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On some analogies between one-criterion decision making under uncertainty and multi-criteria decision making under certainty[1]

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Abstract: One-criterion decision making under uncertainty (1-DM/U) is related to situations in which the decision maker (DM) evaluates the alternatives on the basis of one objective, but e.g. due to numerous uncertain future factors some parameters of the problem are not deterministic. Instead of entirely known paramaters, a set of possible scenarios is available. Multi-criteria decision making under certainty (M-DM/C) concerns cases where the DM assesses particular options in terms of many objectives. The parameters are known. Therefore, scenario planning is redundant. Both issues are investigated by many researchers and practitioners, since real economic decision problems are usually at least uncertain or multi-objective. In the paper, numerous analogies between 1-DM/U and M-DM/C are revealed. Some of them have existed for many decades, but others, so far, have not been developed. A careful examination of all the similarities enables an improvement of existing methods and a formulation of new algorithms for 1-DM/U and M-DM/C. The article presents six pairs of similar procedures and contains the description of three novel approaches created by analogy to existing ones.

Keywords: one-criterion and multi-criteria decision making, certainty, uncertainty, scenario planning, economic problems, optimization, payoff matrix, decision rules, decision maker's preferences.

JEL codes: C01, C02, C44, C61, C7, D81, D9, G11, G21, O1, O2, O3.

Introduction

One-criterion decision making under uncertainty (1-DM/U) is connected with situations in which the decision maker (DM) evaluates a given decision variant on the basis of one objective function, but due to numerous uncertain future

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factors and unsufficient knowledge the parameters of the problem are not deterministic. In this paper decision making approaches are considered in which uncertainties are modelled using a set of possible scenarios. These scenarios may be defined by experts, decision makers or by a person who is simultaneously an expert and a DM (Gaspars-Wieloch, 2020b).

On the other hand, multi-criteria decision making under certainty (M-DM/C) is related to cases where the decision maker assesses particular alternatives in terms of many criteria (at least two). This time the parameters of the problem are supposed to be known. Therefore, scenarios representing diverse potential future situations are redundant.

Both issues (1-DM/U and M-DM/C) have been investigated by many researchers and practitioners since real economic decision problems (e.g. choice of technology, human resource management, chain supply management, selection of marketing strategies, choice of investment project) are usually at least uncertain or at least multi-objective (Bakir, Akan, & Durmaz, 2019; Karvetski & Lambert, 2012). M-DM/C are especially applied to generate rankings of banks, investment funds, companies, universities, countries and so on.

Nevertheless except the multi-criteria decision making under uncertainty (M-DM/U) which combines 1-DM/U with M-DM/C (Bieniek, 2016; Chiao, 2021; Durbach & Stewart, 2012; Eiselt & Marianov, 2014; Hopfe, Augenbroe, & Hensen, 2013; Nelyubin & Podinovski, 2017; Pirvu & Schulze, 2012; Stewart, French, & Rios, 2013), the aforementioned topics are analyzed seperately in the literature without identifying visible analogies.

The research gap results from the fact that essential similarities between 1-DM/U and M-DM/C have not been previously identified. Additionally, some existing procedures designed for 1-DM/U or M-DM/C have diverse and evident drawbacks which could be avoided by referring to analogical methods already improved. The paper reveals numerous analogies between one-criterion indeterministic decision making and multi-criteria deterministic decision making. Some of them have existed for many decades (consciously or unconsciously), but other analogies, so far, have not been developed. The analogies are related both to the construction of the problems and to the procedures designed for them. A careful examination of all the similarities enables an improvement in existing decision rules and the formulion of new algorithms for 1-DM/U and M-DM/C. The article contains a comparative analysis of six pairs of similar methods and a description of three novel approaches created by analogy to existing ones. Of course it can can be stated that the investigation of the analogies between two issues aforementioned and the attempt to construct new procedures on the basis of the observed similarities are useless since the dominant research trend in world literature is multi-criteria decision making under uncertainty. However, it is worth stressing that decision algorithms developed for the last topic are often hybrids of methods designed for 1-DMU and M-DMC, respectively (Durbach, 2019; Ekel, Pedrycz, & Pereira, 2019; Homaei & Hamdy, 2020; Oliveira et al., 2021). Thus with a large variety of efficient procedures for these problems the development of techniques for indeterministic multi-criteria optimization will be easier and more effective.

The paper is organized as follows. Section 1 describes one-criterion indeterministic decision making and multi-criteria deterministic decision making. It also contains some explanations concerning uncertainty levels, scenario planning and payoff matrices. Section 2 uses illustrative examples to demonstrate numerous analogies between methods applied to both issues. Section 3 proposes three novel ideas based on existing methods. Conclusions are offered in the last section.

