STATISTICS IN TRANSITION new series, March 2021
Vol. 22, No. 1, pp. 55–73, DOI 10.21307/stattrans-2021-003
Received – 28.03.2020; accepted – 22.12.2020

Measurement of enterprise mobility among size classes, taking into account business demography

Camilla Ferretti[1]

ABSTRACT

A extensive body of literature is devoted to the production of mobility measures based on transition matrices. The applications often involve panel data, and yet the impact of demographic events on enterprise mobility is not considered. The article aims provide a definition of enterprise mobility, in terms of the capability to create or liquidate jobs. Moreover, some existing mobility measures are modified so that they also take into account newborn and exiting firms. The proposed index has all the relevant basic properties which make it a rigorous descriptive statistics. The mobility of Italian capital-owned enterprises in the years 2010 – 2017 is analysed in the case study.
What we propose may be an alternative tool for practitioners to measure the degree of mobility in the presence of demographic events. It may be considered an initial step in future research regarding its different applications (e.g. labour market flows or movements among income classes), also considering more complex theoretical backgrounds.

Key words: mobility measure; transition matrix; firm size; business demography.

1. Introduction

The importance of having available some descriptive statistics for measuring the mobility in an evolving sample has been widely recognized both in the past and today. Typically, a set of k non-overlapping and discrete states (also said *classes* or *categories*) is given, based on an economically relevant variable (e.g. employment or unemployment state, firm size, income classes). Movements among such classes happened between time t and time $t+1$ are usually recorded into a $k \times k$ *transition matrix*. Measuring the degree of mobility corresponds to choosing a suitable indicator able to summarize in a unique real number the global amount of movements. There are mainly two distinct ways to face this issue: $\imath$) by comparing two distributions among states at time t_0 and t_1 and measuring their "distance", as in Shorrocks (1982) and Fields and Ok (1996) (among others); $\imath\imath$) by applying a suitable function $I : \mathbb{R}^{k\times k} \rightarrow \mathbb{R}$ on the $k \times k$ transition matrix $P = \{p_{ij}\}$ as, for example, in Prais (1955); Shorrocks (1978); Bourguignon and Morrisson (2002) (see Ferretti, 2012, for an exhaustive survey). Proposals in the more recent literature include also Ferretti and Ganugi (2013); Chen and Cowell (2017); Cowell and Flachaire (2018); Paul (2019). Furthermore, the relationship of the mobility measures with some theoretical stochastic processes, possibly underlying the dynamics under study, and the sampling properties of a given mobility index have been treated in Geweke et al. (1986); Schluter (1998); Formby et al. (2004) and Ferretti (2014).

[1]DISES, Univ. Cattolica del Sacro Cuore, Piacenza, Italy. E-mail: camilla.ferretti@unicatt.it; cami.ferretti@gmail.com. ORCID: https://orcid.org/0000-0003-4508-5127.

In this work, we focus specifically on the Firms Size and Business Demography, and on the issue of measuring firms mobility using transition matrices (TMs herein). It is an attempt to construct an indicator able to unify two relevant features in any set of firms evolving with respect of time: on the one hand we aim to measure the global mobility, intended as capacity to move among size classes (a more refined definition will be provided in the following sections); on the other hand we would like to measure the effect of demographic events (births and deaths) on the mobility as a whole. As a matter of fact, these two features are usually treated separately: both for descriptive and estimating purposes, movements among different states are often stored in a transition matrix; whereas national statistical bureaus usually provide the birth and death rates without considering the mobility among size classes, as in the technical report ISTAT (2019).

In the following we propose a step-by-step procedure to define mobility in the Firm Size framework, accounting at the same time for the effect of Business Demography. Starting from the fact that the existing mobility indices are additively decomposable in k contributions, representing the mobility in different parts of the size range, we introduce some ad-hoc modifications to obtain a new index which measures both the prevalent tendency towards upsizing/downsizing, and the number of created/destroyed job places. The new index represents for now an alternative and quite easy-to-handle descriptive measure for practitioners, because it can be evaluated having at disposal the sole aggregated data (transition matrix, birth rate and death rate).

This work is organized as follows: Section 2 provides a short survey about mobility indices measured with transition matrices; Section 3 introduces our concept of mobility in the Firm Size framework; Section 4 recalls the usual way to set TMs when births and deaths are considered and proposes a formalization of the effect of newborns and exiting enterprises on the mobility; in Section 5 the main properties of the new index are analysed (in particular the decomposition in terms of birth and death events); Section 6 contains the empirical example regarding the mobility of Italian capital-owned Employer Enterprises belonging to B – E sectors in NACE Rev. 2 (Industry except construction) for the years 2010 – 2017. The last Section concludes.

2. Measuring mobility with TMs

Considering a set of k non-overlapping size classes (for example, firms can be divided according the official definition of ”micro”, ”small”, ”medium” and ”large”), we observe the size of firms in two consecutive instants t and $t+1$. Note that a set of *Incumbents* is required[2], intended as a group of firms being already active at time t and still active at time $t+1$, to count the number of transitions among the size classes. We choose here to work in a non-parametric framework, in the sense that we aim to build a mobility indicator which does not require underlying theoretical assumptions. In consequence of that, p_{ij} will be considered as a relative frequency (or empirical probability) instead of a theoretical probability. The empirical transition matrix (TM) is thus defined as usual by $P = \{p_{ij}\}_{i,j=1,\dots,k}$

[2]The term *Incumbents* is generally referred to a set of firms already in position in a market. For extension we use the same term to indicate firms observed for the whole considered interval of time.

such that

$$p_{ij} = \frac{n_{ij}}{\sum_{l=1}^{k} n_{il}}, \tag{1}$$

where n_{ij} is the number of firms moved from the i-th to the j-th class, for every couple $i, j = 1, \dots, k$.[3] Consequently, p_{ij} is the relative frequency of movements between i and j, conditioned to the starting size class i.

In this framework, a *mobility index* is a function $I : [0,1]^{k\times k} \to \mathbb{R}$. Given two generic groups A and B of enterprises, and the corresponding TMs P_A and P_B, by definition A is said to have a higher degree of mobility than B if and only if $I(P_A) > I(P_B)$. Main proposals in the literature are, among others:

- the trace index $I_{tr}(P) = \frac{1}{k}\sum_i (1 - p_{ii})$ (Prais, 1955; Shorrocks, 1978);
- the index $I_b(P) = \frac{1}{k}\sum_{i,j} p_{ij}|j - i|$ (Bartholomew, 1982);
- the up/downward index $I_{up}(P) = \sum_i f_i \sum_{j>i} p_{ij}$ and $I_{down}(P) = \sum_i f_i \sum_{j<i} p_{ij}$, where f_i is the percentage of firms moving from i (Bourguignon and Morrisson, 2002);
- the directional index $I^v_{dir}(P) = \sum_i f_i \sum_j p_{ij} \cdot sign(j-i) \cdot v(|j-i|)$, where $sign(j-i) \cdot v(|j-i|)$ is included to grasp both the direction and the magnitude of jumps from i to j for every possible couple i, j (Ferretti and Ganugi, 2013).[4]

As explained in Ferretti (2012), mobility has many facets, each one measured by one or more specific indices. To better illustrate this fact, we consider the trivial and fictitious case of 300 firms with 1, 2 or 3 employees, and we suppose to observe the following numbers of transitions:

		Size at time $t+1$			
	$N = \{n_{ij}\}$	1	2	3	TOT at time t
Size at time t	1	50	40	10	100
	2	20	60	20	100
	3	10	20	70	100
	TOT at time $t+1$	80	120	100	TOT = 300

N provides many different pieces of information about the firms mobility: for example, we may be interested in measuring 1) the tendency to move one class up from the lowest class; or 2) the tendency to remain in the same size class; or 3) the global tendency to downsize. The goal of mobility indices is twofold: to provide a summarizing measure for the mobility facet we are interested in (choosing the correct index) and to furnish a rigorous indicator which is a pure number and does not depend on the total amount of considered

[3] Note that n_{ij} depends on t. The time label is discarded for the sake of simplicity, when possible.

