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Estimation of finite population mean using two auxiliary variables under stratified random sampling

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ABSTRACT

This paper addresses the problem of an alternative approach to estimating the population mean of the study variable with the help of the auxiliary variable under stratified random sampling. The properties of the suggested estimator have been studied under large sample approximation. It has been demonstrated that the suggested estimator is more efficient than other considered estimators. To judge the merits of the proposed estimator, an empirical study has been carried out to support the present study.

Key words: Study variable, auxiliary variable, stratified random sampling, dual to ratio estimator, bias and mean squared error.

1. Introduction

It is a well-known fact that the supplementary or auxiliary information always increases the precision of the estimators for the population parameters of the study variable. Ratio, product, regression and ratio-cum-product type of estimators are good examples in this context. Cochran (1940) proposed the ratio estimator assuming that the study variable (y) and auxiliary variable (x) are positively correlated, and the population mean of the auxiliary variable is known. However, when the study variable (y) and the auxiliary variable (x) are negatively correlated then the ratio estimator does not perform well. In that situation, the product estimator envisaged by Robson (1957) is appropriate.

Many authors including Murthy (1964), Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh and Tailor (2003), Singh et al. (2004), Upadhyaya et al. (2011), etc., proposed different ratio type estimators for the population mean $\overline{Y}$ in simple random sampling. Singh and Tailor (2005), Tailor and Sharma (2009), Upadhyaya et al. (2011) and Yadav et al. (2012) proposed different ratio-cum-product estimators of a

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finite population mean of the study variable using the values of known parameters of the auxiliary variables in simple random sampling. Srivenkataramana (1980) first proposed dual to ratio estimator, Bandopadhyaya (1980) suggested dual to product estimator for the population mean using transformation on auxiliary variable under simple random sampling.

As we know, the stratified random sampling can provide greater precision than a simple random sampling of the same size and it often requires a smaller sample, which saves money. Due to these shortcomings under simple random sampling, many authors like Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Kadilar and Cingi (2003, 2005), Singh et al. (2004), Singh and Vishwakarma (2008, 2010), Koyuncu and Kadilar (2009), Tailor (2009), Tailor el al. (2012), Yadav et al. (2014), Gupta and Shabbir (2015), Tailor et al. (2015) and Mishra et al. (2017) defined ratio estimators and ratio-cum-product estimators under stratified random sampling, which perform better than usual ratio and product estimators in simple random sampling under certain limitations. Motivated by them, an attempt is made to develop an efficient dual to ratio-cum-product estimator of the population mean of the study variable using the knowledge of coefficient of kurtosis of the auxiliary variable under stratified random sampling.

Let the population of size N be equally divided into L strata with N_h elements in the h^{th} stratum such that $N=\sum_{h=1}^{L} N_h$. Let y be the study variable and x and z be two auxiliary variables assuming values y_{hi}, x_{hi} *and* z_{hi} for the i^{th} unit in h^{th} stratum. Let n_h be the size of the sample drawn from h^{th} stratum of size N_h by using simple random sampling without replacement (SRSWOR) such that sample size $n=\sum_{h=1}^{L} n_h$.

We define

$$\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi} : \quad h^{th} \text{ stratum mean for the study variate y}$$

$$\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi} : \quad h^{th} \text{ stratum mean for the study variate x}$$

$$\bar{Z}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} z_{hi} : \quad h^{th} \text{ stratum mean for the study variate z}$$

$$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi} = \frac{1}{N}\sum_{h=1}^{L} N_h \bar{Y}_h = \sum_{h=1}^{L} W_h \bar{Y}_h : \text{ population mean of the study variate y}$$

$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi} = \frac{1}{N}\sum_{h=1}^{L} W_h \bar{X}_h$: population mean of the auxiliary variate x

$\bar{Z} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} z_{hi} = \frac{1}{N}\sum_{h=1}^{L} W_h \bar{Z}_h$: population mean of the study variate z

$\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$: sample mean of the study variate y for h^{th} stratum

$\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}$: sample mean of the auxiliary variate x for h^{th} stratum

$\bar{z}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} z_{hi}$: sample mean of the auxiliary variate z for h^{th} stratum

$W_h = \frac{N_h}{N}$: stratum weight of h^{th} stratum

Hansen et al. (1946) defined the classical combined ratio estimator for the population mean of the study variable y under stratified random sampling as

$$\bar{y}_{RC} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right) \tag{1.1}$$

Here, it is assumed that the study variable y and auxiliary variable x are positively correlated.

