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# Process Approach to Learning and Teaching Mathematics

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**Abstract**

In the research, a quasi-experimental model was applied and the experimental group received the process approach to learning and teaching mathematics, which builds on the cognitive-constructivist findings of educational profession about learning and teaching mathematics. In the control group, the transmission approach prevailed.

In the research, the question was answered of what impact the implementation of the process approach to learning and teaching mathematics has on the learner's knowledge, which can be tested and assessed.

Students in the experimental group (EG) performed significantly better in basic and conceptual knowledge, in solving simple mathematical problems, and in complex knowledge than those in the control group. Results of the research have also shown that there are statistically significant correlations between individual areas of mathematical knowledge. The correlations between the areas of knowledge are from medium high to high, indicating that conceptual knowledge correlates significantly with solving simple mathematical problems and with complex knowledge.

**Keywords:** *process approach to learning and teaching, mathematics, basic and conceptual knowledge, solving simple mathematical problems, complex knowledge*

## Introduction

The purpose of teaching mathematics is not just to transmit mathematical knowledge – the opposite is true: the basic purpose is to make students discover mathematics, think, and build it. To learn mathematics means doing mathematics by solving and exploring it. But the findings of international evaluations point to deficient knowledge of mathematics and poorly developed competences, because of which the question of the quality of learning and teaching mathematics is persistently raised. The findings also warn that in the practice of mathematical education formal teaching prevails, oriented to techniques of memorising rules, which students often do not understand. Students do not manage to see the links between new knowledge and previously acquired concepts, they are not able to connect mathematics with everyday life, in their work they are not autonomous, and they often just repeat certain activities or procedures (UNESCO, 2012).

Although it has been emphasised since the eighties of the past century that the teaching of mathematics should include solving problems and point to the use of mathematics in everyday life, in reality it seems that this kind of teaching has not actually come to life (Dindyal et al., 2012) and that this continues to be one of the unattainable goals of teaching mathematics (Stacey, 2005).

Basic mathematics education is still too often boring because: it is designed as formal teaching, centred on learning techniques and memorizing rules, whose rationale is not evident to pupils; pupils do not know which needs are met in the mathematics topics introduced or how they are linked to the concepts familiar to them; links to the real world are weak, generally too artificial to be convincing and applications are stereotypical; there are few experimental and modelling activities; technology is quite rarely used in a relevant manner; pupils have little autonomy in their mathematical work and often merely reproduce activities (UNESCO, 2012).

To overcome the above-mentioned challenges, changes in teaching practices must be made consistently with the stated goals. As early as 1987, Shulman (1987) found that the teacher needs not only a good methodological and substantive knowledge of the topics he teaches, but also a substantive pedagogical knowledge, i.e., awareness of how students construct knowledge of individual contents. The teacher who knows how the student constructs knowledge, the teacher who possesses substantive pedagogical knowledge prepares activities that build on students' *pre-knowledge*, on *linking* knowledge, he introduces concepts and content *gradually.* The notions of both learning and teaching, in turn, significantly influence the individual's understanding, perspective or interpretation of the context of learning or teaching. The basic assumption of the teacher's operation is promoting the quality of learning, which leads to students' quality knowledge.

## Research Methodology

### The purpose of the research

In the research, we sought to answer the question of how the implementation of the *process approach to learning and teaching mathematics*, which had been produced on the basis of the theoretical knowledge of children's mental development, also of recent findings about the child's thinking, and the knowledge of social cognition, of learning and teaching mathematics, influences the student's knowledge that can be tested and assessed. In this we based on the theory of developmental psychology, which studies the development of concepts from the point of view of the developmental stage of the child's thinking (Vygotsky, 1978; Labinowicz, 1989; Gilly et al., 1988) and took into account more recent cognitive-constructivist findings of learning, which emphasise learners' activity in the learning process (Maričić et al., 2013; Van de Walle et al., 2013; Břehovský et al., 2015). The *process approach to learning and teaching* mathematics is characterised by experiential learning, discovering and exploring mathematics through mathematical and life challenges, and by developing reading learning strategies as the integrating activity of learning and teaching.