[4] The function $sign(x)$ is defined to be equal to +1 if $x \geq 0$ and to -1 if $x < 0$. Note also that v is required to satisfy $v(0) = 0$, see Ferretti and Ganugi (2013) for more details.

firms (using P instead of N). For example, p_{12} is a possible index for the case 1) in the previous example, whereas the trace index and the downward index can be suitable for the cases 2) and 3). To conclude the example, P is easily obtained dividing the N's rows by 100, and here we display some indices evaluated following the definitions listed above:

$$I_{tr} = 0.4, \quad I_{up} = 0.233, \quad I_{down} = 0.167, \quad I_b = 0.467.$$

2.1. Additively decomposable mobility measures

To better explain the concept of mobility, we first recall the fact that mobility measures are additively decomposable, with few exceptions[5]. All the aforementioned indices, together with the most part of the existing ones, can be indeed written in the following general form:

$$I(P) = \sum_{i=1}^{k} \omega_i \cdot I_i(P_i), \tag{2}$$

where ω_i are weights such that $0 \leq \omega_i \leq 1$ for every $i = 1, \dots, k$ and $\sum_i \omega_i = 1$. Given the i-th size class, the function $I_i : [0,1]^k \to \mathbb{R}$ measures the mobility of individuals leaving from such class, whose transitions are ruled by the i-th row $P_i = (p_{i1}, \dots, p_{ik})$ of P. We also point out that I_i may be not well defined for vectors not belonging to the set

$$\Delta^{k-1} := \{(x_1, \dots, x_k) \in \mathbb{R}^k : \sum_{i=1}^{k} x_i = 1, x_i \geq 0, \forall i = 1, \dots, k\},$$

which contains all the possible probability mass functions on k discrete states.

Consequently, Equation 2 mirrors the fact that every individual starting from i has a certain degree of mobility depending on the empirical probability mass function given by P_i. All the individuals starting from the same category are supposed to have the same degree of mobility: this is a reasonable approximation because the use of TMs implies that categories are homogeneous enough to include like-minded individuals. Thus, mobility indices on TMs basically measure the mobility in different parts of the firms size' distribution, defined by the selected categories, and furnish a weighted mean of such contributions.

3. Definition of firms' mobility

Decomposability helps to give a suitable interpretation to the aforementioned mobility measures, when the firm size framework is considered. Let P be the TM obtained in the previous example, and consider the trace index decomposed as follows:

$$I_{tr}(P) = \omega_1 \underbrace{(1 - p_{11})}_{I_1(P_1)} + \omega_2 \underbrace{(1 - p_{22})}_{I_2(P_2)} + \omega_3 \underbrace{(1 - p_{33})}_{I_3(P_3)}.$$

[5]For example, the index $I(P) = det(P)$ examined in Shorrocks (1978), which is not directly decomposable in k terms related to the k size classes. However, it is implicitly based on the Markov assumption and consequently goes beyond the scope of this work.

Note that $\omega_i = 1/3$ for every i, by definition. Given i, the function I_i results to be the empirical probability to move away from i. Consequently, the trace index is a (weighted) mean probability that a generic firm will leave its starting class. If $\omega_i = f_i$ (the empirical probability to move from i), the trace index is exactly equal to the probability that a generic firm will move from its starting class. Analogously, I_{up} and I_{down} are mean probabilities that a generic firm will respectively upsize or downsize.

More relevant is the interpretation of the Bartholomew's index I_b. Considering again the aforementioned 3×3 matrix P, for every i the function I_i is defined by

$$I_i(P_i) = p_{i1}|1-i| + p_{i2}|2-i| + p_{i3}|3-i|.$$

In the previous example the quantity $|j-i|$ measures exactly the amount of job places involved by an enterprise moving from i to j, counted without considering whether they are created or destroyed. Given i, the term $|j-i|$ is equal to 0, 1, or 2 with probability p_{ij}, thus $I_i(P_i)$ is the expected number of job places which "move" together with a generic firm starting from i, under the simplistic assumption that firms hire/fire only one or two employees at once. Lastly, $I_b(P)$ is the analogous mean number considering all the starting classes. In the previous example we obtain that every firm moves on average 0.467 job places. It is worth noting that I_b results to be always not negative but it has no upper bound, because every moving firm may move potentially more than one job place. On the other hand, I_{tr}, I_{up} and I_{down} are bounded by definition in $[0,1]$.

As a last step, we revise the directional index I_{dir} from the firms' mobility point of view. The quantity $sign(j-i) \cdot v(|j-i|)$ is here included, instead of $|j-i|$, where $v(|j-i|)$ is a generalized measure for the number of involved job places by firms moving from i to j, and $sign(j-i)$ indicates job creation if $j > i$ and job destruction if $j < i$. If we set $v(|j-i|) = |j-i|$ in the previous example, we obtain:

$$I_1(P_1) = p_{12} + 2p_{13} = 0.6; \quad I_2(P_2) = -p_{21} + p_{23} = 0; \quad I_3(P_3) = -2p_{31} - p_{32} = -0.4.$$

Thus, on average, a firm in the first class creates 0.6 job places and a firm in the last class destroys 0.4 job places. Note that firms in the intermediate class move and cause both job creation and destruction, which however cancel one another. The global index is $I_{dir} = 0.067$, which indicates a modest tendency to upsize, as signalled by the positive sign of the index, and every firm creates 0.067 job places on average, under the same simplistic assumption as before.

To conclude this Section, we propose the following concept of mobility regarding Firm Size:

A suitable mobility measure should quantify the tendency to leave the starting size class, together with the direction towards up/downsize and the number of job places created/destroyed.