Using the information on two auxiliary variables x and z, Tailor et al. (2012) proposed a ratio-cum-product estimator of the population mean $\bar{Y}$ in stratified random sampling as

$$\hat{\bar{Y}}_{RP}^{ST} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right) = \sum_{h=1}^{L} W_h \bar{Y}_h \left(\frac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h \bar{x}_h}\right)\left(\frac{\sum_{h=1}^{L} W_h \bar{z}_h}{\sum_{h=1}^{L} W_h \bar{Z}_h}\right) \tag{1.2}$$

Tailor et al. (2015) utilized the information of the coefficient of kurtosis of the auxiliary variables x and z, and proposed a ratio-cum-product estimator $\hat{\bar{Y}}_{RP1}^{ST}$ of the population mean $\bar{Y}$ under stratified random sampling as

$$\hat{\bar{Y}}_{RP1}^{ST} = \bar{y}_{st}\left[\frac{\sum_{h=1}^{L} W_h\left\{\bar{X}_h + \beta_{2h}(x)\right\}}{\sum_{h=1}^{L} W_h\left\{\bar{x}_h + \beta_{2h}(x)\right\}}\right]\left[\frac{\sum_{h=1}^{L} W_h\left\{\bar{z}_h + \beta_{2h}(z)\right\}}{\sum_{h=1}^{L} W_h\left\{\bar{Z}_h + \beta_{2h}(z)\right\}}\right] \tag{1.3}$$

where $\beta_{2h}(x)$ *and* $\beta_{2h}(z)$ are the coefficients of kurtosis of the auxiliary variates x and z, respectively in h^{th} stratum.

The mean squared errors (MSE) of the combined ratio estimator $\bar{y}_{RC}$, Tailor et al. (2012) estimator $\hat{\bar{Y}}_{RP}^{ST}$ and Tailor et al. (2015) estimator $\hat{\bar{Y}}_{RP1}^{ST}$, defined in (1.1), (1.2) and (1.3) up to the first order of approximation, are respectively given by

$$MSE\left(\bar{y}_{RC}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2 + R_1^2 S_{xh}^2 - 2R_1 S_{yxh}\right) \tag{1.4}$$

$$MSE\left(\hat{\bar{Y}}_{RP}^{ST}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2 + R_1^2 S_{xh}^2 + R_2^2 S_{zh}^2 - 2R_1 S_{yxh} + 2R_2 S_{yzh} - 2R_1 R_2 S_{xzh}\right) \tag{1.5}$$

$$MSE\left(\hat{\bar{Y}}_{RP1}^{ST}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + R_{12}^2 S_{xh}^2 + R_{13}^2 S_{zh}^2 - 2R_{12} S_{yxh} + 2R_{13} S_{yzh} - 2R_{12} R_{13} S_{xzh}\right] \tag{1.6}$$

where $\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h$, $\quad \bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h$, $\quad \bar{z}_{st} = \sum_{h=1}^{L} W_h \bar{z}_h$,

$$S_{yh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}_h\right)^2, \qquad S_{xh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(x_{hi} - \bar{X}_h\right)^2,$$

$$S_{zh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(z_{hi} - \bar{Z}_h\right)^2, \qquad S_{yxh} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}_h\right)\left(x_{hi} - \bar{X}_h\right),$$

$$S_{yzh} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}_h\right)\left(z_{hi} - \bar{Z}_h\right), \quad S_{xzh} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(x_{hi} - \bar{X}_h\right)\left(z_{hi} - \bar{Z}_h\right),$$

$$\gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right), \qquad R_1 = \frac{\bar{Y}}{\bar{X}}, \qquad R_2 = \frac{\bar{Y}}{\bar{Z}},$$

$$R_{12} = \frac{\bar{Y}}{\sum_{h=1}^{L} W_h\left\{\bar{X}_h + \beta_{2h}(x)\right\}} = \frac{\bar{Y}}{\bar{X}_{1h}} \quad \textit{and} \quad R_{13} = \frac{\bar{Y}}{\sum_{h=1}^{L} W_h\left\{\bar{Z}_h + \beta_{2h}(z)\right\}} = \frac{\bar{Y}}{\bar{Z}_{1h}}.$$

