

## The Model of Projection of a Fuzzy Individual Professional Educational Trajectory

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### Abstract

The article suggested considering certain components of the professional competence as fuzzy sets and strictly the professional competence as a universal fuzzy set. A model of the projection of a fuzzy individual professional educational trajectory (FIPET) on the basis of modalities is “essentially” and “possibly” presented. The mathematical solution to the problem of the optimum choice of courses of the career enhancement on the basis of the expert estimation of the level of professional competence and one’s own preferences of a trainee is proposed.

**Keywords:** *professional competence, fuzzy state, fuzzy modeling, linguistic variable, mathematical properties*

### Introduction

The reform of the postgraduate pedagogical education system in Ukraine contemplates, first of all, de-monopolization, which enables the teacher to choose the content, form and place of their qualification improvement. Simultaneously, an efficient influence on the pedagogical activity productivity is insured only if the content of teacher studying takes into account not only the current level of professional competence, but also peculiarities of professional activity in the workplace. The presented materials are a result of research aimed at the creation of a certain model of an individual professional educational trajectory of pedagogical staff, which will bring into coherence contradictions concerning the peculiarities of the professional competence level assessment on the basis of linguistic variables; the necessity of providing the freedom of choice of professional development on the basis of modern modeling methods.

In the last twenty years, a true explosion of investigations into fuzzy modeling and its applications in control, diagnostics, decision making, optimization, pattern recognition, robotics, etc. has been observed. The main cause of the usage of fuzzy logics was formulated by Zadeh L.A., 1975a, 1975b, 1975c: through the lack of surplus aspiration for accuracy many classes of important problems, in which data, purposes and restrictions are too complicated or insufficiently determined to provide correct mathematical analysis, have been kept out because they cannot be mathematically interpreted. The usage of fuzzy logic as a basis for the modeling of different pedagogical phenomena and processes to some degree represents the development of anthropocentric technologies in education, since fuzzy logic is closest to human thinking.

### **Research Problem**

The usage of fuzzy logics as a methodological basis for pedagogical research is related to the fact that both final goals and points of the current level of different competences are verbally expressed in the form of linguistic variables. The main statements, a terminological dictionary and the mathematical solution of different applied problems are presented in scientists' papers (Zadeh L.A., 1976, Borisov A.N., Krumberg O.A., Fedorov I.P., 1990, etc.). Robert Babuska, 1998, addresses fuzzy modeling from the system and control engineering point of view and focuses on the selection of appropriate model structures, on the acquisition of dynamic fuzzy models from process measurements (fuzzy identification), and on the design of nonlinear controllers based on fuzzy models. Andrzej Piegat, 2001, claims that the attraction of fuzzy modeling results from its intelligibility and the high effectiveness of the models obtained in any field process modeling, systems modeling, e.g., economics, business, medicine, agriculture, meteorology. Nowadays, there are various opportunities of the application of fuzzy logic in the course of modeling in education (Michael Gr. Voskoglou, 2007, 2012). Programming with the application of fuzzy sets is studied by international research groups (Tavana Madjid, Marbini Adel Hatami, Saati Saber, Hajiahkondi Elham, 2012). Minghuang Li and Fusheng Yu from Beijing Normal University, China (Li, Minghuang; Yu, Fusheng, 2013) are involved in the sphere of fuzzy programming.

Fuzzy modeling was not used in the planning or organization of training of pedagogical workers in the past. The scientific novelty of research consists in the creation of mathematical models, expansion of Mamdani's algorithm for the solution of practical tasks: free (but effective) choice of teacher-training courses (in systems of formal, informal or non-formal education). Problems related to

the improvement of professional competence are generally not presented in papers on fuzzy logics dedicated to the solution of applied problems. At the same time, taking into account the mathematical properties of fuzzy sets in modeling and projecting the FIPET will enhance the accuracy of the choice of an educational course and, as a result, the efficiency of the improvement of professional competence.

