STATISTICS IN TRANSITION new series, December 2021
Vol. 22, No. 4 pp. 101–119, DOI 10.21307/stattrans-2021-040
Received – 26.08.2020; accepted – 16.02.2021

A study on exponentiated Gompertz distribution under Bayesian discipline using informative priors

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ABSTRACT

The exponentiated Gompertz (EGZ) distribution has been recently used in almost all areas of human endeavours, starting from modelling lifetime data to cancer treatment. Various applications and properties of the EGZ distribution are provided by Anis and De (2020). This paper explores the important properties of the EGZ distribution under Bayesian discipline using two informative priors: the Gamma Prior (GP) and the Inverse Levy Prior (ILP). This is done in the framework of five selected loss functions. The findings show that the two best loss functions are the Weighted Balance Loss Function (WBLF) and the Quadratic Loss Function (QLF). The usefulness of the model is illustrated by the use of real-life data in relation to simulated data. The empirical results of the comparison are presented through a graphical illustration of the posterior distributions.

Key words: exponentiated Gompertz distribution, loss functions, informative priors, Bayes estimators, posterior risks.

1. Introduction

The Gompertz distribution was named after Benjamin Gompertz. It is an exponentially increasing, continuous probability distribution. Exponentiated Gompertz (EGZ) distribution is basically a truncated extreme value distribution (Johnson et al. 1994 and Chaturvedi et al. 2012) which ranges from zero to positive infinity. In early days EGZ was used in the area of insurance to measure life expectancy and human mortality rates with a range of 0 to ≅ 100. However, recently these distributions have been used in a wide range of other applications in various areas of human endeavours, including risk management, economic and finance, cancer treatment, medical and biological sciences and demography. Recently, the various

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applications and properties of EGZ distribution have been studied by Chaturvedi et al. (2000), Abu-Zinadah et al. (2017), Hoseinzadeh et al. (2019), Mazucheli et al. (2019), Alrajhi et al. (2020), Dey et al. (2018), Leren et al. (2019), Anis and De (2020), Anis (2020), Jha et al. (2020), Shrivastava et al. (2019) and Obeidat et al. (2020), among others. For example, Abu-Zinadah et al. (2017), Dey et al. (2018) developed some theoretical properties of EGZ, which are being used by economists, financiers and practitioners. For example, Jha et al. (2020) applied EGZ distribution in reliability whereas Hoseinzadeh et al. (2019) employed in financial markets and risk management. Moreover, Mazucheli et al. (2019) introduced the unit-Gompertz (UG) distribution and studied some important properties. Anis and De (2020) pointed out a flaw of an error term in Mazucheli et al.'s (2019) paper and proposed a new type of UG-distribution with additional interesting properties. Moreover, Alraihi et al. (2020) tackled hybrid censored sample issue of complexity in a fuzzy system and artificial intelligence, whereas Leren et al. (2020) applied EGZ model-based distribution to bladder cancer patient's data and observed interesting properties in bioinformatics.

In some early studies, El-Gohary et al. (2013) suggested EGZ's interesting properties. Sherpiency et al. (2013) introduced a new distribution called bivariate generalized Gompertz (BGG) distribution, whose marginals are generalized Gompertz distributions (GGD) and discussed some of its properties. Zinadah and Oufi (2014) studied the EGZ distribution and its properties like, quantiles, median, mode, mean residual lifetime, mean deviations, Rényi entropy, density, survival and hazard functions were derived. Zinadah (2014a) derived the expressions for reliability and failure rate functions of the EGZ distribution. Saraçoğlu et al. (2014) considered the Maximum Likelihood Estimators (MLE) and Bayes Estimators (BE) for unknown parameters of GGD. Moreover, Zinadah (2014) also worked on three goodness of fit test statistics, namely Kolmogorov Smirnov (KS), Anderson Darling (AD) and Cramer Von Mises (CVM) for EGZ distribution utilizing complete and type-II censored data. Jafari et al. (2014) introduced a new four parameter generalized version of Gompertz distribution called Beta-Gompertz (BG) distribution. Zinadah (2014) examined the EGZ distribution, for estimating the shape parameter θ, considering five different estimation methods. Namely Maximum Likelihood method, method of Moments, method of Percentiles, Least Square method and Weighted Least Square method. Damcese et al. (2015) demonstrated a new lifetime model called Odd Generalized Exponential Gompertz (OGE-G) distribution.