2. The suggested estimator

Motivated by Srivenkataramana (1980) and assuming that the parameters of the auxiliary variables x and z are known, we propose the dual to ratio-cum-product estimator t_{st}^{*} of Tailor et al. (2015) estimator $\hat{\bar{Y}}_{RP1}^{ST}$ of the population mean $\bar{Y}$ of the study variable y, which is defined as

$$t_{st}^{*}=\bar{y}_{st}\left[\frac{\sum_{h=1}^{L}W_h\left\{\bar{x}_h^{*}+\beta_{2h}(x)\right\}}{\sum_{h=1}^{L}W_h\left\{\bar{X}_h+\beta_{2h}(x)\right\}}\right]\left[\frac{\sum_{h=1}^{L}W_h\left\{\bar{Z}_h+\beta_{2h}(z)\right\}}{\sum_{h=1}^{L}W_h\left\{\bar{z}_h^{*}+\beta_{2h}(z)\right\}}\right] \tag{2.1}$$

where $\bar{x}_h^{*}=\left(\frac{N_h\bar{X}_h-n_h\bar{x}_h}{N_h-n_h}\right)$ and $\bar{z}_h^{*}=\left(\frac{N_h\bar{Z}_h-n_h\bar{z}_h}{N_h-n_h}\right)$.

Using the transformation on $\bar{x}_h^{*}$ *and* $\bar{z}_h^{*}$ of the auxiliary variables x and z, the suggested estimator t_{st}^{*} in (2.1) can be written as

$$t_{st}^{*}=\left(\sum_{h=1}^{L}W_h\bar{y}_h\right)\left[\frac{\sum_{h=1}^{L}W_h\left\{\left(\frac{N_h\bar{X}_h-n_h\bar{x}_h}{N_h-n_h}\right)+\beta_{2h}(x)\right\}}{\sum_{h=1}^{L}W_h\left\{\bar{X}_h+\beta_{2h}(x)\right\}}\right]\left[\frac{\sum_{h=1}^{L}W_h\left\{\bar{Z}_h+\beta_{2h}(z)\right\}}{\sum_{h=1}^{L}W_h\left\{\left(\frac{N_h\bar{Z}_h-n_h\bar{z}_h}{N_h-n_h}\right)+\beta_{2h}(z)\right\}}\right] \tag{2.2}$$

Let $\bar{y}_h=\bar{Y}(1+e_{0h})$, $\bar{x}_h=\bar{X}_h(1+e_{1h})$ *and* $\bar{z}_h=\bar{Z}_h(1+e_{2h})$
such that $E(e_{0h})=E(e_{1h})=E(e_{2h})=0$

$$E(e_{0h}^2)=\gamma_h C_{yh}^2,\qquad E(e_{1h}^2)=\gamma_h C_{xh}^2,\qquad E(e_{2h}^2)=\gamma_h C_{zh}^2,$$

$$E(e_{0h}e_{1h})=\gamma_h\rho_{yxh}C_{yh}C_{xh}=\gamma_h\frac{S_{yxh}}{\bar{Y}_h\bar{X}_h},\qquad E(e_{1h}e_{2h})=\gamma_h\rho_{xzh}C_{xh}C_{zh}=\gamma_h\frac{S_{xzh}}{\bar{X}_h\bar{Z}_h}\quad \textit{and}$$

$$E(e_{0h}e_{2h})=\gamma_h\rho_{yzh}C_{yh}C_{zh}=\gamma_h\frac{S_{yzh}}{\bar{Y}_h\bar{Z}_h}$$

Expressing (2.2) in terms of e's, we get

$$= \bar{Y}\left(1+e_0\right)\left(1-e_1\right)\left(1-e_2\right)^{-1}$$

where $e_0 = \dfrac{\sum_{h=1}^{L} W_h \bar{Y}_h e_{0h}}{\sum_{h=1}^{L} W_h \bar{Y}_h} = \dfrac{\sum_{h=1}^{L} W_h \bar{Y}_h e_{0h}}{\bar{Y}}$, $\quad e_1 = \dfrac{\sum_{h=1}^{L} W_h g_h \bar{X}_h e_{1h}}{\sum_{h=1}^{L} W_h \left\{\bar{X}_h + \beta_{2h}(x)\right\}} = \dfrac{\sum_{h=1}^{L} W_h g_h \bar{X}_h e_{1h}}{\bar{X}_{1h}}$,