### Research Focus

The main method of scientific research at this stage is the method of theoretical modeling on the basis of fuzzy logics (fuzzy modeling), using such general scientific methods as analysis, synthesis, upward movement from an abstract one to a concrete one, deduction, analogy, and generalization. Fuzzy modeling is staged creation of an informational and logical model with the use of linguistic variables to describe its state. Stages of the creation of the fuzzy model include: analysis of a problem situation; structuring of a researched area, formation of the fuzzy model; performance of computing experiments with the fuzzy model; application of the obtained results; and correction and adaption of the fuzzy model. The obtained informational and logical model expressing the most sufficient stages and peculiarities of the process of formation of the individual professional educational trajectory and taking into account the structure of professional competence is grounded in the theory of fuzzy sets and fuzzy logics (Leonenkov A.V., 2005). Such a peculiarity allows for identification of the model as a model of projecting the fuzzy individual professional educational trajectory (FIPET).

The majority of pedagogical studies using fuzzy logics as a methodological basis are grounded in defining fuzzy sets as a result of a gradual approach to fuzzy grouping or grouping of objects being in a fuzzy state (Novak V., Perfil'eva I., Mochkorzh I., 2006). At the same time, a fuzzy state is a state of an object described with the use of linguistic variables expressing subjective categories. The uncertain choice of a priority direction of further education by a subject is not a less important moment of FIPET projecting. Fuzzy choice is defined as a lack of knowledge on the possibility of one or another choice of a direction of changes in the bifurcation point during a determined moment of time. Consequently, the diagnostics of a professional competence fuzzy state (**problem 1**) and the uncertainty of choice of a further direction of education (**problem 2**) are the main problems of projecting the FIPET requiring a theoretical solution as well as a practical one.

## **Research Methodology**

### **Research General Background**

The phenomenon of “professional competence”, which is a complicated open dynamic system in itself, simultaneously expresses two aspects – a system of functions compulsory for the representative of a certain profession and the personal content of a certain system expressed as the performance of functions in a concrete workplace. Being a subject or object to a set of studies, professional competence remains in the area of scientific interests due to its own complicity and ambiguity of approaches to its examination. Analysis of a set of papers dedicated to the examination of different aspects of professionalism (Kuz'mina N.V., 1990; Vasil'ev A.F., Vegeera A.S., Myslovec E.N., 2012; Grebnev I.V., 2007) allows for emphasizing further peculiarities of the professional competence of pedagogical staff, conducive to the identification of it as a fuzzy set:

- determining the level of professional competence, researchers use value judgments;
- the mentioned value judgments are expressed by linguistic variables and are of subjective character;
- the value judgments, which are used to estimate the level of the professional competence of pedagogical staff, express the gradual approach being a distinctive peculiarity of a fuzzy set.

### **Research Sample**

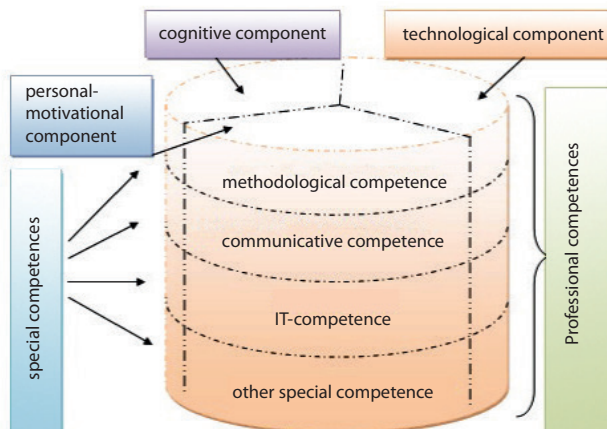
The model (Figure 1) of professional competence is based on a three-dimensional structure containing cognitive, technological, and personal-motivational components.

The quantity and sense of special competences are determined on the basis of functions performed by an employee in a concrete workplace and of a scientifically substantiated model of a special competence chosen as the background. The structural model of the professional competence of pedagogical staff can be used for the planning of professional development. The algorithm of planning has to consider the level of professional competence and freedom of choice.

### **Instrument and Procedures**

The algorithm of making a decision on the priority of one or another content of education based on fuzzy logics is grounded in Mamdani's fuzzy inference method (Mamdani, E.H., 1977) and consists of logic blocks: a basis for knowledge (linguistic variables and fuzzy rules), a system of data estimation; a block of fuzzification;