We wanted to determine whether the students in the experimental group (EG), who had received the *process approach to learning and teaching mathematics*, performed better in basic and in conceptual knowledge (PR), in solving simple mathematical problems (EP) in complex knowledge (ZP) than the control group. Three research hypotheses were formulated.

### Research hypotheses:

$H_1$: In the selected contents block in basic and conceptual knowledge (PR), the experimental group will perform significantly better than the control group.

$H_2$: In the selected content blocks in solving simple mathematical problems (EP), the experimental group will perform significantly better than the control group.

$H_3$: In the selected content blocks in complex knowledge (ZP), the experimental group will perform significantly better than the control group.

### Research method

The model of quasi-experiment was applied and the experimental model *process approach to learning and teaching mathematics* was introduced in the experimental group, whereas in the control group the transmission approach prevailed. Because the model without randomisation was applied—opportunities for the use of models with randomisation are rather limited in schools—the students' most relevant factors were controlled at the beginning (overall learning performance, marks in Slovenian and in mathematics, education level of parents).

### Research sample

In the experimental group (EG), there were 190 eighth grade pupils and in the control group (CG), 220 eighth grade pupils of Slovenian basic schools. All the students participating in the research were at the age between 13 and 14 years.

### Data gathering and processing

The students' performance in dependent variables was assessed with knowledge tests, the content structure of which was: dependent and independent quantities, percentage, direct proportion, inverse proportion, and equation. The situation before and after the introduction of the experimental factor was recorded empirically, namely with initial and final tests of knowledge. The knowledge tests that had been adapted as a measurement instrument were used to determine basic and conceptual knowledge, solving simple mathematical problems, and complex knowledge. The initial and final tests of knowledge were preliminarily first attributed measurement characteristics: objectivity, difficulty, reliability, discriminativeness, and validity. Results of the initial and of final tests were processed with the use of multivariate factor analysis. The Guttman split-half coefficient of reliability for the initial test was 0.82 and for the final test 0.87. The discriminative coefficients for individual items at the initial test ranged from 0.38 to 0.700; while the discriminative coefficients for individual items at the final test ranged from 0.29 to 0.72.

To determine the significance of the differences between the students of the experimental and control groups and to determine the significance of differences within the experimental group at the end of the experiment, the following statistical techniques were applied in data processing: descriptive statistics, testing the homogeneity of the sample, factor analysis, one-way analysis of variance, and multivariate analysis of variance.

## Results and interpretation

The results of the research show that the experimental group performed statistically significantly better in basic and conceptual knowledge, in solving simple mathematical problems and in complex knowledge than the control group (Table 1, Table 2).

**Table 1.** Average performance of students according to areas of knowledge in the initial and final tests

| | | INITIAL TEST | | | | FINAL TEST | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | N | performance in % | x | SD | N | performance in % | x | SD |
| **EG** | PR | 101 | 61 % | 16.4 | 5.40 | 101 | 68 % | 6.8 | 2.50 |
| | EP | 101 | 39 % | 12.6 | 6.90 | 101 | 83 % | 9.9 | 2.21 |
| | ZP | 101 | 25 % | 3.1 | 1.92 | 101 | 54 % | 15.9 | 7.59 |
| **CG** | PR | 130 | 52 % | 13.5 | 6.50 | 130 | 55 % | 5.5 | 1.60 |
| | EP | 130 | 31 % | 10.0 | 6.50 | 130 | 73 % | 8.8 | 3.16 |
| | ZP | 130 | 23 % | 2. 8 | 2.56 | 130 | 38 % | 11.7 | 6.90 |

Legend: x – average number of points, SD – standard deviation, N – number of students, PR – basic and conceptual knowledge, EP – solving simple mathematical problems, ZP – complex knowledge.

**Table 2.** The significance of performance differences between the control group and experimental groups by areas of knowledge

| | Sum of squares (dif. III) | df | Average of squares | F | Sig. |
|---|---|---|---|---|---|
| **IT PR** | 3.092E-05 | 1 | 3.092E-05 | .000 | .993 |
| **IT EP** | 1.046 | 1 | 1.046 | 2.420 | .121 |
| **IT ZP** | .792 | 1 | .792 | 1.733 | .190 |
| **FT PR** | 6.134 | 1 | 6.134 | 36.345 | **.000** |
| **FT EP** | 7.785 | 1 | 7.785 | 16.769 | **.000** |
| **FT ZP** | 3.162 | 1 | 3.162 | 6.369 | **.010** |

Legend: initial test, basic and conceptual knowledge (IT PR), final test, basic and conceptual knowledge (FT PR), initial test, solving simple mathematical problems (IT PR), final test solving simple mathematical problems (FT PR), initial test, complex knowledge (IT ZP), final test, complex knowledge (FT ZP).