With this aim we will focus on the indices able to measure the tendency to upsize or downsize: I_{up} and I_{down} as defined in Section 2. We also propose a modified version of such indices, which mixes together the original definition proposed by Bourguignon and Morrisson (2002) and the concept of ”directional mobility” introduced in Ferretti and Ganugi (2013):

$$\begin{cases} I^v_{up} = \sum_i f_i \sum_{j>i} p_{ij} v(|j-i|); \\ I^v_{down} = \sum_i f_i \sum_{j<i} p_{ij} v(|j-i|). \end{cases} \quad (3)$$

As explained before, v is a suitable function used to assign different weight to firms making movements of different magnitude (note that $|j-i|$ is the number of size classes crossed in moving from i to j). Formulas in Equation 3 mirror the concept of mobility we aim to use in the firm size analysis. In addition, being I_{up}, I_{down} and I_{tr} as defined in the previous section, it is easy to prove that:

1. $I_{up} + I_{down} = I_{tr}$, and $I^v_{up} + I^v_{down} = I^v_{tr}$, which is the trace index modified to take into account the number of job places;

2. $I_{up} - I_{down} = I_{dir}$, the directional index with $v \equiv 1$, and $I^v_{up} - I^v_{down} = I^v_{dir}$.

Hence, given v, it is worth noting that summing I^v_{up} and I^v_{down} we obtain a measure of the absolute mobility, without regarding at the direction, that recalls the *business churn* as defined in the Eurostat database (birth rate + death rate). On the other hand, subtracting I^v_{down} from I^v_{up} we obtain a net measure of the tendency to upsize or downsize, which can be considered as a mobility turnover (the *net turnover rate* is usually defined as birth rate - death rate).

4. Business Demography and Firms Mobility

Demographic events considered in a general way (appearing and disappearing individuals that can be persons, workers or firms) clearly represent main features to be analyzed in many research fields. Among others, we recall 1) the credit ratings dynamics and the Default probability estimation (Jafry and Schuermann, 2004; Violi, 2008; Ferretti et al., 2019); 2) the analysis of labour forces and the estimation of flows in and out the unemployment state (Gomes, 2012).

In the analysis of enterprises development Business Demography is relevant so that it is monitored yearly by many national statistical bureaus (see Business Demography Database, Eurostat or the Italian last official report ISTAT, 2019). Quoting the Eurostat main page about this topic: *”Business demography statistics present data on the active population of enterprises; their birth; survival ... and death. Special attention is paid to the impact of these demographic events on employment levels.”* In addition, they claim that *”Business demography data can be used to analyse ... the entrepreneurship in terms of the propensity to start a new business, or the contribution of newly-born enterprises to the creation of jobs.”*

Defining the mobility as the tendency to move and create job places we perfectly mirror the scope of the official statistics about Business Demography. We now revise the definition of TM in the presence of demographic events and we propose a way to merge together concepts coming from the official Business Demography and the analysis of mobility. Thus, together with the aforementioned Incumbents, we consider for every year t the enterprises which are active for the first time (*Newborns*) and the enterprises which are active for the last time (*Exiting* firms). In evaluating movements between t and $t+1$, demographic events are treated according to the seminal papers by Adelman (1958) and Schoen (1988): the additional category O is considered and, together with the aforementioned n_{ij}, we define the number n_{io} of deaths from $i=1,\dots,k$ (i.e. firms active at t and no longer active at $t+1$) and the number n_{oj} of births in $j=1,\dots,k$ (firms not active at t and active at $t+1$).

Crudely, the mobility could be measured on the augmented $(k+1)\times(k+1)$ TM $P^*=\{p^*_{ij}\}_{i,j=1,\dots,k,o}$ such that:

$$\begin{cases} p^*_{ij} = \frac{n_{ij}}{\sum_{l=1}^{k} n_{il}+n_{io}} = \text{ empirical probability to move towards } j, \text{ starting from } i; \\ p^*_{io} = \frac{n_{io}}{\sum_{l=1}^{k} n_{il}+n_{io}} = \text{ empirical probability to definitely exit from } i; \\ p^*_{oj} = \frac{n_{oj}}{\sum_{l=1}^{k} n_{ol}+n_{oo}} = \text{ empirical probability to newly enter in } j; \end{cases} \tag{4}$$

for every $i,j=1,\dots,k$.[6] The $(k+1)\times(k+1)$ matrices $N^*=\{n_{ij}\}_{i,j=1,\dots,k,o}$ and P^* indeed represent the usual and easy-to-handle way to consider births and deaths and to measure mobility in empirical applications (see Violi, 2008; Macchiarelli and Ward-Warmedinger, 2014, among others). Note that n_{ij} is not affected by entries or exits, for every couple of regular classes i and j, whereas the number of firms in every class at time t or $t+1$ is changed (for example, the number of firms leaving i is now equal to $\sum_{j=1}^{k} n_{ij}+n_{io}$).

In evaluating the mobility using P^* two main drawbacks arise. First, n_{oo} is often missing, being the non-observable number of potential newborns "waiting outside" (the unbiased estimation of the probability to be outside and to be born in the future is not trivial and goes beyond the scope of this work). Consequently, the values $p_{o1},\dots,p_{ok},p_{oo}$ may be biased and mislead the mobility measurement. Second, the outer state O is often not ordered with respect to the other categories $1,\dots,k$, and loses its exceptional nature if treated as a regular category. Hence, mobility measures evaluated on P^* may be ambiguous in the case of indices considering the ordering or the distance among categories, as in the firm size analysis. Hence, in the following paragraphs we will propose some modifications to avoid such drawbacks, with the aim to find the better choice for the weight ω^*_i and the function $I^*_i(\cdot)$, $i=1,\dots,k$, to obtain a mobility measure in line with the context of Business Demography.

Mobility and Newborn Enterprises Official statistics usually contain information about the starting number of employees in the cohort of enterprises newly born at time t and their gain/loss of employees at $t+s$ (often with s =1,2,3,4,5). We now remark that, according with Equations 1 and 4, the number of births between time t and time $t+1$ does not affect the values p^*_{ij}, if $i\neq O$. Instead, it will affect the future mobility because it increases the number of individuals moving at time $t+1$. Therefore, the O-th row $(p_{o1},\dots,p_{ok},p_{oo})$ of P^* is not useful for measuring the mobility: it indeed represents the percentage of births in every class with respect to the total amount of newborns, quantifying the "attractiveness" of a given class, rather than its impact on the mobility as a whole.

For this reason, we propose to mirror the official statistics and to divide active enterprises

[6]From now on, the superscript * will indicate objects such as TMs or mobility indices measured considering also the demographic events.

into *long-term Incumbents* (firms active at $t-1$, t, and $t+1$) and *Newborns* (firms not active at $t-1$ and active at t and $t+1$). Deaths are temporarily ignored. The following decomposition holds: let $n_i^*(t)$ be the total number of firms being active in i at time t, which includes both the long-term Incumbents arrived from the regular classes (indicated with $n_{li}(t-1)$) and the newly born enterprises which were not active at time $t-1$ ($n_{oi}(t-1)$). Consequently, the percentage of firms in i at time t is given by:

$$f_i^* = \frac{n_i^*(t)}{n(t)} = \frac{\sum_{l=1}^k n_{li}(t-1) + n_{oi}(t-1)}{n(t)} = f_{i,inc}^* + f_{i,new}^*,$$

where $n(t)$ is the total number of firms active at time t, $f_{i,inc}^*$ is the fraction composed by Incumbents and the remaining part $f_{i,new}^*$ regards Newborns. Following the Eurostat birth rate's definition (*"number of enterprise births in the reference period t divided by the number of enterprises active in t - percentage"*), we can write:

$$f_i^* = f_{i,inc}^* + \frac{n_{oi}(t-1)}{n(t)} = f_{i,inc}^* + \frac{n_i^*(t)}{n(t)}\frac{n_{oi}(t-1)}{n_i^*(t)} = f_{i,inc}^* + f_i^* b_i^*,$$

where $\frac{n_{oi}(t-1)}{n_i^*(t)} = b_i^*$ is by definition the birth rate in i at time t. Thus, assuming to have a suitable function $I^*(P_i^*)$ for every i, we propose to modify Equation 2 in the following way:

$$I^*(P^*) = \sum_{i=1}^k (f_{i,inc}^* + f_i^* b_i^*) \cdot I_i^*(P_i^*). \tag{5}$$