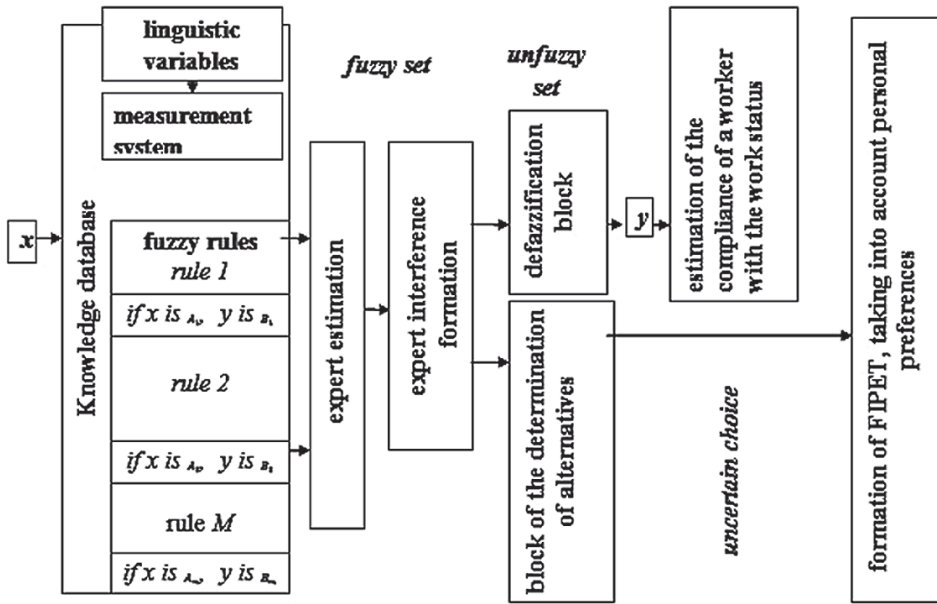
**Figure 1.** Structural model of the professional competence of pedagogical staff



a block of solutions; a block of defuzzification. Input data are correlated with output data. To provide a correlation of these two types of data, there is a need to involve a fuzzy system with a so-called fuzzificator (which transforms sets of input data into a fuzzy set) at the input and a defuzzificator (which transforms fuzzy sets into the concrete value of an output variable) at the output. The fuzzificator transforms an unfuzzy set of input data into a fuzzy set determined by the membership function and the defuzzificator solves an inverse problem, that is to say, it forms an unambiguous solution regarding an input variable on the basis of numerous fuzzy interferences produced by the block of solutions.

The model proposed by Mamdani should be modified according to the task – the estimation of the professional competence level. Moreover, the developed model should solve problems related to the projection of the fuzzy professional educational trajectory on the basis of the obtained fuzzy estimations. It is possible to additionally use unfuzzy estimations obtained after passing the stage of defuzzification to make a decision on the compliance of a worker with the work status in passing attestation. The solution of a complex of problems is possible in terms of the application of modifications such as the usage of a multi-criteria system of professional competence indices, the development of a measurement system, the inclusion of a block of the determination of alternatives; the usage of the uncertain choice and modalities, “essentially” and “possibly,” for the formation of the fuzzy individual professional educational trajectory (Figure 2).

Figure 2. Model of the projection of the fuzzy professional educational trajectory.



The obtainment of the input data is provided due to a specially developed system of the three-level expert estimation of a level of indices of professional competence. Using a previously formed set of indices, which was obtained by the comparison of one or another linguistic variable with the real state of the estimated object, an expert forms a complex estimation of the level of his own professional competence.

An output signal of the interference module has the form of  $P$  of fuzzy sets, which determine a range of changes of the output variable, before it comes to the defuzzificator. The defuzzificator transforms this range into one concrete value perceived as an input signal of the whole system. The obtained unfuzzy value can be used in the process of attestation of a pedagogical worker to determine the compliance of the worker with the work status. The final expert estimation of the professional competence level presented in the form of a fuzzy set is the solution to **problem 1** and the basis for the further projecting of FIPET.

A next stage is the solution to **problem 2** – the choice of the content of education. To form an individual educational root as a component of the fuzzy professional educational trajectory, there is a need to use the modalities “essentially” and “possibly”. The choice of modality depends on the level of usefulness of a certain

educational course and contains 4 alternatives: courses of career enhancement, methodological work at an education institution, self-education (informal education), and training within the system of non-formal education (distance courses of professional network communities). Ranking of the alternatives is pre-formed in accordance with the estimation of an influence on a certain component of professional competence. The computation of the optimum choice of courses of career enhancement is performed on the basis of the method described by Borisov A.N., Krumberg O.A., Fedorov I.P., 1990, for cases of the choice of alternatives under the additiveness of criteria.