$e_2 = \dfrac{\sum_{h=1}^{L} W_h g_h \bar{Z}_h e_{2h}}{\sum_{h=1}^{L} W_h \left\{\bar{Z}_h + \beta_{2h}(z)\right\}} = \dfrac{\sum_{h=1}^{L} W_h g_h \bar{Z}_h e_{2h}}{\bar{Z}_{1h}}$ and $g_h = \dfrac{n_n}{N_h - n_n}$

such that

$$E\left(e_0\right) = E\left(e_1\right) = E\left(e_2\right) = 0$$

and

$$E\left(e_0^2\right) = \frac{1}{\bar{Y}^2}\sum_{h=1}^{L} W_h^2 \gamma_h S_{yh}^2, \qquad E\left(e_1^2\right) = \frac{1}{\bar{X}_{1h}^2}\sum_{h=1}^{L} W_h^2 \gamma_h g_h^2 S_{xh}^2,$$

$$E\left(e_2^2\right) = \frac{1}{\bar{Z}_{1h}^2}\sum_{h=1}^{L} W_h^2 \gamma_h g_h^2 S_{zh}^2, \qquad E\left(e_0 e_1\right) = \frac{1}{\bar{Y}\,\bar{X}_{1h}}\sum_{h=1}^{L} W_h^2 \gamma_h g_h S_{yxh},$$

$$E\left(e_1 e_2\right) = \frac{1}{\bar{X}_{1h}\bar{Z}_{1h}}\sum_{h=1}^{L} W_h^2 \gamma_h g_h^2 S_{xzh}, \qquad E\left(e_0 e_2\right) = \frac{1}{\bar{Y}\,\bar{Z}_{1h}}\sum_{h=1}^{L} W_h^2 \gamma_h g_h S_{yzh}$$

To the first degree of approximation, the bias and mean squared error of the suggested estimator t_{st}^* are given by

$$B\left(t_{st}^*\right) = \bar{Y}\sum_{h=1}^{L} W_h^2 \gamma_h g_h \left[\frac{g_h}{\bar{Z}_{1h}}\left(\frac{S_{zh}^2}{\bar{Z}_{1h}} - \frac{S_{xzh}}{\bar{X}_{1h}}\right) + \frac{1}{\bar{Y}}\left(\frac{S_{yzh}}{\bar{Z}_{1h}} - \frac{S_{xyh}}{\bar{X}_{1h}}\right)\right] \tag{2.3}$$

$$MSE\left(t_{st}^*\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + g_h^2 R_{12}^2 S_{xh}^2 + g_h^2 R_{13}^2 S_{zh}^2 - 2 g_h R_{12} S_{yxh} + 2 g_h R_{13} S_{yzh} - 2 g_h^2 R_{12} R_{13} S_{xzh}\right] \tag{2.4}$$

3. Efficiency comparison

Since we know that the variance of the usual unbiased estimator of the study variable y in stratified random sampling is defined as

$$V\left(\bar{y}_{st}\right)=\sum_{h=1}^{L}W_h^2\gamma_h S_{yh}^2 \tag{3.1}$$

From equations (1.4), (1.5), (1.6), (2.4) and (3.1), we have

(i) $MSE\left(t_{st}^*\right)<MSE\left(\bar{y}_{st}\right)$ if and only if:

$$\sum_{h=1}^{L}W_h^2\gamma_h g_h^2\left\{R_{12}^2S_{xh}^2+R_{13}^2S_{zh}^2-2R_{12}R_{13}S_{xzh}\right\}-2\sum_{h=1}^{L}W_h^2\gamma_h g_h\left\{R_{12}S_{yxh}-R_{13}S_{yzh}\right\}<0 \tag{3.2}$$

(ii) $MSE\left(t_{st}^*\right)<MSE\left(\bar{y}_{RC}\right)$ if and only if:

$$\sum_{h=1}^{L}W_h^2\gamma_h g_h^2\left\{R_{12}^2S_{xh}^2+R_{13}^2S_{zh}^2-2R_{12}R_{13}S_{xzh}\right\}-2\sum_{h=1}^{L}W_h^2\gamma_h g_h\left\{R_{12}S_{yxh}-R_{13}S_{yzh}\right\} -\sum_{h=1}^{L}W_h^2\gamma_h R_1\left\{R_1S_{xh}^2-2S_{yxh}\right\}<0 \tag{3.3}$$