Furthermore, an important work of Zinadah and Oufi (2016) on the four estimators, namely: ML, Least Squares (LS), Weighted Least Squares (WLS), and Percentiles (PC) for the EGZ distribution is some extra contribution in the literature. Cordeiro et al. (2016) investigated a new distribution called Exponentiated Gompertz Generated (EGG) distribution. Bassiouny et al. (2017) proposed a new model, namely

Exponentiated Generalized Weibull-Gompertz (EGWG) distribution. Ade et al. (2017) developed a distribution known as Generalized Exponentiated Gompertz Makeham (EGGM) distribution, consisting of five parameters. Bakouch et al. (2017) introduced a new distribution called the Weighted Gompertz (WGO) distribution.

From the above studies one can see that the literature review revealed that none of the authors have worked on attaining BEs and PRs of the EGZ distribution and have only studied its properties. Hence, this paper is an attempt to fill this gap in the literature. It attempts to analyze the unknown shape parameter of the EGZ distribution. The rest of the paper is organized as follows. In Section 2, we define the pdf of EGZ Distribution and derive its likelihood function. In Section 3, analysis is done on the unknown shape parameter of EGZ Distribution when the rest of the parameters are known. We have derived its posterior distribution, Bayes Estimators and PRs utilizing various loss functions. It considers GP and ILP which are contemplated as informative priors to acquiring the posterior distribution. In Section 4, simulation study is conducted, and comparison of the estimates is presented along with graphical illustration. A real life data set is considered for the analysis purpose in Section 5 and its results are discussed and compared with that of simulation using tabulation, and graphics of the posterior distribution are demonstrated to show that the best loss function is the WBLF followed by QLF. The final section contains some concluding remarks.

2. EGZ distribution and its likelihood function

The pdf of EGZ of variable X is given as:

$$f(x,\lambda,\alpha,\theta) = \theta\lambda e^{\alpha x} e^{\frac{-\lambda}{\alpha}(e^{\alpha x}-1)}\{1 - e^{\frac{-\lambda}{\alpha}(e^{\alpha x}-1)}\}^{\theta-1}, \quad x,\lambda,\alpha,\theta > 0. \tag{1}$$

EGZ has the following likelihood function for random sample x = $x_1,\ldots,x_n$:

$$L(x,\alpha,\lambda,\theta) = (\theta\lambda)^n \prod_{i=1}^{n} e^{\alpha x_i} \prod_{i=1}^{n} e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)} \prod_{i=1}^{n} \{1 - e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\}^{\theta-1},$$

$$L(x,\alpha,\lambda,\theta) = (\theta\lambda)^n \prod_{i=1}^{n} e^{\alpha x_i} \prod_{i=1}^{n} e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)} e^{(\theta-1)\sum_{i=1}^{n}\ln\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\}}, \tag{2}$$

Then (2) becomes:

$$L(x,\alpha,\lambda,\theta) = (\theta\lambda)^n \prod_{i=1}^{n} e^{\alpha x_i} \prod_{i=1}^{n} e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)} e^{(\theta-1)m_1},$$

The likelihood function of EGZ with known scale parameter α , known shape parameter λ and unknown shape parameter θ is:

$$L(x,\theta) \propto \theta^n e^{-\theta(-m_1)}, \tag{3}$$

where, $m_1 = \sum_{i=1}^{n} \ln\{1 - e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\}.$

3. Analysis of shape parameter of EGZ distribution

3.1 Posterior distribution using informative priors

Here, in this subsection we consider GP and ILP which are contemplated as informative priors to acquiring the posterior distribution.