### Data Analysis

Let us consider the solution to this problem (problem 2) on the basis of a particular example. Primarily, four alternatives, the points of which are presented in Table 1, are ranked.

**Table 1.** Points of the usefulness of alternatives.

Components of professional competence	Alternative			
	1	2	3	4
technological	bad	good	satisfactory	bad
cognitive	good	good	satisfactory	bad
individual and incentive	bad	good	good	satisfactory

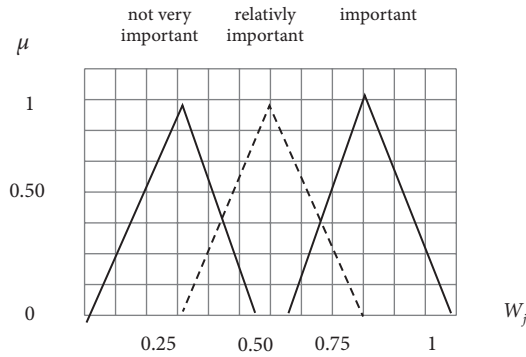
The components in the table are ranked, taking into account their importance – from the most important to the least important, depending on the results of the expert estimation obtained at the previous stage. The first component is determined as **IMPORTANT**, the second one – **RELATIVELY IMPORTANT**, the third one – **NOT VERY IMPORTANT** (Figure 3 – for components and Figure 4 – for alternatives).

The membership functions of weighed points for the alternatives are expressed by the following formulas:

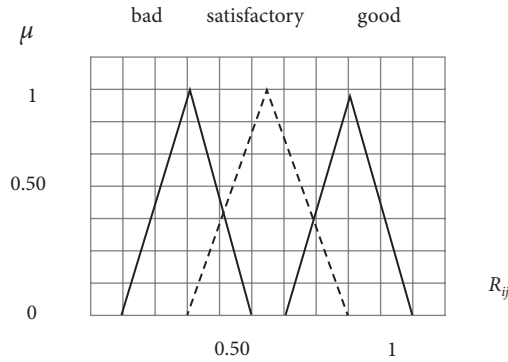
$$\begin{aligned}
 R'_1 &= R'_{11} & W'_1 + R'_{12} & & W'_2 + R'_{12} & & W'_{13} \\
 R''_1 &= R''_{11} & W''_1 + R''_{12} & & W''_2 + R''_{12} & & W''_{13} \\
 R^*_1 &= R^*_{11} & W^*_1 + R^*_{12} & & W^*_2 + R^*_{12} & & W^*_{13}
 \end{aligned}$$

where  $R'_1$  – a left threshold of the membership function;  $R''_1$  – a right threshold;  $R^*_1$  – a top of the membership function.

**Figure 3.** The membership functions of the indices of importance  $W_1, W_2,$  and  $W_3$



**Figure 4.** The membership functions of points of the usefulness of alternatives.



$$R'_1 = R'_{11} W'_1 + R'_{12} W'_2 + R'_{13} W'_{13} = 0.55 \cdot 0.1 + 0.25 \cdot 0.6 + 0.1 = 0.055 + 0.15 + 0 = 0.205;$$

$$R''_1 = R''_{11} W''_1 + R''_{12} W''_2 + R''_{13} W''_{13} = 1 \cdot 0.5 + 0.75 \cdot 0.8 + 0.45 \cdot 0.5 = 0.5 + 0.6 + 0.225 = 1.325;$$

$$R^*_1 = R^*_{11} W^*_1 + R^*_{12} W^*_2 + R^*_{13} W^*_{13} = 0.25 \cdot 0.3 + 0.5 \cdot 0.8 + 0.75 \cdot 0.3 = 0.75 + 0.4 + 0.225 = 0.7.$$