(iii) $MSE\left(t_{st}^*\right)<MSE\left(\hat{\bar{Y}}_{RP}^{ST}\right)$ if and only if:

$$\sum_{h=1}^{L}W_h^2\gamma_h g_h^2\left\{R_{12}^2S_{xh}^2+R_{13}^2S_{zh}^2-2R_{12}R_{13}S_{xzh}\right\}-2\sum_{h=1}^{L}W_h^2\gamma_h g_h\left\{R_{12}S_{yxh}-R_{13}S_{yzh}\right\} -\sum_{h=1}^{L}W_h^2\gamma_h\left\{R_1^2S_{xh}^2+R_2^2S_{zh}^2-2R_1S_{yxh}+2R_2S_{yzh}-2R_1R_2S_{xzh}\right\}<0 \tag{3.4}$$

(iv) $MSE\left(t_{st}^*\right)<MSE\left(\hat{\bar{Y}}_{RP1}^{ST}\right)$ if and only if:

$$\sum_{h=1}^{L}W_h^2\gamma_h g_h^2\left\{R_{12}^2S_{xh}^2+R_{13}^2S_{zh}^2-2R_{12}R_{13}S_{xzh}\right\}-2\sum_{h=1}^{L}W_h^2\gamma_h g_h\left\{R_{12}S_{yxh}-R_{13}S_{yzh}\right\} -\sum_{h=1}^{L}W_h^2\gamma_h\left\{R_{12}^2S_{xh}^2+R_{13}^2S_{zh}^2-2R_{12}S_{yxh}+2R_{13}S_{yzh}-2R_{12}R_{13}S_{xzh}\right\}<0 \tag{3.5}$$

From equations (3.2), (3.3), (3.4) and (3.5), we obtained the conditions under which the suggested estimator performed better than the usual unbiased estimator, combined ratio estimator $\overline{y}_{RC}$, Tailor et al. (2012) estimator $\hat{\overline{Y}}_{RP}^{ST}$ and Tailor et al. (2015) estimator $\hat{\overline{Y}}_{RP1}^{ST}$.

4. Empirical study

To judge the efficiency of the proposed estimator over the usual unbiased estimator, combined ratio estimator $\overline{y}_{RC}$, Tailor et al. (2012) estimator $\hat{\overline{Y}}_{RP}^{ST}$ and Tailor et al. (2015) estimator $\hat{\overline{Y}}_{RP1}^{ST}$, the following data set is taken. The description of the population is given below:

Population [Source: Murthy (1967), p. 228]

z: Number of workers

y: Output and

x: Fixed capital

	n_1=2	n_2=2	N_1=5	N_2=5
	$\overline{Z}_1$=51.80	$\overline{Z}_2$=60.60	$\overline{X}_1$=214.4	$\overline{X}_2$=333.8
N=10	$\overline{Y}_1$=1925.8	$\overline{Y}_2$= 3115.6	S_{z_1}=0.75	S_{z_2}=4.84
	S_{x_1}= 74.87	S_{x_2}=66.35	S_{y_1}=615.92	S_{y_2}=340.38
n=4	S_{zx_1}=-38.08	S_{zx_2}= - 287.92	S_{yz_1}= -411.16	S_{yz_2}=-1536.24
	S_{yx_1}=39360.68	S_{yx_2}=22356.50	C_{x_1}=0.35	C_{x_2}=0.20
	C_{z_1}=0.01	C_{z_2}=0.08	$\beta_{21}(x)$=1.88	$\beta_{22}(x)$=2.32
	$\beta_{21}(z)$=1.84		$\beta_{22}(z)$=1.49	

For the purpose of the efficiency comparison of the proposed estimator, we have computed the percent relative efficiencies (PREs) of the estimators with respect to the usual unbiased estimator $\overline{y}_{st}$ using the formula:

$$PRE(t, \overline{y}_{st}) = \frac{MSE(\overline{y}_{st})}{MSE(t)} \times 100; \qquad where\ t = \overline{y}_{st},\ \overline{y}_{RC},\ \hat{\overline{Y}}_{RP}^{ST},\ \hat{\overline{Y}}_{RP1}^{ST}\ and\ t_{st}^{*}$$

The findings are given in Table 1.