3.1.1 Gamma prior

The gamma prior of θ with hyperparameters 'v' and 'w' is given by:

$$p(\theta) = \frac{w^v}{\Gamma v}\theta^{v-1}e^{-w\theta}, \qquad v, w, \theta > 0. \tag{4}$$

The posterior distribution using equations (3) and (4) is given by:

$$p(\theta|\mathbf{x}) \propto \theta^{v-1}e^{-w\theta}\theta^n e^{-\theta(-m_1)},$$

$$p(\theta|\mathbf{x}) \propto \theta^{\Phi_2 - 1}e^{-\theta\Psi_2},$$

where, $\Phi_2 = v + n$ and $\Psi_2 = w - m_1$.

which is the density kernel of gamma distribution having parameters Φ_2 and Ψ_2.

Hence, the posterior distribution $\theta|\mathbf{x}$ is Gamma (Φ_2, Ψ_2).

3.1.2 Inverse levy prior

The inverse Levy prior of θ with hyperparameter 'c' is given by:

$$p(\theta) = \sqrt{\frac{c}{2\pi}}\theta^{-\frac{1}{2}}e^{\frac{-c\theta}{2}}, \qquad c, \theta > 0. \tag{5}$$

The posterior distribution using equations (3) and (5) is given by:

$$p(\theta|\mathbf{x}) \propto \theta^{\frac{-1}{2}}e^{\frac{-c\theta}{2}}\theta^n e^{-\theta(-m_1)},$$

$$p(\theta|\mathbf{x}) \propto \theta^{\Phi_3 - 1}e^{-\theta\Psi_3},$$

where, $\Phi_3 = n + \frac{1}{2}$ and $\Psi_3 = \frac{c}{2} - m_1$.

which is the density kernel of gamma distribution having parameters Φ_3 and Ψ_3.

Hence, the posterior distribution $\theta|\mathbf{x}$ is Gamma (Φ_3, Ψ_3).

3.2. BEs and PRs under different loss functions

The general expressions for loss functions along with the expressions of their Bayes Estimators and PRs are given as follows.

3.2.1 Squared error loss function

The expression for squared error loss function is given as:

$$L(\theta,\theta^*) = (\theta-\theta^*)^2, \tag{6}$$

The Bayes estimator and posterior risk of SELF are:

$$\theta^* = E_{\theta|\mathbf{x}}(\theta), \qquad \rho(\theta^*) = E_{\theta|\mathbf{x}}(\theta)^2 - \{E_{\theta|\mathbf{x}}(\theta)\}^2. \tag{7}$$

3.2.2 Weighted squared error loss function

The expression for weighted squared error loss function is given as:

$$L(\theta,\theta^*) = \frac{(\theta-\theta^*)^2}{\theta}, \tag{8}$$

The Bayes estimator and posterior risk of WSELF are:

$$\theta^* = \{E_{\theta|\mathbf{x}}(\theta^{-1})\}^{-1}, \qquad \rho(\theta^*) = E_{\theta|\mathbf{x}}(\theta) - \{E_{\theta|\mathbf{x}}(\theta^{-1})\}^{-1}. \tag{9}$$

3.2.3 Precautionary loss function

The expression for precautionary loss function is given as:

$$L(\theta,\theta^*) = \frac{(\theta-\theta^*)^2}{\theta^*}, \tag{10}$$

The Bayes estimator and Posterior risk of PLF are:

$$\theta^* = \sqrt{E_{\theta|\mathbf{x}}(\theta^2)}, \qquad \rho(\theta^*) = 2\{\sqrt{E_{\theta|\mathbf{x}}(\theta^2)} - E_{\theta|\mathbf{x}}(\theta)\}. \tag{11}$$

3.2.4 Weighted balance loss function

The expression for weighted balance loss function is given as:

$$L(\theta,\theta^*) = \left(\frac{\theta-\theta^*}{\theta^*}\right)^2, \tag{12}$$

The Bayes estimator and posterior risk of WBLF are:

$$\theta^* = \frac{E_{\theta|\mathbf{x}}(\theta^2)}{E_{\theta|\mathbf{x}}(\theta)}, \qquad \rho(\theta^*) = 1 - \frac{\{E_{\theta|\mathbf{x}}(\theta)\}^2}{E_{\theta|\mathbf{x}}(\theta^2)}. \tag{13}$$