1. 1-DM/U and M-DM/C—problem description and analogies

Before demonstrating the difference between 1-DM/U and M-DM/C there is a brief explanation of several notions used in the article. The term "alternative" ("decision variant", "option", "course of action") is understood here as a possible way the decision maker may choose and perform. "Scenario" means a possible way in which the future might unfold. The notions "criterion" and "objective" are usually related to the intention to maximize or minimize a measure representing an attribute (feature) which is important for the decision maker. Nevertheless sometimes the goal may consist of reaching a specific value (neither the maximal nor the minimal one). The words "outcome", "result", "payoff" signify the effect gained by the DM if he or she selects a given option and a given scenario occurs. The "payoff matrix" constitutes a table representing possible alternatives, possible future scenarios and outcomes for each pair: option / scenario. If scenario probabilities are known the payoff matrix may contain an additional column with these values.

1.1. One-criterion decision making under uncertainty

Within one-criterion decision making under uncertainty (1-DM/U), as the name suggests, the decision is made on the basis of one objective (profit maximization, time minimization, cost minimization, etc.).

"Under uncertainty" means that at least one parameter of the decision problem is not known exactly. Durbach and Stewart (2012) enumerate diverse techniques enabling the handling of uncertainty, i.e. fuzzy numbers, probabilities, probability-like quantities and explicit risk measures. However, in their opinion, »uncertainties become increasingly so complex that the elicitation of those measures becomes operationally difficult for DMs to comprehend and virtually impossible to validate«. Therefore they encourage the construction of scenarios describing possible ways in which the future might unfold and that is why, in this work the main interest is in methods integrating scenario planning

(SP) into 1-DM/U. SP is a tool frequently used in the decision-making process (Gaspars-Wieloch, 2020b, 2021; Silber, 2017; Vilkkumaa, Liesio, Salo, & Imola-Sheppard, 2018). It supports the identification of risks understood as uncertain and uncontrolled factors influencing the consequences of chosen strategies. It is useful for government planners and military analysts, companies, scientific communities, futurists and educational institutions (Mietzner & Reger, 2005; Silber, 2017). Scenario planning supports inventory management, sales forecasting or project selection. The scenarios may represent possible alternatives (methods to achieve an objective) or potential future situations (independent on behaviour). In this paper scenarios are treated according to the second approach. The guidelines concerning the construction of scenarios are presented in Michnik (2013). Project managers eagerly use SP since it gives the opportunity to analyse the problems in a more deterministic way than continuous probability distributions or fuzzy numbers (Maciel, Ballini, & Gomide, 2018). Thanks to SP the organisations are better prepared to handle new situations and promote proactive leadership initiatives as scenario planning recognises technological discontinuities or disruptive events and includes them in long-range planning (Mietzner & Reger, 2005). Note that SP is inadequate for decisions made in the short-term and for people without sufficient skills to collect, interpret and monitor data from different sources (Mietzner & Reger, 2005). Roxburgh (2009) stresses that scenarios do not cover the full range of future possibilities. SP has also a psychological and behavioural aspect since payoffs connected with particular scenarios may be estimated by: (1) experts, (2) decision makers or (3) people being both experts and decision makers. In the first approach the outcomes are generated in the most objective way while the second approach may lead to the most subjective predictions (Gaspars-Wieloch, 2021). Regardless of the estimation method it is recommended to convert initial values into numbers reflecting the DM's preferences (utilities).

It is worth underlining that there are diverse uncertainty levels (Gaspars-Wieloch, 2020b). According to one of the possible divisions, there are four uncertainty categories. The first level (uncertainty with known probabilities) occurs when the DM knows the alternatives, scenarios, scenario probabilities and particular outcomes. The second level (uncertainty with partially known probabilities) is related to situations where the DM knows the alternatives, scenarios, partial scenario probabilities and particular outcomes. Probabilities may be given as interval values. Sometimes scenarios are ordered according to their approximate chance to occur. Within the third level (uncertainty with unknown probabilities) the DM knows the alternatives, scenarios and particular outcomes. Scenario probabilities are not known. The fourth level (uncertainty with unknown scenarios) concerns cases where the DM knows alternatives only.

In the article the third level (i.e. payoff matrices without any information on scenario probabilities) is mainly investigated since in connection with the fact that the set of scenarios in SP does not need to be exhaustive the use of probabilities seems to be unjustifiable (Michnik, 2013; Stewart et al., 2013). Additionally, von Mises (1949) adds that the probability of a single event should not be expressed numerically as probabilities only concern repetitive situations which are not frequent in real economic problems usually related to investments in turbulent times and innovation or innovative projects, etc. (Denkowska & Wanat, 2020; Gaspars-Wieloch, 2020b).

The results of the use of scenario planning in 1-DM/U with unknown probabilities can be gathered in the payoff matrix presented in Table 1.

Table 1. Payoff matrix for 1-DM/U with unknown probabilities

	Alternatives				
Scenarios	A_1	…	A_j	…	A_n
S_1	$a_{1,1}$	…	$a_{1,j}$	…	$a_{1,n}$
⋮	⋮	⋱	⋮	⋱	⋮
S_i	$a_{i,1}$	…	$a_{i,j}$	…	$a_{i,n}$
⋮	⋮	⋱	⋮	⋱	⋮
S_m	$a_{m,1}$	…	$a_{m,j}$	…	$a_{m,n}$

Note: n—number of alternatives, m—number of scenarios, $a_{i,j}$—payoff obtained if option A_j is selected and scenario S_i occurs.