Table 1

Percent relative efficiencies of the estimators $\overline{y}_{st}$, $\overline{y}_{RC}$, $\hat{\overline{Y}}_{RP}^{ST}$, $\hat{\overline{Y}}_{RP1}^{ST}$ and t_{st}^{*} with respect to $\overline{y}_{st}$

Estimators	$\overline{y}_{st}$	$\overline{y}_{RC}$	$\hat{\overline{Y}}_{RP}^{ST}$	$\hat{\overline{Y}}_{RP1}^{ST}$	t_{st}^{*}
Population	100.00	239.8632589	141.9128961	146.8036738	361.4516525

5. SIMULATION STUDY

In the paper, we generated two populations for two auxiliary variables x and z. Population I has equal size stratum and Population II has unequal size stratum. We calculated the variance and MSE's values of $\overline{y}_{st}$, $\overline{y}_{RC}$, $\hat{\overline{Y}}_{RP}^{ST}$, $\hat{\overline{Y}}_{RP1}^{ST}$ and t_{st}^{*} respectively, for different values of the sample size viz. 500, 700, 900; obtained from different stratum using proportional allocation. The variance and MSE's of the estimators are represented in Table 2 and Table 3.

Population I: N **=2500** N_1 **=500** N_2 **=500** N_3 **=500** N_4 **=500** N_5 **=500**

Table 2

Estimators / n	$\overline{y}_{st}$	$\overline{y}_{RC}$	$\hat{\overline{Y}}_{RP}^{ST}$	$\hat{\overline{Y}}_{RP1}^{ST}$	t_{st}^{*}
500	23.6416	0.004104	0.006670	0.001786	0.001283
700	23.6416	0.001785	0.003674	0.015174	0.000766
900	23.6416	0.001793	0.001898	0.005736	0.001114

Population II: N **=2500** N_1 **=500** N_2 **=300** N_3 **=700** N_4 **=600** N_5 **=400**

Table 3

Estimators / n	$\overline{y}_{st}$	$\overline{y}_{RC}$	$\hat{\overline{Y}}_{RP}^{ST}$	$\hat{\overline{Y}}_{RP1}^{ST}$	t_{st}^{*}
500	22.9679	0.000955	0.002192	0.002793	0.000432
700	22.9679	0.001202	0.001678	0.011525	0.000514
900	22.9679	0.001018	0.001101	0.0052308	0.000476

From Table 2 and Table 3, we came up with a conclusion that MSE of the proposed estimator is less than all the other considered estimators. So, we can say that the performance of our proposed estimator is better than the usual unbiased estimator, combined ratio estimator $\overline{y}_{RC}$, Tailor et al. (2012) estimator $\hat{\overline{Y}}_{RP}^{ST}$ and Tailor et al. (2015) estimator $\hat{\overline{Y}}_{RP1}^{ST}$.

5. Conclusion

This paper has suggested a dual to ratio-cum-product estimator to estimate the population mean of the study variable using the knowledge of the population mean as well as the coefficient of kurtosis of two auxiliary variables x and z under stratified random sampling. Its properties have been studied under large sample approximation. Section 3 reveals the conditions under which the suggested estimator has less MSE than the usual combined ratio estimator $\overline{y}_{RC}$, Tailor et al. (2012) estimator $\hat{\overline{Y}}_{RP}^{ST}$ and Tailor et al. (2015) estimator $\hat{\overline{Y}}_{RP1}^{ST}$. This means that the proposed estimator is more efficient than other considered estimators under certain limitations. Table 1 shows that the suggested dual to ratio-cum-product estimator has more percent relative efficiency as compared to the usual combined ratio estimator $\overline{y}_{RC}$, Tailor et al. (2012) estimator $\hat{\overline{Y}}_{RP}^{ST}$ and Tailor et al. (2015) estimator $\hat{\overline{Y}}_{RP1}^{ST}$. In addition, the simulation study has also been carried out to show the efficiency of the suggested estimator, whose results are displayed in Table 2 and Table 3. Therefore, it can be concluded that if information on the coefficient of kurtosis of the auxiliary variables is available for each stratum then the suggested estimator performs well and more efficiently than other considered estimators. Thus, the suggested estimator can be recommended as an alternative use of the estimation of the population mean of the character under study.

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