3.2.5 Quadratic loss function

The expression for quadratic loss function is:

$$L(\theta,\theta^*)=\left(\frac{\theta-\theta^*}{\theta}\right)^2, \tag{14}$$

The Bayes estimator and posterior risk of QLF are:

$$\theta^*=\frac{E_{\theta|\mathbf{x}}(\theta^{-1})}{E_{\theta|\mathbf{x}}(\theta^{-2})}, \qquad \rho(\theta^*)=1-\frac{\{E_{\theta|\mathbf{x}}(\theta^{-1})\}^2}{E_{\theta|\mathbf{x}}(\theta^{-2})}. \tag{15}$$

3.3. Expressions for BEs and PRs under different loss functions

This section derives and summarize the expressions for BEs and PRs under SELF in the presence of priors based on GP and ILP distributions. This is done in tabular form below in Tables 3.1. to 3.5., which represent five loss functions, respectively.

Table 3.1. Expressions for BEs and PRs under SELF

Priors	BEs	PRs
GP	$(v+n)\left[w-\sum_{i=1}^{n}\ln\left\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\right\}\right]^{-1}$	$(v+n)\left\{w-\sum_{i=1}^{n}\ln[1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}]\right\}^{-2}$
ILP	$(2n+1)\left[c-2\sum_{i=1}^{n}\ln\left\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\right\}\right]^{-1}$	$2(2n+1)\left\{c-2\sum_{i=1}^{n}\ln[1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}]\right\}^{-2}$

Table 3.2. Expressions for BEs and PRs under WSELF

Priors	BEs	PRs
GP	$(v+n-1)\left[w-\sum_{i=1}^{n}\ln\left\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\right\}\right]^{-1}$	$\left[w-\sum_{i=1}^{n}\ln\left\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\right\}\right]^{-1}$
ILP	$(2n-1)\left[c-2\sum_{i=1}^{n}\ln\left\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\right\}\right]^{-1}$	$(2)\left[c-2\sum_{i=1}^{n}\ln\left\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\right\}\right]^{-1}$

Table 3.3. Expressions for BEs and PRs under PLF

Priors	Bes	PRs
GP	$\left(\sqrt{(v+n)(v+n+1)}\right)\left[w-\sum_{i=1}^{n}\ln\left\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\right\}\right]^{-1}$	$2\left(\sqrt{(v+n)(v+n+1)}-(v+n)\right)\left[w-\sum_{i=1}^{n}\ln\left\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\right\}\right]^{-1}$
ILP	$\left(\sqrt{(2n+1)(2n+3)}\right)\left[c-2\sum_{i=1}^{n}\ln\left\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\right\}\right]^{-1}$	$2\left(\sqrt{(2n+1)(2n+3)}-(2n+1)\right)\left[c-2\sum_{i=1}^{n}\ln\left\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\right\}\right]^{-1}$

Table 3.4. Expressions for BEs and PRs under WBLF

Priors	Bes	PRs
GP	$(v+n+1)\left[w-\sum_{i=1}^{n}\ln\left\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\right\}\right]^{-1}$	$(v+n+1)^{-1}$
ILP	$(2n+3)\left[c-2\sum_{i=1}^{n}\ln\left\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\right\}\right]^{-1}$	$2(2n+3)^{-1}$

Table 3.5. Expressions for BEs and PRs under QLF

Priors	Bes	PRs
GP	$(v+n-2)\left[w-\sum_{i=1}^{n}\ln\left\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\right\}\right]^{-1}$	$(v+n-1)^{-1}$
ILP	$(2n-3)\left[c-2\sum_{i=1}^{n}\ln\left\{1-e^{\frac{-\lambda}{\alpha}(e^{\alpha x_i}-1)}\right\}\right]^{-1}$	$2(2n-1)^{-1}$