Source: (Gaspars-Wieloch, 2020b).

1.2. Multi-criteria decision making under certainty

Multi-criteria decision making under certainty (M-DM/C) concerns cases where the DM assesses particular options in terms of many objective functions which may be quite complicated since particular criteria can indicate differ-

Table 2. Payoff matrix for M-DM/C

	Alternatives				
Criteria	A_1	…	A_j	…	A_n
C_1	$b_{1,1}$	…	$b_{1,j}$	…	$b_{1,n}$
⋮	⋮	⋱	⋮	⋱	⋮
C_k	$b_{k,1}$	…	$b_{k,j}$	…	$b_{k,n}$
⋮	⋮	⋱	⋮	⋱	⋮
C_p	$b_{p,1}$	…	$b_{p,j}$	…	$b_{p,n}$

Note: n—number of alternatives, p—number of criteria, $b_{k,j}$—performance of criterion C_k if option A_j is selected.

Source: (Gaspars-Wieloch, 2020b).

ent optimal solutions. In such circumstances it is suggested that a compromise course of action is found. “Under certainty” means that all the parameters of the problem are known. Hence, the scenario identification is useless. Table 2 shows the payoff matrix related to M-DM/C.

1.3. Comparison

On the basis of Tables 1-2 the following conclusions can be formulated. On one hand there is a significant difference between 1-DM/U and M-DM/C consisting of the fact that within 1-DMU, if A_j is chosen, the final outcome ($a_{i,j}$) is single and depends on the real scenario which will occur, meanwhile within M-DM/C, if A_j is selected, there are p final outcomes, i.e. $b_{1,j}, \ldots, b_{k,j}, \ldots, b_{p,j}$ (Gaspars-Wieloch, 2020b), because particular decision variants are evaluated in terms of p essential objectives. On the other hand some similarities between both issues are also very visible. The construction of both payoff matrices is extremely similar. In both cases there is a set of potential options and the set of possible scenarios in 1-DM/U can correspond to the set of significant objectives in M-DM/C. Another analogy, not discussed so far is related to the final step of the decision making process. The decision maker, in both decision problems, can select and execute only one option (such a solution is called “pure strategy”) or a combination of several options (such a solution is called “mixed strategy”). Mixed strategies are especially common in portfolio construction and cultivation of different plants (Latoszek & Ślepaczuk, 2020).

2. Analogies in existing methods applied to 1-DM/U and M-DM/C

In the previous section some strong similarities between one-criterion decision making under uncertainty and multi-criteria decision making under certainty have been presented. These analogical features lead to the formulation of similar procedures for both issues, but curiously this phenomenon is not discussed in the litterature. In this part of the paper six pairs of analogical decision rules are revealed. In each case analyzed it is assumed that the subsequent alternatives signify different investment strategies considered by the company. All the criteria taken into consideration are maximized: in the cases related to 1-DMU the only essential criterion is the annual profit maximization of the company (in million euros); in the cases connected with M-DM/C the decision maker assesses potential variants by taking into account three objectives: annual profit maximization (in million euros), market share maximization (in %) and annual efficiency maximization (in million pieces of a given product). Due to the fact that the objectives aforementioned are not comparable (units and scales are different), usually initial data need to be normalized and then the transformed data belong to the interval [0,1] where 0 means that a given option performs the worst in the objective analyzed (among all the decision variants considered) and 1 signifies that an alternative performs the best in the objective taken into consideration.

2.1. Wald rule and max-min rule (pure strategies)

The Wald rule is designed for 1-DM/U and extreme pessimists. It consists of finding the minimal payoff for each option and selecting the alternative with the highest minimal outcome. Table 3a shows a simple example to which this procedure has been applied. The minimal values connected with particular decision variants are equal to 2, –1 and 3 (million euros), so the best option (investment strategy) is A_3.

On the other hand the max-min rule used in M-DM/C consists of indicating criteria that are perfomed the worst by particular courses of action and then in choosing the option with the best realization of its weakest criterion. As it can be seen in Table 3b, the max-min rule for multi-criteria decision problems, from the mathemathical point view, has the same construction as the Wald rule has. As a matter of fact, only the interpretation of the final results is different. In the first case, if the company performs the recommended option (A_3), depending on the real scenario, the outcome may be expected to equal 8, 3 or 6 (million euros). In the second case, if the company executes the suggested investment strategy (also A_3), the profit will be relatively the highest, the market share will equal 30% of the maximum possible and the efficiency will amount to 60% of the maximum.

Table 3a–3b. Payoff matrices for 1-DM/U (Wald rule) and M-DM/C (max-min rule)

Table 3a	**Alternatives**		
Scenarios	A_1	A_2	A_3
S_1	5	–1	8
S_2	10	0	3
S_3	2	13	6
Min	2	–1	3

Source: Prepared by the author.

Table 3b	**Alternatives**		
Criteria	A_1	A_2	A_3
C_1	0.5	0.0	1.0
C_2	1.0	0.0	0.3
C_3	0.0	1.0	0.6
Min	0.0	0.0	0.3

Source: Prepared by the author.