Mobility and Declining Enterprises We now temporarily discard Newborns and we consider Exiting firms, intended as firms which are active at time t and no longer active at time $t+1$. As noted before, calculating $I_i^*(P_i^*)$ we lump deaths from i with regular transitions, potentially losing the peculiar information provided by p_{io}^*.

To rightly consider mortality, we first note that the effect of *Exits* is quite different from the effect of Newborns, mainly because $p_{io}^* = \frac{n_{io}(t)}{n_i^*(t)}$ corresponds by definition to the Eurostat's death rate d_i^* in $i = 1, \dots, k$ (*"number of enterprise deaths in the reference period t divided by the number of enterprises active in t - percentage"*). Thus, the death rate is still comprised in the TM P^*. In addition, we observe that:

$$p_{ij}^* = \frac{n_{ij}(t)}{n_i^*(t)} = \frac{n_{ij}(t)}{\sum_{l=1}^k n_{il}(t) + n_{io}(t)} = \frac{n_{ij}(t)}{\sum_{l=1}^k n_{il}(t)} \frac{\sum_{l=1}^k n_{il}(t)}{\sum_{l=1}^k n_{il}(t) + n_{io}(t)} = p_{ij}(1 - p_{io}^*).$$

The same result is obtained through probabilistic facts, noting that p_{ij}^* is a conjoint probability that two events happen: 1) to not exit between t and $t+1$ and 2) to move towards j, given the starting state i. Analogously, p_{ij} is the (empirical) probability to move towards j, conditioned on both the starting state i and the survival until $t+1$. Consequently, under the same p_{ij}, the value p_{ij}^* increases when the death rate d_i^* decreases and vice-versa. Lastly, we observe that the function I^* used in the existing indices is in most cases linear with respect to its variables. Thus, to rightly consider the effect of mortality on the whole mobility, we propose the following substitution: instead of using P^*, we measure the mobility considering the sole-Incumbents $k \times k$ matrix P and we use the death rate as a rescaling factor. The resulting formula is:

$$I^*(P) = \sum_{i=1}^{k} (1 - d_i^*) \cdot I_i(P_i). \quad (6)$$

5. A measure of mobility including birth and death events

Merging Equations 5 and 6, we propose to measure mobility of enterprises using the following formula:

$$I^{bd}(P) = \sum_{i=1}^{k} (f_{i,inc}^* + f_i^* b_i^*)(1 - d_i^*) \cdot I_i(P_i), \quad (7)$$

where $I_i(P_i)$ can be retrieved from existing indices.

Equation 7 can be motivated in terms of probabilities and expected number of job places, in analogy with Section 3. Considering the trace index, and given the starting state i, only surviving firms contribute to the mobility among regular classes and their contribution is equal to the probability to move away from i, given the survival: $I_i = 1 - p_{ii}$. Such term is multiplied by the empirical probability to be a firm moving from i at t and surviving until $t+1$: $f_i^*(1 - p_{io}^*)(1 - p_{ii})$. The sum over i finally provides the global probability to leave the current size class. The same explanation holds if we are interested in the mean number of job places created/destroyed, adding a suitable measure v.

5.1. Main properties

To be a suitable descriptive measure, a generic index I is required to satisfy some properties, such as those described in Shorrocks (1978), Ferretti and Ganugi (2013) and Paul (2019). We first highlight the fact that the proposed index does not require the knowledge of the individual path of every enterprise under analysis, which are often not publicly available. It can indeed be calculated only using the transition matrix and the birth and death rates, possibly retrieved from different sources.

More rigorous properties are analyzed in the following list, reminding that the term I_i derives from an existing decomposable measure such as the trace index. Hereon we will specifically refer to the up/down mobility measures I_{up}^{bd} and I_{down}^{bd} obtained by setting I_i as displayed in Equation 3.

P1) *Immobility:* $I(Id) = 0$. Immobility requires to choose I so that the identity matrix Id, which obviously describes the absence of any type of movement, corresponds to the value 0 of mobility. Given the i-th row e_i of Id, we have $I_i(e_i) = 0$ for both I_{up}^v and I_{down}^v. Immobility is consequently proved.

P2) *Boundedness:* $I(P) \leq M$ for every TM P. Boundedness is related to the existence of a TM P corresponding to the case of maximum mobility M. For example when we measure the tendency to upsize, mobility is maximal if all the firms start from the lowest size class and jump immediately in the highest class. The maximal TM thus exists for I_{up}^{bd}, as well as for I_{down}^{bd}, and can be retrieved as suggested in Ferretti and

Ganugi (2013). Nevertheless, given two groups A and B of enterprises, it is more relevant to compare $I(P_A)$ with $I(P_B)$ instead of evaluating their distance from the maximum mobility scenario, which is quite unrealistic.

P3) *Normalization:* $0 \leq I(P) \leq 1$, for every TM P. Normalization is useful to express any mobility index as a percentage, improving its readiness. If we consider mobility as the pure probability to up/downsize, the mobility index is normalized by definition. Otherwise, as said before, I^{bd}_{up} and I^{bd}_{down} are bounded and consequently they could be re-scaled to satisfy normalization. Nevertheless, such indices can be used to measure the mean number of job places created/destroyed, which may reasonably be greater than one: therefore, normalization can be discarded without consequences.

5.2. Additional specific properties

By construction, I^{bd}_{up} and I^{bd}_{down} are equipped with some additional properties as listed in the following:

A1) *Newborns make the mobility to increase.* If the birth rate is equal to zero then $f^*_i = f^*_{i,inc}$, otherwise $f^*_i = f^*_{i,inc} + f^*_i b^*_i > f^*_{inc}$. Thus, it proved that Newborns bring a gain in the mobility, because they increase the number of firms potentially moving from every size class.

A2) *Exits make the mobility to decrease.* In analogy with Newborns, we observe that if the death rate is greater than zero then $f^*_i(1-d^*_i) < f^*_i$. Thus, deaths cause a loss in the mobility because they reduce the number of transitions among the regular size classes.