Compared to its initial state, after the introduction of the experimental factor into the learning process, the experimental group progressed significantly in solving simple mathematical problems and in complex knowledge (Table 3).

**Table 3.** The significance of differences on the initial and final tests by areas of knowledge in the experimental group

| | Sum of squares (dif. III) | df | Average of squares | F | Sig. |
|---|---|---|---|---|---|
| **PR** | 0.464 | 1 | 0.464 | 0.582 | 0.447 |
| **EP** | 46.470 | 1 | 46.481 | 60.960 | 0.00 |
| **ZP** | 5.682 | 1 | 5.660 | 6.480 | 0.01 |

Legend: basic and conceptual knowledge (PR), knowledge that allows for solving simple mathematical problems (EP), complex knowledge (ZP)

The first hypothesis, $H_1$, was confirmed with the results of the research, which show that in basic and conceptual knowledge after the introduction of the experimental factor, the experimental group obtained statistically higher results than the control group (EG: 68 %, CG: 55 %, p=0.00).

Basic and conceptual knowledge, which covers the knowledge and understanding of mathematical concepts, was tested with the recognition of concepts, determining the relations between data, analysing, proposing examples and counterexamples, etc. In verifying understanding, attention was paid to composing the task in such a way that allowed the student to really demonstrate his/her knowledge. This is the reason why in this kind of tasks mathematical procedures were, as a rule, not included.

At the time of the experiment, the students of the experimental group built their knowledge and deepened understanding through various activities of representing concepts, which includes pictures, diagrams, symbols, concrete material, language, realistic situations, shaping conceptual networks, etc. As early as in 1991, also Novak & Musonda (1991) attracted attention to the significance of conceptual networks in shaping concepts with understanding, emphasising that based on students' correct and wrong presentations, the teacher can analyse their knowledge and determine wrong and correct conceptual images (ibid.) and based on the findings, guide students in upgrading and transforming knowledge.

Similarly, Griffin & Case (1997) and after them Duval (2002) stated that the teaching of mathematics that is based on exploring diverse representations of a definite mathematical concept and that encourages students to fluently and flexibly transit between a variety of representations is more efficient and allows for a better understanding of mathematical concepts than the teaching that does not enable this. De Jong et al. (1998) emphasise that in teaching mathematics handling diverse representations fluently and also transiting between them (e.g., knowing how with concrete material to compute a given calculation and to "translate" the calculation into symbolic record) and from offered representations selecting the one appropriate for the representation of a definite concept (e.g., representation of adding three-digit numbers with tens units is a more appropriate representation than representing computing in the 1 000 range with non-structured material) is important for the student's successful and productive interaction with diverse representations. In addition to what has been mentioned, the use of diverse representations of mathematical concepts satisfies the needs of learners with different styles of learning (Mallet, 2007).

The second hypothesis, $H_{2,}$ was confirmed with the results of the research, which show that after the introduction of the experimental factor in solving simple mathematical problems the experimental group obtained statistically significantly higher results than the control group (EG: 83 %, CG: 73 %, $p = 0.00$). In solving simple mathematical problems, the experimental group also significantly progressed in relation to its initial state (Table 3).

Taking into account the results concerning progress in solving simple mathematical problems, where mathematical procedures had to be meaningfully applied such as computing procedures, drawing diagrams, production of tables, solving simple one-stage textual tasks, the fact must be emphasised that the students in both the experimental group (83 %) and the control group (73 %) demonstrated satisfactory knowledge.