2.2. Wald rule and max-min rule (mixed strategies)

The first pair of examples was related to pure strategies, but it is worth emphasizing that the Wald rule for 1-DM/U and the max-min rule for M-DM/C can be also applied to find the best mixed strategy. Equations (1)–(4) concern the first optimization problem and equations (5)–(8) allow the solution of the second (Table 4).

Table 4. Optimization models for 1-DM/U (Wald rule) and M-DM/C (max-min rule)

1-DM/U, Wald rule			M-DM/C, max-min rule		
$y \rightarrow \max$		(1)	$v \rightarrow \max$		(5)
$\sum_{j=1}^{n} a_{i,j} x_j \geq y$	$i = 1, \ldots, m$	(2)	$\sum_{j=1}^{n} g_{k,j} x_j \geq v$	$k = 1, \ldots, p$	(6)
$x_j \in SFS$	$j = 1, \ldots, n$	(3)	$x_j \in SFS$	$j = 1, \ldots, n$	(7)
$x_j \geq 0$	$j = 1, \ldots, n$	(4)	$x_j \geq 0$	$j = 1, \ldots, n$	(8)

Source: Prepared by the author.

In the model (1)–(4) symbol y denotes the minimal guaranteed outcome, x_j signifies the share of a given option in the mixed strategy, SFS is the set of feasible solutions (the remaining symbols are explained in Section 1). In the second model v denotes the minimal guaranteed performance degree and $g_{k,j}$ signifies the performance degree (normalized value) connected with criterion C_k if alternative A_j is chosen. The solution recommended by the first model maximizes the minimum guaranteed outcome (i.e. the payoff gained regardless of the real scenario). The plan obtained after solving the second model guarantees that the performance degree of each criterion will be equal to at least v, where v must be as high as possible. These models are well-known by scientists and often presented in the literature, but despite obvious similarities they are not described together as analogical optimization problems.

2.3. Bayes rule and Simple Additive Weighting method (pure strategies)

In the previous examples the methods applied were focused on the worst scenarios (1-DM/U) or the worst criteria (M-DM/C). Now the procedures which take into account all the payoffs connected with a given alternative will be examinated.

Table 5a shows computations performed in the case of the Bayes rule. This algorithm is designed for multi-shot decisions and for situations where the DM has no knowledge about the chance of the occurrence of particular scenarios. That is why the Bayes rule consists of calculating the arithmethical average of all the payoffs for each option—such a way allows the assignment of the same importance (weight) to each outcome (i.e. the same probability to each scenario). In the here example alternative A_1 is the best according to the Bayes rule. Another similar approach used in 1-DM/U, but designed for situations where the scenario probabilities are exactly known (uncertainty with known probabilities), is the expected value maximization within which it is required to compute for each alternative the sum of the products concerning

Table 5a–5b. Payoff matrices for 1-DM/U (Bayes rule) and M-DM/C (SAW method)

Table 5a	Alternatives			p_i
Scenarios	A_1	A_2	A_3	
S_1	5	−1	4	0.33
S_2	10	0	3	0.33
S_3	2	13	6	0.33
Average	5.6	4.0	4.3	

Source: Prepared by the author.

Table 5b	Alternatives			w_k
Criteria	A_1	A_2	A_3	
C_1	0.5	0.0	1.0	0.2
C_2	1.0	0.0	0.3	0.3
C_3	0.0	1.0	0.6	0.5
SAW_j	0.40	0.50	0.59	

Source: Prepared by the author.

particular outcomes and their probabilities. Then the option with the highest expected value is selected.

If the methods worked out for M-DM/C are carefully examined it may be noticed that there also exists a procedure based on the sum of products. It is the Simple Additive Weighting method (SAW) where outcomes (performance degrees) are multiplied by the importance (weight) of subsequent criteria. Table 5b contains fictitious weights and synthetic measures for each variant. The SAW method suggests the selection of A_3 due to its highest final total value.

As can be seen the computation steps to be performed coincide in both techniques (Bayes rule and SAW), but, again, the interpretation of the outcomes and the synthetic measures are different.

2.4. Bayes rule and Simple Additive Weighting method (mixed strategies)

The Bayes rule and the SAW method can be also applied to find the optimal mixed strategy. Models (9)–(11) and (12)–(14) in Table 6 concern the continuous version of both problems: 1-DMU and M-DM/C respectively. In the first model parameter b_j denotes the Bayes index calculated for alternative A_j, x_j still signifies the share of a given option in the mixed strategy, *SFS* is the set of feasible solutions (the remaining symbols are explained in Section 2). In the

Table 6. Optimization models for 1-DM/U (Bayes rule) and M-DM/C (SAW method)

1-DM/U, Wald rule	M-DM/C, max-min rule
$\sum_{j=1}^{n} b_j x_j \to \max$ (9)	$\sum_{k=1}^{p} w_k f_k(x) \to \max$ (12)
$x_j \in SFS$ $\quad j = 1, \ldots, n$ (10)	$x_j \in SFS$ $\quad j = 1, \ldots, n$ (13)
$x_j \geq 0$ $\quad j = 1, \ldots, n$ (11)	$x_j \geq 0$ $\quad j = 1, \ldots, n$ (14)

Source: Prepared by the author.

second model w_k is the weight connected with criterion C_k and $f_k(x)$ represents the objective function related to this criterion where x is the vector of decision variables concerning particular options. Although both models have been proposed for totally different purposes, the mathematical form is extremely similar.