4. Simulation study

In this section, we conduct a simulation study using the expression of the loss functions from above tables, which are obtained by the BEs and PRs for the shape parameter θ, using two informative priors, namely: GP and ILP, under **five** different loss functions, namely SELF, WSELF, PLF, WBLF and QLF. Various sample sizes such as 20, 30,100, 300, 500, 1000 are used for simulation purposes, taking 10,000 replications in 'R'. Several values of the scale and shape parameters are considered. α is taken as 2, λ is taken as 1 and 3 and θ is taken as 1, 2 and 3. The estimated values of the parameters for all BEs and PRs are tabulated in Table 4.1. below. It is important to note that corresponding to selected samples of GP's and ILP's values are given. BEs are without parenthesis while estimates of PRs are enclosed in parenthesis for each prior and loss function under different sample sizes.

Note from the above table, as the sample size, n, increases then the GP and ILP of BE also goes up but PRs decreases. Similarly, once we change the value of θ=1 to θ=2 GP of BE and ILP follow the similar pattern with one exception. The same pattern is observed from the graphics demonstration via Bayes estimates and posterior risks for selected values of α, λ, w, v, and θ given below in graphs Figures 4.1. to fig. 4.5. and figures 5.1. to 5.3. Rest of the graphs are not presented due to similar observations.

Table 4.1. BEs and PRs under SELF using various priors

Priors	α =2, λ=1, θ=1, v=1, $w=1$, $c=1$					
N	20	30	100	300	500	1000
GP	0.97626 (0.056405)	0.97803 (0.03374)	0.98101 (0.012509)	1.02273 (0.00316)	1.08835 (0.00192)	1.12402 (0.00095)
ILP	0.82894 (0.10345)	0.94924 (0.02252)	0.979703 (0.00896)	0.99385 (0.00362)	1.04427 (0.00197)	1.45631 (0.00095)
Priors	α =2, λ=1, θ=2, v=1, $w=1$, $c=1$					
GP	2.34054 (0.26086)	2.22234 (0.15931)	2.08343 (0.04102)	2.03552 (0.01181)	1.91313 (0.007305)	1.88565 (0.00433)
ILP	2.12046 (0.159907)	2.09176 (0.11592)	1.97831 (0.04474)	1.89975 (0.01201)	1.88035 (0.00781)	1.81055 (0.00437)
Priors	α =2, λ=1, θ=3, v=1, $w=1$, $c=1$					
GP	2.65453 (0.33554)	2.67769 (0.23129)	2.84175 (0.08995)	2.99406 (0.02682)	3.01425 (0.01858)	3.05168 (0.00895)
ILP	2.62072 (0.39717)	2.85344 (0.27311)	2.86699 (0.10028)	2.88619 (0.02285)	2.93718 (0.01642)	3.17466 (0.00862)
Priors	α =2, λ=3, θ=1, v=1, $w=1$, $c=1$					
GP	0.74424 (0.02637)	0.84441 (0.0230009)	0.858404 (0.00729)	1.01196 (0.00352)	1.03065 (0.00217)	1.04442 (0.00102)
ILP	0.65127 (0.04646)	0.92084 (0.013906)	0.94216 (0.00843)	0.97597 (0.00295)	1.02088 (0.00208)	1.03285 (0.00106)
Priors	α =2, λ=3, θ=2, v=1, $w=1$, $c=1$					
GP	1.720105 (0.22553)	1.85295 (0.09544)	1.98521 (0.03399)	1.99369 (0.01309)	2.03254 (0.00824)	2.17628 (0.00397)
ILP	1.83514 (0.17915)	1.90751 (0.16993)	1.91639 (0.03351)	2.00648 (0.01411)	2.05984 (0.00726)	2.27661 (0.00402)
Priors	α =2, λ=3, θ=3, v=1, $w=1$, $c=1$					
GP	2.49229 (0.35855)	2.74402 (0.20037)	2.79417 (0.077301)	2.94542 (0.03222)	3.11463 (0.02063)	3.215501 (0.00866)
ILP	2.23795 (0.364107)	2.727302 (0.16421)	2.73206 (0.15021)	3.00286 (0.03603)	3.29053 (0.01486)	3.88547 (0.00901)
Priors	α =2, λ=1, θ=1, v=1, w=2, $c=2$					
GP	0.87387 (0.04876)	0.90072 (0.02463)	0.90842 (0.00817)	0.95581 (0.00269)	0.98606 (0.00182)	1.01198 (0.00097)
ILP	1.20682 (0.06767)	1.17785 (0.03656)	1.05607 (0.01449)	0.98644 (0.00311)	0.98438 (0.00193)	0.96682 (0.00097)