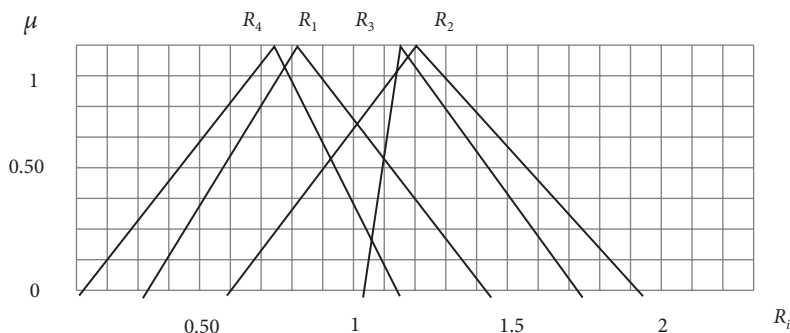
Values of  $R'_2, R''_2, R^*_2$  and also  $R'_3, R''_3,$  and  $R^*_3$  are analogically computed. In this case, the weighed points account for the following values:

$$\begin{array}{lll} R'_2 = 0.48 & R'_3 = 0.915 & R'_4 = 0.08 \\ R''_2 = 1.76 & R''_3 = 1.675 & R''_4 = 1.01 \\ R^*_2 = 1.2 & R^*_3 = 1.01 & R^*_4 = 0.6375 \end{array}$$



Results of computations in the form of the membership functions of the weighed points are presented in Figure 5.

**Figure 5.** Membership functions of the weighed points  $R_1, R_2, R_3, R_4$



Having obtained the weighed points  $R_i$ , a researcher should compare the alternatives on the basis of these points. To achieve this goal there is a need to apply the fuzzy set  $I$  determined on the basis of the set of indices of the alternatives  $\{1, 2, \dots, m\}$ . (Borisov A.N., Krumberg O.A., Fedorov I.P. (1990, p. 91)). The value  $\mu_I(i)$ , determining this index, is calculated according to the following formula:

$$\mu_I(i) = \sup \min_{r_1, r_2, \dots, r_m \geq r_j; V_j}$$

Consequently, there is an ordered series of alternatives, which is expressed in the following way: 2,3,1,4.

## **Research Results**

The mathematical solution to the problem of the choice of the most useful alternative allows for determining of a course of education, which provides an optimum influence on each of the components of professional competence, taking into account the priority. Further choice is performed by a pedagogical worker with the use of recommendations after the choice of modality.

The choice of modality is “essentially” and “possibly” performed in accordance with the ordered series of alternatives (a variant 1) or after the additional computation of indices “the priority of a course” and “personal preferences of a pedagogue” (variant 2). In the first case, the modality is “essentially” used for the alternative

with the maximum coefficient of priority. The modality is “possibly” used for the next alternative. The obtained FIPET is considered to be a recommendation, since it is exclusively based on the results of expert estimation and does not take into account personal preferences of a pedagogical worker.

In the second case, the mathematical computations are performed with the use of the formulas mentioned above before decision making. However, since the “personal preferences of a pedagogue” are of individual character, the obtained ordered series allows for determining of a course of education taking into consideration both the expert estimation of the professional preference level of a pedagogical worker, the usefulness of one or another alternative and the pedagogue’s personal preferences. This is the final aim of the formation of the FIPET.

## **Discussion**

The application of the developed model of the projection of the fuzzy professional educational trajectory allows for revealing of a group of difficulties that appeared in the process of implementation. These difficulties require solution before a mass formative experiment will be performed:

- the complicated procedure of the expert estimation and the formation of the final expert value;
- the necessity of preliminary calibrating of all used verbal estimations;
- a significant number of mathematical computations at different stages of modeling.

The creation of a computer program providing automation of preliminary calibrating of the verbal estimations of all persons who make decisions at different stages of the FIPET projection and, also, the calculation of mathematical operations are effective ways of the solution to all the mentioned problems.

The model of the projection of the FIPET is substantiated by the mathematical properties of the fuzzy sets. It solves the two main applied problems: the diagnostics of the fuzzy state of professional competence and the determination of the choice of a further direction of education.

Peculiarities of the presented model of the process of projecting the FIPET consist in taking into account mathematically proved properties of fuzzy sets, while determining the professional competence level as well as forming the content of education; the mathematical substantiation of the choice of alternatives, the usage of the modalities “essentially” and “possibly”, while making the uncertain choice of an educational root, and, also, the adaptation of Mamdani’s inference model to solve this problem.

## Conclusions

As a result of the research, the main mathematical properties of the structure components of professional competence, which are presented in the form of fuzzy sets and which directly influence the selection of the content of education, were revealed. Despite the fact that the professional competence of pedagogical workers is considered as an example, fuzzy logics may be applied in the field of professional competence enhancement and the improvement of skills of any specialists. Further research should be aimed at revealing and analyzing of indices of additiveness and the emergency of the system; the specification of the optimum correlation between formal, non-formal, and informal education.

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