2.5. Pareto-optimal solutions

So far the methods providing one concrete solution have been discussed. However, sometimes the decision maker is interested in generating the set of Pareto-optimal solutions, just to know which potential alternatives are non-dominated, i.e. which reduced set of options is worth further consideration. Pareto-optimality is a concept of efficiency used in the social sciences, including economics and political sciences. Usually Pareto-optimality is investigated in the context of multi-criteria analysis, but this feature can be also significant when making decisions under uncertainty.

Table 7a–7b. Payoff matrices for 1-DM/U and M-DM/C (Pareto-optimality)

Table 7a	**Alternatives**		
Scenarios	A_1	A_2	A_3
S_1	5	–1	4
S_2	10	0	3
S_3	2	13	2

Source: Prepared by the author.

Table 7b	**Alternatives**		
Criteria	A_1	A_2	A_3
C_1	5	–1	4
C_2	10	0	3
C_3	2	13	2

Source: Prepared by the author.

Tables 7a–7b contain data concerning three options. Note that this time the outcomes in the table connected with M-DM/C (Table 7b) do not need to be transformed into normalized values (i.e. performance degrees) since, when searching Pareto-optimal solutions, particular variants do not obtain a synthetic index on the basis of which the final decision is made.

In order to find the Pareto set, regardless of the issue (1-DM/U or M-DM/C) graphs or matrices can be applied. Here, the illustrative examples are very simple. That is why the comparison of payoffs for each pair of courses of action is also possible. Such an analysis leads to the conclusion that only the first two variants (A_1 and A_2) are Pareto-optimal whereas the investment strategy A_3 is dominated by A_1 ($5 \geq 4$, $10 \geq 3$, $2 \geq 2$). In the context of 1-DMU it means that if scenario S_1 occurs A_3 will offer a worst profit than A_1, if scenario S_2 occurs, again, A_3 will offer a worst profit than A_1, if the real scenario is S_3, then A_3 will offer the same profit as the profit connected with A_1. In the context of M-DM/C it signifies that there is no criterion (among three objectives analyzed, i.e. profit maximization, market share maximization and efficiency maximization) within which the investment strategy A_3 would be better than A_1.

2.6. Savage rule (Hayashi rule) for 1-DM/U and performance degrees for M-DM/C

The last analogy concerning the methods developed for both analyzed decision problems is connected with the assessment of a given payoff in comparison with the remaining ones. Note that such an evaluation enables the definition of the relative position (attractiveness) of particular outcomes. Furthermore, it can be done both within scenarios (1-DM/U) and within criteria (M-DM/C). How is it perfomed in both issues? Tables 8a–8b present initial ficticious outcomes. This time data in Table connected with M-DM/C do not belong to the interval [0,1], since it will be explained in detail how primary values are transformed into secondary outcomes.

Table 8a–8b. Initial payoff matrices for 1-DM/U (Savage rule) and M-DM/C (performance degrees)

Table 8a	**Alternatives**		
Scenarios	A_1	A_2	A_3
S_1	5	–1	8
S_2	10	0	3
S_3	2	13	6

Source: Prepared by the author.

Table 8b	**Alternatives**		
Criteria	A_1	A_2	A_3
C_1	5	–1	8
C_2	10	0	3
C_3	2	13	6

Source: Prepared by the author.

In order to indicate the position of each payoff within 1-DM/U the Savage rule may be applied which consists of computing the matrix of relative losses (the differences between a given value and the best one for the scenario analyzed) and selecting the alternative that minimizes the maximal loss (Table 9a). According to the Savage rule option A_3 is optimal.

It is worth emphasizing that this approach assumes that each payoff ought to be compared with the highest outcome. However, another method exists which is also designed for 1-DM/U and which also consists of indicating the position of each outcome in comparison to a reference point within a given scenario. Nevertheless, the other procedure is based on relative profits (not relative losses) which means that secondary data represent the differences between particular outcomes and the worst (lowest) payoff. The aforementioned rule is described in Hayashi (2008). Its first step involves generating the relative profit matrix and the second is to find the variant which maximizes the minimal relative profit.