A3) *Weights do not sum up to one.* In Equation 7 we use the peculiar weights $\omega^*_i = (f^*_{i,inc} + f^*_i b^*_i)(1-d^*_i)$ such that $\sum_i \omega^*_i = 1 - \sum_i f^*_i d^*_i \leq 1$. Obviously, it is possible to substitute ω^*_i with $\frac{\omega^*_i}{1-\sum_i f^*_i d^*_i}$. Nevertheless, reminding that the considered indices are defined to be the empirical probability to upsize/downsize (if $v \equiv 1$) or the expected number of job places created/destroyed, in our opinion the weights normalization is not useful to improve the index readiness. In addition, the missing fraction $\sum_i f^*_i d^*_i$ is related to the loss caused by exiting enterprises, as explained by property A2.

A4) *The index is decomposable in terms of births and deaths effects.* Indeed, we can write:

$$I^{bd}(P) = \sum_{i=1}^{k} (f^*_{i,inc} + f^*_i b^*_i)(1-d^*_i) \cdot I_i(P_i) = M_{inc} + M_{new} + M_{ex} + M_{newex}.$$

In details:

- $M_{inc} = \sum_{i=1}^{k} f^*_{i,inc} \cdot I_i(P_i)$ is the mobility due to the long-term Incumbents (enterprises active at $t-1$, t and $t+1$);
- $M_{new} = \sum_{i=1}^{k} f^*_i b^*_i \cdot I_i(P_i)$ is the gain in the mobility due to Newborns (enterprises active at t and $t+1$);

- $M_{ex} = -\sum_{i=1}^{k} f_{i,inc}^{*} d_i^{*} \cdot I_i(P_i)$ is the loss in the mobility due to exiting Incumbents (enterprises active at $t-1$ and t);
- $M_{newex} = -\sum_{i=1}^{k} f_i^{*} b_i^{*} d_i^{*} \cdot I_i(P_i)$ is the loss due to immediately exiting Newborns (enterprises active only at t).

Note that the above decomposition is meaningful is we use the not-normalized weights ω_i^{*}, as explained by property A3. Lastly, the same decomposition can be based also on a wider time window, considering for example transitions between t and $t+5$, instead of $t+1$, with the aim to mirror the official statistics, which often provide information about the cohort of Newborns after five years.

6. The mobility of Italian capital-owned manufacturing firms

6.1. Data description

Our source is the AIDA database by Bureau van Dijk (https://aida.bvdinfo.com), which collects annual accounts of the Italian enterprises, with a strong prevalence of capital-owned firms. To our knowledge, no other databases are publicly available to observe firms transitions, thus we will focus the analysis on this specific legal form. The last release covers the years 2009 – 2018, and we discard the first and the last year, which are potentially biased. In addition, to reduce the computational effort and for the sake of homogeneity, we select only firms belonging to sectors B - E in NACE Rev. 2 (Industry except Construction) which is considered separately also by the Italian Statistical Office in the annual report (see ISTAT, 2019). Data are cleaned considering only firms having a defined number of employees for every year of activity. As further check, we also exclude enterprises with missing values in the Annual Turnover or in the Balance Sheet Total. Lastly, we discard reactivated firms and firms that ceased by merging or division, with the aim to reproduce the official technique for identifying true births and deaths.

The resulting dataset covers eight years 2010 – 2017 and 52937 enterprises. To make possible the comparison with the official indicators, size categories are defined according to the Eurostat definition in terms of employees: no employees, from 1 to 4, from 5 to 9, more than 10 employees (see Business Demography Database, Eurostat). The observed numbers of transitions are displayed in Table 8 in the Appendix. Unfortunately, we note that enterprises with no employees are severely under-represented in the AIDA dataset with respect to the official data, given that they cover around 14% of the set against 40% on average resulting from the Eurostat database (see section (b) of Table 8). To avoid drawbacks, we thus choose to work with the subset of *Employer Enterprises*, defined as firms with at least one employee, as explained in Eurostat-OECD manual (2007). According to the same manual:

- *“an Employer Enterprise Birth occurs either as an enterprise birth with at least one employee in the year of birth, or as an entry by growth reaching the threshold of one employee”*;
- *“an Employee Enterprise Death occurs either as an enterprise death with at least one employee in the year of death or as an exit by decline, moving below the threshold of one employee”*.

Table 1 contains the main indicators about the employer enterprises. Panel A displays the annual fraction of firms moving from each size category. We can see that, except for the first two years, the percentage of "1 – 4" firms (51% on average) is similar to official data (56%), whereas percentages regarding the other two classes are reversed, given that "5 – 9" firms represent on average 31% of the AIDA set and less than 20% in the Eurostat data.

Panels C and D contain the annual birth and death rates. Years 2010 and 2011 show exceptional and not explainable values which are consequently excluded in evaluating the average with respect of time. In the remaining years, capital-owned firms result to have a different demography with respect to Eurostat data, being the birth rate very high for small firms and generally higher than the death rate. In opposition, the official Italian turnover rate is always negative, as revealed by the Eurostat's averages (see also ISTAT, 2016, which provides rates disaggregated by size class). Consequently, if only the birth and death rates are considered, Italian capital-owned enterprises seem to enjoy good health, and the economic crisis seems to solely cause the negative turnover of the largest firms since 2012. Reminding that we are working with a specific set of firms (capital-owned employer enterprises belonging to Industry except Constructions), the analysis of the determinants of this quite surprising behaviour goes beyond the scope of this work.

Table 1: Main indicators for the Italian capital-owner Employer Enterprises.

	2010	2011	2012	2013	2014	2015	2016	2017	Average	EUROSTAT^{+}
	PANEL A: Percentage of firms by size class									
From 1 to 4	44.60%	47.99%	49.61%	51.67%	52.48%	52.39%	53.62%	53.40%	50.72%	55.81%
From 5 to 9	28.13%	33.00%	32.21%	30.65%	30.08%	30.89%	29.92%	30.49%	30.67%	19.31%
10 or more	27.28%	19.02%	18.17%	17.69%	17.43%	16.73%	16.46%	16.11%	18.61%	24.88%
Tot Firms	17777	29994	31360	31731	32476	34225	35165	36454	31148	257407
	PANEL B: Mean number of employees by size class									
From 1 to 4	2.44	2.47	2.43	2.41	2.38	2.39	2.37	2.37	2.41	1.83
From 5 to 9	6.48	6.48	6.44	6.38	6.36	6.37	6.37	6.41	6.41	6.63
10 or more	46.67	42.01	42.48	42.72	42.38	42.49	43.05	43.20	42.62	46.11
Tot Employees	278034	339347	344984	341366	342697	353389	360946	371102	350547	3520079
	PANEL C: Birth rate by size class									
From 1 to 4	20.17%	53.79%	12.45%	12.24%	14.73%	17.60%	13.13%	12.93%	13.85%*	9.35%
From 5 to 9	18.20%	44.97%	4.33%	5.18%	5.52%	7.56%	4.55%	4.96%	5.35%*	1.90%
10 or more	5.26%	14.78%	1.74%	1.51%	1.78%	1.76%	0.97%	0.95%	1.45%*	0.75%
TOT	15.55%	43.46%	7.89%	8.18%	9.70%	11.85%	8.56%	8.57%	9.12%*	5.78%
	PANEL D: Death rate by size class									
From 1 to 4	7.06%	5.32%	10.39%	11.28%	10.55%	9.16%	7.89%	9.07%	9.72%*	10.90%
From 5 to 9	4.40%	2.80%	4.85%	4.67%	4.10%	3.24%	2.60%	2.43%	3.65%*	2.53%
10 or more	0.80%	1.12%	2.05%	1.84%	1.89%	1.48%	1.28%	1.14%	1.61%*	1.13%
TOT	4.61%	3.69%	7.09%	7.58%	7.10%	6.05%	5.22%	5.77%	6.47%*	6.86%
	PANEL E: Net Turnover (birth rate - death rate)									
From 1 to 4	13.11%	48.47%	2.06%	0.96%	4.18%	8.44%	5.25%	3.86%	4.12%*	-1.54%
From 5 to 9	13.80%	42.17%	-0.52%	0.51%	1.41%	4.31%	1.95%	2.53%	1.70%*	-0.63%
10 or more	4.45%	13.66%	-0.32%	-0.32%	-0.11%	0.28%	-0.31%	-0.19%	-1.61%*	-0.38%
TOT	10.94%	39.77%	0.79%	0.60%	2.60%	5.80%	3.34%	2.80%	2.66%*	-1.08%