Simulation Results' Graphs

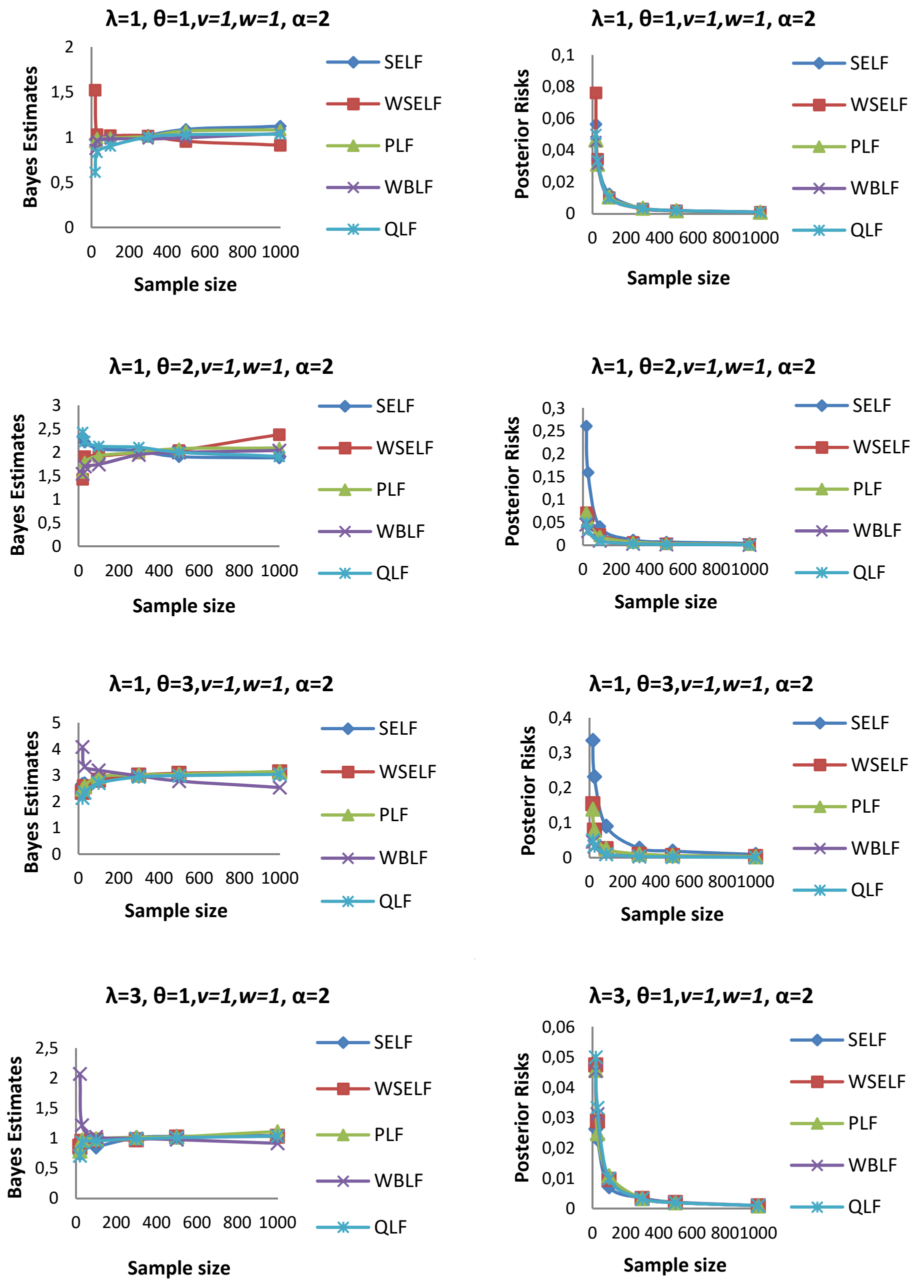

Figure 4.1. Graphs of