Now how the position of a payoff within a given criterion is calculated in M-DM/C will be examined. As a matter of fact this approach has been already mentioned in this paper—in multi-criteria decision making use of performance degrees (normalized values) is very popular since they allow a comparison of objectives expressed in different units and scales. Nevertheless, in this case the position is not indicated on the basis of the maximal outcome or the minimal one only. The performance degrees are computed by taking into consideration two reference points simultatenuously, i.e. both extreme values (maximum and minimum). If the criterion is maximized the normalization is done by means of Equation (15). For objectives being minimized the application of Equation (16) is recommended. In the example all the criteria are maximized so in each case the use of the first equation is required. The performance degrees are shown in Table 9b. Note that the normalized values are applied to numerous M-DM/C methods (SAW, max-min rule, goal programming, interactive programming, TOPSIS, etc.). That is why in this subsection the analysis of the example presented in Table 9b is not continued—the focus is on the data transformation.

$$b(n)_{k,j} = \frac{b_{k,j} - \min_j b_{k,j}}{\max_j b_{k,j} - \min_j b_{k,j}} \tag{15}$$

$$b(n)_{k,j} = \frac{\max_j b_{k,j} - b_{k,j}}{\max_j b_{k,j} - \min_j b_{k,j}} \tag{16}$$

Table 9a–9b. Relative losses (Savage rule, 1-DM/U) and performance degrees (M-DM/C)

Table 9a	**Alternatives**		
Scenarios	A_1	A_2	A_3
S_1	3	9	0
S_2	0	10	7
S_3	11	0	7
Max	11	10	7

Source: Prepared by the author.

Table 9b	**Alternatives**		
Criteria	A_1	A_2	A_3
C_1	0.67	0.00	1.00
C_2	1.00	0.00	0.30
C_3	0.00	1.00	0.36

Source: Prepared by the author.

As it can be seen both in 1-DM/U and M-DM/C initial data are sometimes transformed in such a way that it is possible to define their positions absolutely or relatively. Therefore, a visible analogy occurs in this area. However, the differences connected with that aspect are also worth mentioning. In 1-DM/U the transformation is only used in the Savage and Hayashi rules because their goal is to take into account the structure of the payoff matrix—that target is not essential for other uncertain decision rules. In M-DM/C the transformation is always required for incomparable objectives.

3. Novel approaches for 1-DM/U and M-DM/C

The paper contains a review of selected methods applied to 1-DM/U and M-DM/C. It also reveals numerous common features. Now it should be verified whether it would be possible to create new procedures by analogy to existing ones. Hence, the target of this section is to make an attempt to adjust some techniques formulated for one issue (1-DM/U or M-DM/C) to an additional issue (M-DM/C or 1-DM/U).

3.1. A method for 1-DM/U with a neutral criterion

When the objectives are neutral (i.e. they are neither maximized nor minimized—they consist in reaching a specific value), M-DM/C can be supported by the goal programming (GP). Tables 10a–10b present a brief example concerning this approach. Symbol d_k denotes the desired level of criterion C_k, while w_k still signifies the weight of a given objective. The synthetic measure calculated for each variant in the goal programming (GP_j) constitutes the sum of the products of all the absolute deviations from particular desired criteria levels and weights of these objectives, e.g. $GP_1 = 0.1 \cdot 0.5 + 0.2 \cdot 0.1 + 0.9 \cdot 0.4 = 0.43$. The best option according to the aforementioned approach is A_3 since its index minimizes the sum of weighted deviations.

Table 10a–10b. Normalized values and absolute deviations from the desired level—goal programming (M-DM/C)

Table 10a	**Alternatives**			d_k
Scenarios	A_1	A_2	A_3	
C_1	0.5	0.0	1.0	0.6
C_2	1.0	0.0	0.1	0.8
C_3	0.0	0.9	1.0	0.9

Source: Prepared by the author.

Table 10b	**Alternatives**			w_k
Criteria	A_1	A_2	A_3	
C_1	0.1	0.6	0.4	0.5
C_2	0.2	0.8	0.7	0.1
C_3	0.9	0.0	0.1	0.4
GP_j	0.43	0.38	0.31	

Source: Prepared by the author.

Why is it worth applying the idea of goal programming to 1-DM/U? The answer is simple—the existing procedures designed for this issue assume that the objectives are maximized or minimized. The problem of neutral criteria in decision making under uncertainty is not investigated in the literature. However, the need to explore that topic exists especially when considering such attributes as the air temperature, the distance between two places, the number of bedrooms in a potential house to buy, the period of paying off the credit (the term of the loan), the rental time of office space or the duration of the project.

In many decision situations concrete air temperature levels, distances (e.g. between the residence and the city centre) and the numbers of rooms are required, neither the highest / biggest nor the lowest / smallest.

Table 11a–11b. Initial data and absolute deviations from the desired level—1-DM/C

Table 11a	Alternatives			d_i
Scenarios	A_1	A_2	A_3	
S_1	15	5	11	6
S_2	-4	0	7	7
S_3	0	9	3	6

Source: Prepared by the author.

Table 11b	Alternatives		
Criteria	A_1	A_2	A_3
S_1	9	1	5
S_2	11	7	0
S_3	6	3	3

Source: Prepared by the author.

This gap may be bridged easily. It suffices to transform primary data (Table 11a) into absolute deviations from a desired value (Table 11b). Note that if the ranges of payoffs for particular scenarios differ significantly the desired values (d_i) can be different for each scenario. Further steps of the new approach depend on the DM's preferences and nature. If the decision maker intends to execute the chosen variant many times he or she may combine the first step with the Bayes rule (i.e. calculate the averages of all the deviations for each alternative and select the option with the minimal average). If the DM is a strong pessimist it would be recommended to follow the Wald rule, i. e to find the worst (maximal) deviation for each course of action and choose the option with the minimal worst deviation. If the decision maker wants to declare his or her attitude towards risk by using the pessimism coefficient it is suggested to refer to the Hurwicz rule etc. Regardless of the type of hybrid applied there must be an awareness that the transformed matrix contains data which are treated negatively since they represent the deviations from the desired level.

