delta(1-q)p(a^{HL}(q))(1+s)t\pi$ for $q \in [q^0_{HL}, q^0_{LL}]$, and it is the full expression for $q \ge q^0_{LL}$ converging to $((\gamma-\delta(1-q))p(a^{LL}))(1+s)t\pi$ as q approaches unity. We know that $p_a'(q)$ is convex (we can directly compute second derivatives). We also know that $p(p_a')$ is decreasing, but we did not impose anything on its convexity/concavity. Now, $p(q)$ can be written as $p(p_a'(q))$. It is increasing, and it is also concave if $p(p_a')$ is not too convex. Thus, rhs is concave under a mild ansumption on the third derivative of the function $p(a)$.

At q^0_{LL}, the left derivative of rhs is $[(1-q^0_{LL})p_q(a^{HL}(q^0_{LL}))-p(a^{HL}(q^0_{LL}))]\delta(1+s)t\pi$, the right derivative has an additional term $[(\gamma-\delta(1-q^0_{LL}))p_q(a^{LL}(q^0_{LL}))+\delta p(a^{LL}(q^0_{LL}))](1+s)t\pi$, which is positive.

Lhs is linearly increasing in q with the slope $2c$, starting with $\gamma\pi(t-b)-c$. The intersection(s) define Bayesian Nash equilibrium. Because of a jump in the derivative of rhs at q^0_{LL}, we may have up to 6 intersections (with up to 3 locally stable equilibria). However, the more interesting case for us is the stable unique equilibrium, which indeed results for small values of c, if either $(1-q^0_{LL})p_q(a^{HL}(q^0_{LL}))-p(a^{HL}(q^0_{LL}))\ge 0$ or $\gamma(t-b)\le(\delta(1-q^0_{LL})p(a^{HL}(q^0_{LL})))(1+s)t$ or $\gamma(t-b)\ge(1+s)t\max_{q\in[q^0_{HL},q^0_{LL}]}(\delta(1-q)p(a^{HL}(q)))$. At the limit of no coordination costs, the equilibrium is defined by one of the conditions $\gamma(1-b/t)=((\gamma-\delta(1-q))p(a^{LL})+\delta(1-q)p(a^{HL}))(1+s)$ or $\gamma(1-b/t)=\delta(1-q)p(a^{HL})(1+s)$, depending on the auditing technology. Namely, the first happens, if $\gamma(t-b)\pi-(1-2q^0_{LL})c>\delta(1-q^0_{LL})p(a^{HL}(q^0_{LL}))(1+s)t\pi$, and the second otherwise.

This equilibrium is unique and stable. By continuity, the same is true for small values of c.

Note that with increase of c lhs simply rotates around horizontal line given by $\gamma(t-b)\pi$. It retains this value at $q=0.5$, while going down by c at $q=0$ and up by c at $q=1$. This immediately leads us to the following corollary:

With $q=1/2$, the effect of coordination costs is completely neutralized.

This is very intuitive: when the two populations are balanced, there is neither potential gain nor loss in terms of coordination from playing either strategy.

Note that the equilibrium will only be stable, if at the intersection the slope of the evasion costs (rhs) exceeds the slope of the benefits from evasion (lhs). Thus, stability requires the following condition to be satisfied:

$$2c<[((1-q^*)p_q(a^{HL}(q^*))-p(a^{HL}(q^*)))\delta+(\gamma-\delta(1-q^*))p_q(a^{LL}(q^*))+\delta p(a^{LL}(q^*))](1+s)t\pi,$$

where q^* is the equilibrium share of the firms that employ aggressive accounting.

If there is no stable interior equilibrium, the full cheating is stable. A general condition for existence of full cheating equilibrium is $u(A)\ge u(C)$ given $q=1$. This can be rewritten, similarly to (19), as

$$\gamma(t-b)\pi+c\ge\gamma p(a^{LL}(1))(1+s)t\pi, \quad (20)$$

with $p'(a^{LL}(1))=1/(\gamma(1+s)t\pi)$.

Full honesty may also be an option, if the auditing is cheap or payment for evasion high. A general condition for the existence of full honesty euilibrium is $u(A)\le u(C)$ given $q=0$. This can be rewritten as

$$\gamma(t-b)\pi-c\le 0.$$

However, this equilibrium is globally stable only if

$$\gamma(t-b)\pi-(1-2q^0_{HL})c\le 0. \quad (21)$$

C - comparative statics results

By inverse function theorem

$$a_q^{HL} = \left(p^{'-1}\right)'\left(\frac{\delta q+\gamma-\delta}{\delta q(1+s)t\pi}\right)_q' = -\frac{\gamma-\delta}{\delta q^2(1+s)t\pi}\frac{1}{p''\left(a^{HL}\right)}, \tag{22}$$

$$a_q^{LL} = \left(p^{'-1}\right)'\left(\frac{\delta q^2+2q(\gamma-\delta)+1-2\gamma+\delta}{\left(\delta q^2+q(\gamma-\delta)\right)(1+s)t\pi}\right)_q' = \tag{23}$$

$$-\left(\delta\frac{\gamma-\delta}{(\delta q+\gamma-\delta)^2(1+s)t\pi}+(2\delta q+\gamma-\delta)\frac{1-2\gamma+\delta}{\left(\delta q^2+(\gamma-\delta)q\right)^2(1+s)t\pi}\right)\frac{1}{p''\left(a^{LL}\right)}. \tag{24}$$

and

$$a_k^{HL} = \left(p^{'-1}\right)'\left(\frac{\delta q+\gamma-\delta}{\delta q(1+s)t\pi}\right)_q' = -\frac{\gamma-\delta}{\delta q^2(1+s)t\pi}\frac{1}{p''\left(a^{HL}\right)}, \tag{25}$$

$$a_q^{LL} = \left(p^{'-1}\right)'\left(\frac{\delta q^2+2q(\gamma-\delta)+1-2\gamma+\delta}{\left(\delta q^2+q(\gamma-\delta)\right)(1+s)t\pi}\right)_q' = \tag{26}$$

$$-\left(\delta\frac{\gamma-\delta}{(\delta q+\gamma-\delta)^2(1+s)t\pi}+(2\delta q+\gamma-\delta)\frac{1-2\gamma+\delta}{\left(\delta q^2+(\gamma-\delta)q\right)^2(1+s)t\pi}\right)\frac{1}{p''\left(a^{LL}\right)}. \tag{27}$$