BEs and PRs for GP

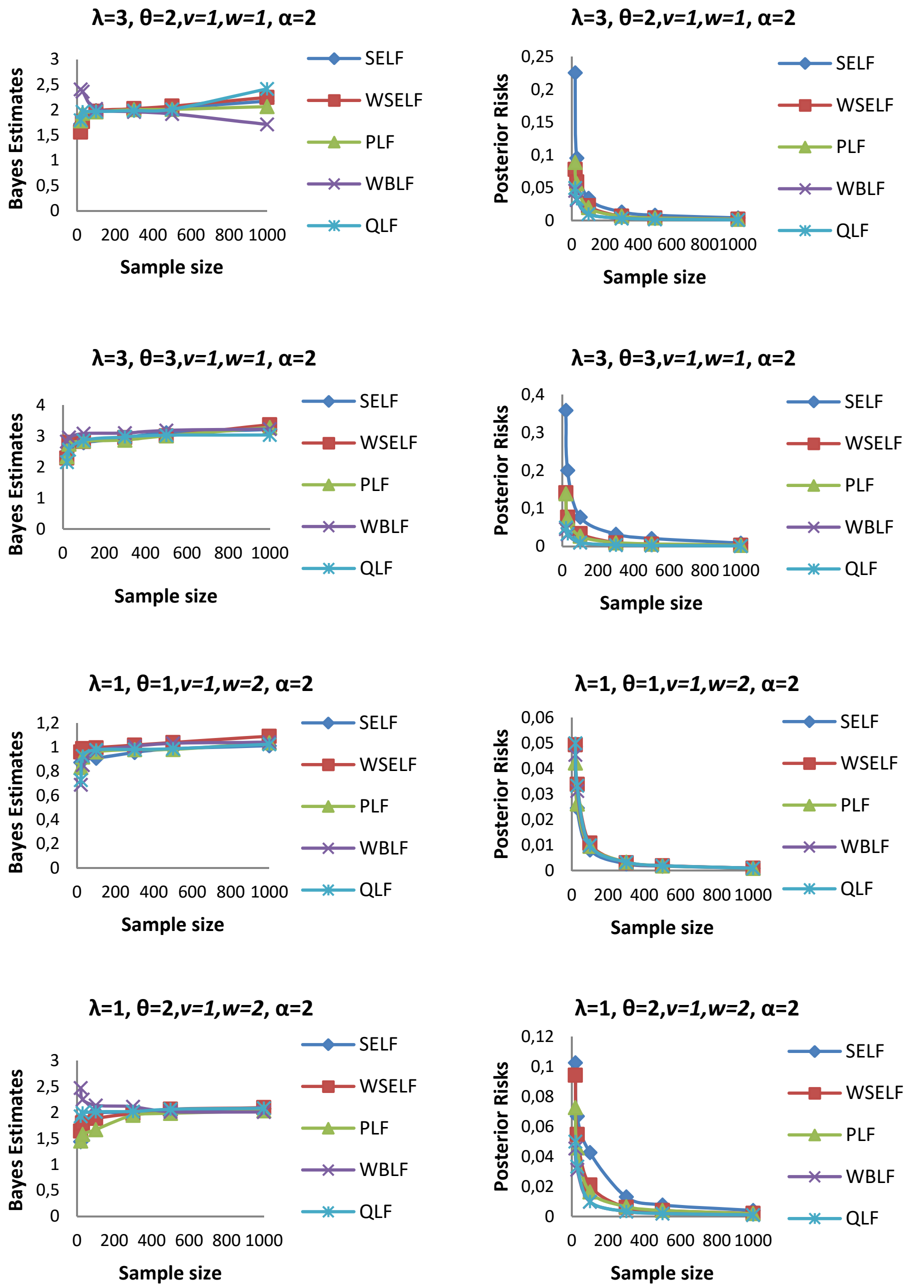

Figure 4.2. Graphs of BEs and PRs for GP