3.2. A method based on scenario hierarchy for 1-DM/U

Another procedure succesfully applied to M-DM/C is the interactive pogramming (IP) which allows the analysis of criteria in a defined sequence and to eliminate sequentially options too weak according to some acceptable levels declared by the decision maker. Within IP the DM is supposed to set an acceptable level for each criterion (AL_k) with the exception of the last objective (Table 12a). The interactive programming similarly to GP is usually based on performance degrees, but in IP the matrix with performance degrees has to be updated after the analysis of each criterion since the sequential reduction of the worst variants within a given objective may change the maximal and minimal values used in Equations (15)–(16), see Table 12b.

Table 12a–12b. Normalized values and updated normalized values—interactive programming (M-DM/C)

Table 12a	Alternatives			AL_k
Scenarios	A_1	A_2	A_3	
C_1	0.5	0.0	1.0	0.4
C_2	1.0	0.0	0.1	0.5
C_3	0.0	0.9	1.0	

Source: Prepared by the author.

Table 12b	Alternatives		AL_k
Criteria	A_1	A_2	
C_1	1.0	0.0	0.5
C_2	0.0	1.0	

Source: Prepared by the author.

In this example, A_2 is eliminated in the first iteration since its performance degree connected with the first objective is lower (0.0) than the acceptable level (0.4). After updating the performance degrees it is necessary to eliminate A_3, because its current performance degree related to the second objective is lower (0.0) than the acceptable level (0.5). The reduced set of options contains one alternative only so A_1 is the variant recommended by interactive programming. In the most general case if the last reduced set of options contains more than one element the choice of the alternative maximizing the performance degree of the last objective is suggested.

How can IP be useful in 1-DM/U? The concept is very simple. Instead of setting the order of criteria a hierarchy for scenarios may be defined. Such a hierarchy is justifiable if the DM is able to subjectively declare which scenarios are more probable than others. It should be pointed out that the article does not involve the cases where the objective probabilities are known. Here the concentration is only on some subjective, individual expectations (predictions) resulting from the DM's nature. The acceptable levels for each scenario may be different, especially if the ranges of payoffs for particular scenarios are different (Table 13a). Contrary to IP designed for M-DM/C the novel approach does not need to use normalized data since in 1-DM/U data are related to one criterion. That means that the data update after each iteration is not required in the proposed method (Table 13b). Hence, the new procedure for 1-DM/U

Table 13a–13b. Initial payoff matrix and reduced payoff matrix (1-DM/U)

Table 13a	Alternatives			AL_k
Scenarios	A_1	A_2	A_3	
S_1	5	0	10	4
S_2	2	8	1	1.5
S_3	9	11	7	

Source: Prepared by the author.

Table 13b	Alternatives		AL_k
Criteria	A_1	A_2	
S_1	2	1	1.5
S_2	9	7	

Source: Prepared by the author.

is even less complex than the original one. In the example the interactive programming adjusted to 1-DM/U leads to the selection of option A_1.

Note that if the acceptable levels are too high the use of IP for M-DM/C will not lead to a concrete answer and the same obstacle occurs when applying IP to 1-DM/U. That is why if the DM intends to take into consideration each scenario given in the payoff matrix he or she should declare subsequent acceptable levels very reasonably.

3.3. A modification of the max-min rule for M-DM/C

The last suggestion presented in this paper and created by analogy to an already existing approach refers to the Wald rule and its modification called "the lexicographic Wald rule". As a matter of fact sometimes it is difficult to find the optimal solution on the basis of the original Wald rule if the minimal payoffs for each alternative are the same. According to that approach each option is equivalent in this case, regardless of the remaining payoffs connected with particular variants.

Fortunately, the lexicographic Wald rule can succesfully solve the aforementioned problem (Gaspars-Wieloch, 2020a; Sen, 1984). It suffices to analyze the second-lowest outcome of each course of action and to select the option with the highest value. If it is still impossible to find the best alternative, the third-lowest outcomes have to be compared and so on (Table 14a).

Table 14a–14b. Payoff matrix (lexicographic Wald rule, 1-DM/U) and performance degrees (lexicographic max-min rule, M-DM/C)

Table 14a	Alternatives		
Scenarios	A_1	A_2	A_3
S_1	5	3	8
S_2	10	2	2
S_3	2	13	6
min(1)	2	2	2
min(2)	5	3	6

Source: Prepared by the author.

Table 14b	Alternatives		
Criteria	A_1	A_2	A_3
C_1	0.00	0.25	1.00
C_2	1.00	0.00	0.50
C_3	0.67	1.00	0.00
min(1)	0.00	0.00	0.00
min(2)	0.67	0.25	0.50

Source: Prepared by the author.