Douglas (2000) states that for learning algorithms and computing procedures as well as for solving problems, the understanding of concepts is crucial. We learn to solve problems faster and better if we understand the basic concepts. A conclusion can be drawn that the advantage of the students of the experimental group in solving simple mathematical problems also lies in the acquisition of diverse experiences in the learning of concepts. Introducing procedures when the student has not yet thoroughly acquired the basic concepts inherent in the procedure implies learning by memorising. In this case, how well the procedure is going to be learnt depends on the number of repetitions of the procedure. Such knowledge is, however, short-lived and quickly forgotten; it is also not transferable and applicable, e.g., to solving problems.

The interweavement among different areas of knowledge is also indicated by the statistically significant correlations between them. The correlations between the areas of knowledge range from medium to high and show that conceptual knowledge is significantly related to solving simple mathematical problems and complex knowledge (Table 4).

**Table 4.** The correlations between basic and conceptual knowledge and between complex knowledge and solving simple mathematical problems

| | | FT EP | FT ZP |
|---|---|---|---|
| FT PR | Pearson coefficient | 0.44** | 0.69** |

Legend: FT PR – basic and conceptual knowledge on final test, FT EP – solving simple mathematical problems on final test, ** the coefficient is statistically significant at the level of 1 % risk, * the coefficient is statistically significant at the level of 5 % risk

It can be concluded that the advantage of the students of the experimental group in solving simple mathematical problems as well as in complex knowledge—as will be shown below—also lies in the acquisition of a variety of experiences in learning concepts, which has a positive impact on efficient learning of procedures and solving problems. Solving problems, in turn, is an important skill that is indispensable in life, as it involves analysis, interpretation, reasoning, anticipation, assessment, and reflection, so it should be the main goal and fundamental component of the mathematical curriculum (Anderson, 2009).

The third hypothesis, $H_{3,}$ was confirmed with the results of the research, which show that after the introduction of the experimental factor in complex knowledge, the experimental group obtained statistically significantly higher results than the control group (EG: 54 %, CG: 38 %, $p = 0.01$). In complex knowledge, the experimental group also significantly progressed in relation to the initial state (Table 3).

Complex knowledge, which covers solving problems, was tested with solving complex tasks (multistage textual problems), analysing the problem situation, generalising, substantiating, etc. Detailed analysis of the results by items shows that neither the students of the control group nor those of the experimental group successfully solved textual tasks, they especially experienced difficulties in solving algebraic problems, generalisation, and using formal mathematical knowledge. With the task *"Compute what percentage of the figure is shaded,"* the ability of solving problems at the symbol level was tested. The text of the textual task was accompanied with a picture of a rectangle, a part of which was shaded. The data of the lengths of the sides were given at the symbol level, with variables. Very few students solved the task at the symbol level. Most students solved the task by choosing concrete data—some by measuring, others by drawing a grid and defining the surface unit, some also came to an approximate result by estimation.

The students' lower results in complex knowledge can partly be explained with the findings of Demetriou et al. (1991), who developed four tests for the determination of the level of development of the cognitive system and understanding of mathematical concepts, among other things also a test for the definition of the stage of formal-logical thinking and algebraic abilities. The essential development of integrating the four calculus operations happens at the age of 13–14, and the development of algebraic abilities at the age of 14–15. The introduction of abstract algebraic concepts (e.g., the concept of a variable) is possible when the development of algebraic abilities has been completed. The introduction of these concepts, though, must still be linked to concrete objects (ibid.).

It can be concluded that the path do deeper knowledge, which is applicable and complex, is neither easy nor fast, it is conditioned both on the student's cognitive development and on the quality learning and teaching.

## Concluding findings

The issue of examining the impact of approaches to learning and teaching on learning performance is an extremely demanding and complex one. In our research, we focused on three levels of mathematical knowledge: basic and conceptual knowledge, solving simple mathematical problems and complex knowledge. As evident from the paper, there are substantiated reasons for the assertion that the implementation of the process approach to learning and teaching mathematics, which we have produced ourselves on the basis of the theoretical knowledge of the mental development of children and recent findings about children's thinking, significantly contributes to the quality of learning and teaching mathematics and to students' academic achievement.

A positive impact of the process approach to learning and teaching mathematics is recorded both in the understanding of concepts and solving problems and in learning algorithms and calculation procedures. The research results show that mathematical conceptual knowledge is significantly related to solving simple mathematical problems and complex knowledge; learning with understanding, however, is a long lasting process associated with the cognitive development of the student and with quality teaching.

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