Source: Own calculations on the AIDA data.* Averages are calculated excluding 2010 and 2011.

$^{+}$Source: Eurostat's database "Employer Business Demography by size class (from 2004 onwards, NACE Rev. 2)"

7. Results

We finally measure the mobility in the set under analysis. Reminding the definition of mobility in the firm size framework, and Equations 3 and 7, we propose to measure the tendency to upsize and create job places in the following way:

$$I_{up}^{bd} = \sum_{i=1}^{k} (f_{i,inc}^* + f_i^* b_i^*)(1 - d_i^*) \sum_{j>i} p_{ij} v(|j-i|).$$

As a first step we set $v(|j-i|) \equiv 1$ to evaluate the probability to move upward. As a second step, in line with the example proposed in Section 3, we choose a suitable v to measure the effort in terms of job places required to upsize. In particular, we propose to define:

$$v(|j-i|) = |\text{(mean number of employees in } j\text{) - (mean number of employees in } i\text{)}|.$$

To better explain this choice, consider for example enterprises in the first and in the second size class. On average, they employ respectively 2.41 and 6.41 workers (see Panel B in Table 1). Thus, the mean distance, intended as the size difference, between firms in the first and in the second class is equal to 4. In other words, a firm in the first class is required to hire four employees on average, to move to the second class. Analogous facts hold for I_{down}^{bd}.

Figure 1 shows the results of the annual mobility indices (the anomalous values in 2010 – 2011 are discarded). Values are equipped with standard errors obtained simulating 1000 bootstrapped copies of the same set (the index sampling distribution results to be Gaussian as proved in Ferretti, 2014). In the left panel we note that the probability to move towards every direction among size classes is very low, being both the upward and the downward indices not higher that 5%. In particular, the probability to move downward is predominant for active enterprises in 2012, which was the year of economic crisis in Italy (Il Sole 24 Ore, 2013). The same negative peak is reached in the right panel, followed by a recovery during which the mean number of job places created per firm (around 0.4) is neatly higher than the amount of job places destroyed (lower than 0.3). Note also that in 2015 firms have equal probability to upsize or downsize, but upsizing firms involve more job places than the downsizing ones.

From now on we will focus only the concept of mobility intended as the tendency to create or destroy job places. Table 2 contains the mobility indices I_{up}^{bd} and I_{down}^{bd} with v defined as the mean difference between size classes, decomposed by size category. For every category we also evaluate the corresponding contribution as a percentage of the whole mobility. Note that being aggregated in a unique class, largest firms cannot become even larger and contribute to the upward mobility, as well as smallest enterprises cannot contribute to the downward mobility. Considering the mobility turnover, defined as (job places created - job places destroyed), it is worth noting that it mirrors the turnover between the birth rate and the death rate, in the sense that it is positive for the first two size classes, and negative for the largest enterprises. Nevertheless, the total turnover is negative in 2012 – 2013, as revealed by Figure 1.

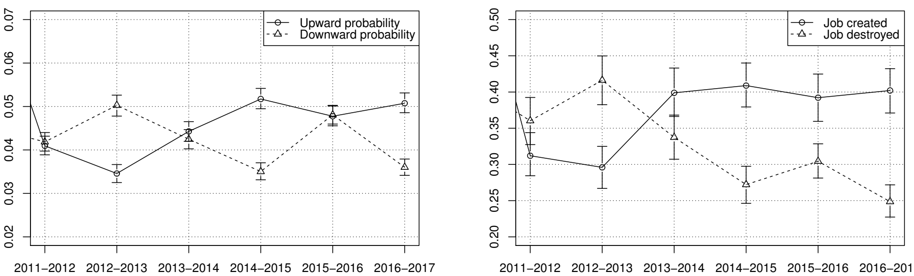

Figure 1: Mobility of Italian capital-owner Employer Enterprises recorded in AIDA. Left panel: yearly probability to move upward (solid line) or downward (dotted line). Right panel: yearly mean number of job places created (solid line) or destroyed (dotted line) per firm.

Table 2: Mean number of job places created and destroyed per firm, disaggregated by size category. In italic, the corresponding contribution as a percentage of the whole mobility.

No. of employees	2010 – 11	2011– 12	2012 – 13	2013 – 14	2014 – 15	2015 – 16	2016 – 17	Average
	PANEL A: Job places created on average per firm							
From 1 to 4	0.3850	0.1650	0.1586	0.1968	0.2050	0.1953	0.1985	0.1865*
	56.08%	*52.91%*	*53.66%*	*49.29%*	*50.18%*	*49.88%*	*49.37%*	*50.67%*
From 5 to 9	0.3015	0.1469	0.1370	0.2025	0.2035	0.1963	0.2036	0.1816*
	43.92%	*47.09%*	*46.34%*	*50.71%*	*49.82%*	*50.12%*	*50.63%*	*49.33%*
10 or more	-	-	-	-	-	-	-	-
TOT	0.6865	0.3119	0.2955	0.3993	0.4084	0.3916	0.4021	0.3681*
	100.00%	*100.00%*	*100.00%*	*100.00%*	*100.00%*	*100.00%*	*100.00%*	*100.00%*
	PANEL B: Job places destroyed on average per firm							
From 1 to 4	-	-	-	-	-	-	-	-
From 5 to 9	0.1568	0.1442	0.1751	0.1498	0.1242	0.1789	0.1315	0.1506*
	37.35%	*40.02%*	*42.02%*	*44.34%*	*45.61%*	*58.92%*	*52.97%*	*46.61%*
10 or more	0.2630	0.2160	0.2417	0.1880	0.1480	0.1248	0.1168	0.1725*
	62.65%	*59.98%*	*57.98%*	*55.66%*	*54.39%*	*41.08%*	*47.03%*	*53.39%*
TOT	0.4199	0.3602	0.4168	0.3377	0.2722	0.3037	0.2483	0.3231*
	100.00%	*100.00%*	*100.00%*	*100.00%*	*100.00%*	*100.00%*	*100.00%*	*100.00%*
	PANEL C: Net turnover (job places created - job places destroyed)							
From 1 to 4	0.3850	0.1650	0.1586	0.1968	0.2050	0.1953	0.1985	0.1865*
From 5 to 9	0.1447	0.0027	-0.0382	0.0527	0.0793	0.0174	0.0721	0.0310*
10 or more	-0.2630	-0.2160	-0.2417	-0.1880	-0.1480	-0.1248	-0.1168	-0.1725*
TOT	0.2666	-0.0483	-0.1213	0.0615	0.1363	0.0879	0.1538	0.0450*

* Averages are calculated excluding 2010 – 2011.