Is this modification valuable for M-DM/C? Undoubtedly, yes, since the Wald rule for 1-DM/U is similar to the max-min rule for M-DM/C. They both have even the same drawbacks! Hence, when using the max-min rule it is also difficult sometimes to select the best alternative, if all the options have got the same (or almost the same) minimal performance degree (Table 14b). Thanks to the implementation of the lexicographic approach to the max-min rule that problem can be successfully avoided. In the example presented in Table 14b alternative A_1 should be chosen.

Discussion and conclusions

The paper reveals numerous analogies occurring between methods formulated for one-criterion scenario-based decision making under uncertainty (1-DM/U) and procedures designed for multi-criteria decision making under certainty (M-DM/C). The aforementioned approaches are often applied by researchers, practitioners and students, but a comparative analysis of these topics has not been investigated yet in the literature. That is why this article gathers diverse existing analogies and similarities.

These analogies result from the fact that both issues can be presented by a very similar payoff matrix (in both tables a set of alternatives is given and the set of scenarios defined in 1-DM/U corresponds to the set of criteria defined in M-DM/C). For both problems the search for both optimal pure and mixed strategies is possible. Thanks to the almost identical structure of the payoff matrices numerous procedures developed for one field have their equivalent methods designed for the other field. This is the case of the following pairs: Wald rule for 1-DM/U and max-min rule for M-DM/C; Bayes rule for 1-DM/U and SAW method for M-DM/C; Pareto solutions for M-DM/C and non-dominated solutions for 1-DM/U; Savage rule for 1-DM/U and performance degrees applied to M-DM/C.

It is worth underlining that the paper does not constitute a review of 1-DM/U and M-DM/C procedures only. It primarily contains the description of three novel approaches which are partially based on other decision rules developed for initially different purposes. The new procedures may prove to be very useful since they bridge the existing gap. The first one (based on goal programming designed for M-DM/C) is devoted to uncertain decision making and neutral criteria. The second one (based on interactive programming designed for M-DM/C), also formulated for 1-DM/U, gives the opportunity to consider particular scenarios within a defined subjective hierarchy based on expectations (predictions). Such a scenario hierarchy may be helpful when the decision maker does not treat each scenario in the same way, but he or she has no information on objective scenario probabilities. The last novel procedure (based on the Wald rule devoted to 1-DM/U) is developed for M-DM/C. It is a modification of the max-min rule which enables the avoidance of the problem of equal (or quasi-equal) minimal performance degrees for each option. The author intends to present in detail the construction of each suggested algorithm in separate scientific papers.

The new approaches may bring some new opportunities. Nevertheless, it is worth emphasizing that if the decision maker's expectations concerning the scenario occurrence is based on an extremely limited knowledge the decision variant suggested by the first or the second method might be unfavourable. That is why it is highly recommended to gather extra knowledge before making the final decision. The third novel technique may also have a limitation since, in practice, the decision maker may be unable to declare the intervals for the minimal performance degree within which the solutions are treated as equivalent. Hence the last suggested new method should be especially applied by rational decision makers.

As it can be observed, the paper strongly recommends developing decision rules by analogy. Such an approach is appreciated by numerous researchers who emphasize that learning and understanding are "most effective when the concepts under consideration can be aligned with our present knowledge. (…) The analogy works well because (…) the relationships between the mapped elements are largerly preserved" (Martin, 2003). The observations aforementioned concern not only students, but also managers. Furthermore, Herstatt and Kalogerakis (2005) and Hey, Linsey, Agogino and Wood (2008) stress that the analogical thinking is extremely useful in creative design and innovating companies and projects which is explored in this article. The analogies found during the investigation have provided two kinds of direct benefits. First, the similatiries detected in the structure of two topics have allowed the discovery of the existence of analogical decision algorithms for both issues. Second, this phenomenon has enabled the development of the concept of novel procedures by analogy to existing ones. However, the analogies revealed in the paper and the presentation of new methods created by analogy to existing procedures lead to the realisation that further analogies between 1-DM/U and M-DM/C are still possible and that their development may contribute to a solution of other essential decision problems. This conclusion is really justified since the article discusses only selected 1-DM/U and M-DM/C classical methods. Hence, the analysis of other extended procedures for both issues could be very fruitful.

Note that possible future research directions are not limited to the issues already described above. After developing new procedures (on the basis of both classical and extended methods) by analogy to existing ones the next step should be connected with the creation of hybrids referring to these novel techniques and designed for uncertain multi-criteria problems (M-DM/U) which occur more frequently in real economic decision situations than deterministic multi-criteria problems (M-DM/C) or indeterministic one-criterion problems (1-DM/U). Thus research findings presented in this paper may enhance the decision process in numerous optimization economic and management problems especially in:

- the choice of investment projects where some criteria are neutral,
- the creation of customized rankings of institutions, countries, investment funds and so on,
- the multi-facet and multi-stage analysis of particular options.

The use of hybrids combining existing approaches with novel ones may be advantageous particularly in the case of innovation and innovative products since the suggested procedures do not require reference to objective probabilities.

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