Lastly, Table 3 shows the main property of the proposed mobility index: the decomposition in terms of births and deaths. It is relevant that the 2012's negative turnover between job places created and destroyed is mainly caused by Incumbents, which hire (resp. fire) 0.29 (resp. 0.41) employees per firm. On the other hand, newborn firms show a certain liveliness for the whole considered period, creating 0.06 job places per firm on average, against the 0.02 job places destroyed. Nevertheless, their contribution is very small, reflecting the fact that Newborns have to wait some years to become capable to create job places. In addition, until 2014 they cannot withstand the loss due to exiting enterprises. It is also worth noting that exits burden more on the tendency to upsize than the opposite tendency, causing a loss of 0.02 on average in the upsize mobility and of 0.008 in the downsize mobility. These last values can be interpreted as the mobility that exiting enterprises would have had if they had survived. Last, the loss due to immediately exiting firms is negligible.

Table 3: Upsize and downsize mobility decomposition in terms of Incumbents, Newborns and Exiting enterprises.

	2010 – 11	2011– 12	2012 – 13	2013 – 14	2014 – 15	2015 – 16	2016 – 17	Average
	Decomposition of the Upward Mobility							
M_{inc}	0.5887	0.1637	0.2927	0.3961	0.3959	0.3647	0.3867	0.3333*
M_{new}	0.1410	0.1617	0.0283	0.0382	0.0455	0.0532	0.0378	0.0608*
M_{ex}	-0.0347	-0.0066	-0.0228	-0.0314	-0.0289	-0.0223	-0.0200	-0.0220*
M_{newex}	-0.0084	-0.0069	-0.0026	-0.0036	-0.0040	-0.0040	-0.0025	-0.0039*
	Decomposition of the Downward Mobility							
M_{inc}	0.3854	0.2678	0.4186	0.3375	0.2705	0.2954	0.2460	0.3173*
M_{new}	0.0438	0.0990	0.0122	0.0110	0.0098	0.0162	0.0073	0.0285*
M_{ex}	-0.0079	-0.0044	-0.0135	-0.0104	-0.0078	-0.0074	-0.0049	-0.0080*
M_{newex}	-0.0014	-0.0022	-0.0005	-0.0004	-0.0003	-0.0005	-0.0002	-0.0008*

* Averages are calculated excluding 2010 – 2011.

8. Conclusion and discussion

Here, we propose a descriptive index for measuring the mobility in an evolving set of enterprises taking into account the impact of demographic events. Firms are subdivided into k size classes, and we define "mobility" as the tendency to upsize (or downsize) together with the capability to create (or destroy) job places: in this sense the proposed index is flexible enough and can be used for measuring both the probability to upsize/downsize and the expected number of job places created/destroyed. The index is characterized by two main parameters: the weights used to measure the contribution of every size class to the mobility as a whole, and the function which quantifies the distance among size classes. Here, we mathematically prove that weights can be chosen as functions of the birth and death rates, which consequently make the mobility respectively increase and decrease. We also propose to measure the distance among size classes as the mean difference in the number of employees. As a case study, we measure the upward and downward mobility of a set of Italian capital-owned Employer Enterprises in the years 2010 – 2017. Results show that if only birth and death rates are considered, the effect of the economic crisis is not perceived, being the turnover always positive, in contrast with the official results, which are used as benchmark. On the other hand, the new index furnishes a description which correctly mirrors the mainstream idea about the effect of the economic crisis in the years 2010 - 2017.

This work is an initial attempt to formalize the effect of birth/death events on the degree of mobility of an evolving set, using transition matrices. We do not aim to analyze the determinants of the observed mobility values, rather we try to propose an alternative tool for practitioners to better measure the mobility in the presence of demographic events. Generally speaking, the construction of a mobility indicator depends on two main issues: 1) the field of application and 2) the specific feature we are interested in (turbulence, prevailing direction, speed of convergence toward an equilibrium, if it exists, see Ferretti, 2012 for more details). For example, in Geweke et al. (1986) time-continuous economic variables are considered, together with an underlying Markov Chain model; in Ferretti and Ganugi (2013) the proposed mobility measure is thought to grasp both the direction and the distance covered moving from the i-th to j-th category; lastly, Paul (2019) refers to the case of income mobility and attaches the highest weight to mobility of poorest workers. Here, we aim to build a suitable tool for empirical applications: with this in mind we avoid specific assumptions about any possible underlying model ruling the transitions among categories. Furthermore, as explained before, our proposal is based on the fundamental idea that mobility strictly depends on the number of moving individuals: consequently, births and deaths represent respectively a gain and a loss in the whole degree of mobility. In addition, we suppose that individuals are independent one from each other in terms of birth's, death's and transition's probabilities, and that the newborns effect on the mobility is one-year lagged. Such hypotheses are considered valid having in mind the firm size dynamics. For future research they may be questioned and generalized to other relevant issues, for example considering the interaction and the competition among firms. In addition, possible further developments will consider a theoretical description of the birth and death probabilities through suitable probabilistic models, as in Duncan and Lin (1972), and a robustness/inferential analysis of the proposed index.

References

ADELMAN, I., (1958). A stochastic analysis of the size distribution of firms. J Am Stat Ass, 53(284), pp. 893–904.

BARTHOLOMEW, D. J., (1982). Stochastic Models for Social Processes, 3rd ed. London, Wiley.

BOURGUIGNON, F., MORRISSON, C., (2002). Inequality among World Citizens. Am Econ Rev, 92(4), pp. 727–744.

EUROSTAT BUSINESS DEMOGRAPHY, retrieved from https://ec.europa.eu/eurostat/web/structural-business-statistics/business-demography, accessed May, 18, 2020.

CHEN, Y., COWELL F., (2017). Mobility in China. Rev Income Wealth, 63, pp. 203–218.

COWELL, F., FLACHAIRE E., (2018) Measuring Mobility. Quant Econ, 9, pp. 865–901.

DUNCAN G., LIN L., (1972). Inference for Markov chains having stochastic entry and exit. J Am Stat Ass, 67(340), pp. 761–767.

EUROSTAT – OECD, (2007). Manual on Business Demography Statistics, Luxembourg: Office for Official Publications of the European Communities, ISBN: 978-92-79-04726-8.

FERRETTI, C., (2012). Mobility as prevailing direction for transition matrices. Stat Appl, 10(1), pp. 67–79 .

FERRETTI, C., (2014). Sampling properties of the directional mobility index and the income of Italian families. Statistica, 4, pp. 455–466.

FERRETTI, C., GANUGI, P., (2013). A new mobility index for transition matrices. Stat Method Appl, 22(3), pp. 403–425.

FERRETTI, C., GABBI, G., GANUGI, P., SIST, F., VOZZELLA, P., (2019). Credit Risk Migration and Economic Cycles. Risks, 7(109), pp. 1–18.

FIELDS, G., OK, E., (1996). The Meaning and Measurement of Income Mobility. J Econ Th, 71(2), pp. 349–377.

FORMBY, J.P., SMITH, W.J., ZHENG, B., (2004). Mobility measurement, transition matrices and statistical inference. J Econometrics, 120, pp. 181–205.

GEWEKE, J., MARSHALL, R., ZARKIN, G., (1986). Mobility Indices in Continuous Time Markov Chains. Econometrica, 54(6), pp. 1407–1423.

GOMES, P., (2012). Labour Market Flows: Facts from the United Kingdom. Labour Economics, 19(2), pp. 165–175.

ISTAT, (2016). Demografia d'impresa 2009–2014. Tech. rep., Centro diffusione dati.

ISTAT, (2019). Demografia d'impresa 2012–2017. Tech. rep., Centro diffusione dati.

IL SOLE 24 ORE, 26th March 2013, https://st.ilsole24ore.com/art/notizie/2013-03-26/italia-paese-colpito-crisi-123024.shtml#comments.

JAFRY, Y., SCHUERMANN, T., (2004). Measurement, estimation and comparison of credit migration matrices. J Bank Financ, 28(11), pp. 2603–2639.

MACCHIARELLI, C., WARD-WARMENDINGER, M., (2014). Transitions in labour market status in EU labour markets. IZA J Eur Labour Stud, 3(1), pp. 1–17.

PAUL, S., (2019). A measure of Income Mobility based on Transition Matrices and application to China and the United States. J Asia Pacific Econ., 25(3), pp. 389–401.

PRAIS, S. J., (1955). Measuring social mobility. J Roy Stat Soc, Series A, part I, 188, pp. 56–66.

SHOEN, R. (1988). Practical uses of Multistate Population Models. Ann Rev Sociol, 14, pp. 341–361.

SCHLUTER, C., (1998). Statistical inference with mobility indices. Econ Lett, 59, pp. 157–162.

SHORROCKS, A., (1978). The measurement of Mobility. Econometrica, 46(5), pp. 1013 – 1024.

SHORROCKS, A., (1982). On the distance between income distributions [inequality measures between income distributions with applications]. Econometrica 50(5), pp. 1337–1339.

VIOLI, R., (2008). Credit Ratings Transition in Structured Finance. J Financ Transf, 22, pp. 1–36.

APPENDIX

A. AIDA's transition matrices

Table A: (a) Observed transitions among size classes and births/deaths per size class in the AIDA dataset. (b) AIDA's and Eurostat's initial frequencies per size class. (c) Transition matrix for the subset of incumbent Employer Enterprises.

	(a)						(b)		(c)		
	No emp.	From 1 to 4	From 5 to 9	10 or more	Deaths	TOT	Obs. %	Eurostat %[+]	TM for employer enterprises		
2010 – 2011											
No employees	2946	6968	4140	794	409	15257	46.19%	40.63%			
From 1 to 4	282	5923	1416	29	278	7928	24.00%	32.74%	80.39%	19.22%	0.39%
From 5 to 9	88	697	3937	146	132	5000	15.14%	11.65%	14.58%	82.36%	3.05%
10 or more	12	31	93	4686	27	4849	14.68%	14.97%	0.64%	1.93%	97.42%
Births	905	774	311	49	17864	19903					
TOT	4233	14393	9897	5704	18710	52937					
2011 – 2012											
No employees	2632	1198	175	33	195	4233	12.81%	39.57%			
From 1 to 4	370	12518	1095	14	396	14393	43.57%	33.97%	91.86%	8.04%	0.10%
From 5 to 9	62	1081	8419	120	215	9897	29.96%	11.61%	11.24%	87.52%	1.25%
10 or more	16	23	151	5466	48	5704	17.27%	14.86%	0.41%	2.68%	96.91%
Births	877	739	262	66	16766	18710					
TOT	3957	15559	10102	5699	17620	52937					
2012 – 2013											
No employees	2570	999	155	36	197	3957	11.98%	39.68%			
From 1 to 4	846	12974	938	30	771	15559	47.10%	34.16%	93.06%	6.73%	0.22%
From 5 to 9	143	1373	8122	117	347	10102	30.58%	11.53%	14.28%	84.50%	1.22%
10 or more	29	41	161	5380	88	5699	17.25%	14.64%	0.73%	2.88%	96.38%
Births	1078	1007	349	49	15137	17620					
TOT	4666	16394	9725	5612	16540	52937					
2013 – 2014											
No employees	2831	1301	168	38	328	4666	14.12%	40.71%			
From 1 to 4	999	13314	1195	36	850	16394	49.63%	33.67%	91.54%	8.22%	0.25%
From 5 to 9	138	1188	7908	175	316	9725	29.44%	11.25%	12.81%	85.30%	1.89%
10 or more	31	32	127	5350	72	5612	16.99%	14.36%	0.58%	2.31%	97.11%
Births	1170	1210	371	63	13726	16540					
TOT	5169	17045	9769	5662	15292	52937					
2014 – 2015											
No employees	2741	1669	270	44	445	5169	15.65%	40.96%			
From 1 to 4	1003	13747	1481	18	796	17045	51.60%	33.20%	90.17%	9.71%	0.12%
From 5 to 9	119	1008	8180	180	282	9769	29.57%	11.32%	10.76%	87.32%	1.92%
10 or more	27	18	111	5426	80	5662	17.14%	14.52%	0.32%	2.00%	97.68%
Births	799	1487	529	57	12420	15292					
TOT	4689	17929	10571	5725	14023	52937					
2015 – 2016											
No employees	2852	1175	136	21	505	4689	14.19%	40.06%			
From 1 to 4	900	14835	1427	24	743	17929	54.27%	33.88%	91.09%	8.76%	0.15%
From 5 to 9	94	1531	8514	183	249	10571	32.00%	11.37%	14.97%	83.24%	1.79%
10 or more	28	12	103	5525	57	5725	17.33%	14.69%	0.21%	1.83%	97.96%
Births	911	1301	343	35	11433	14023					
TOT	4785	18854	10523	5788	12987	52937					
2016 – 2017											
No employees	2815	1226	184	26	534	4785	14.49%	40.93%			
From 1 to 4	816	15778	1572	17	671	18854	57.07%	32.37%	90.85%	9.05%	0.10%
From 5 to 9	69	1156	8898	195	205	10523	31.86%	11.61%	11.28%	86.82%	1.90%
10 or more	17	17	93	5604	57	5788	17.52%	15.08%	0.30%	1.63%	98.07%
Births	852	1291	367	30	10447	12987					
TOT	4569	19468	11114	5872	11914	52937					

Source: Own calculation on AIDA's data 2010 – 2017

[+]Source: Eurostat database "Business Demography by size class (from 2004 onward, NACE Rev. 2)"