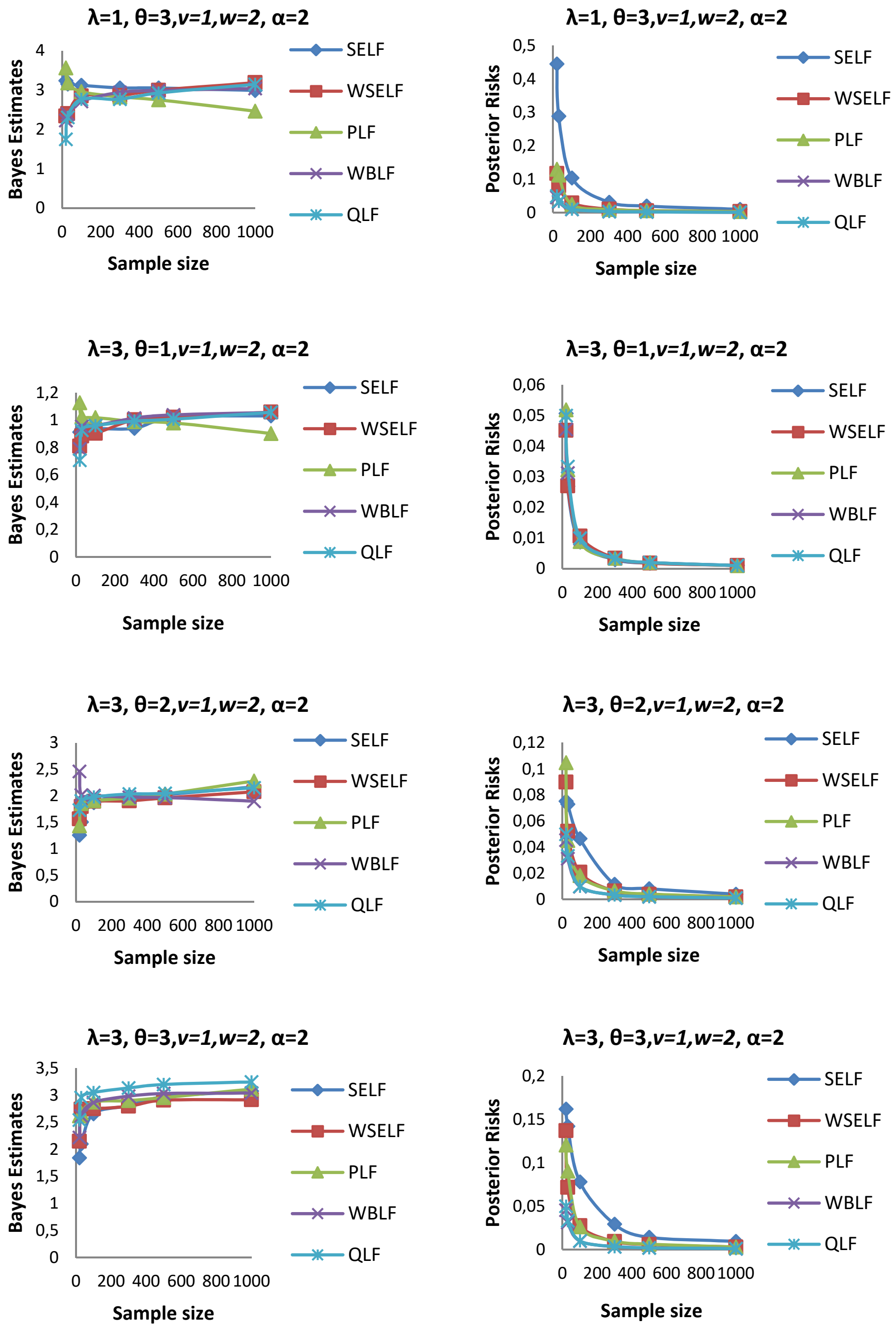

Figure 4.3. Graphs of BEs and PRs for GP

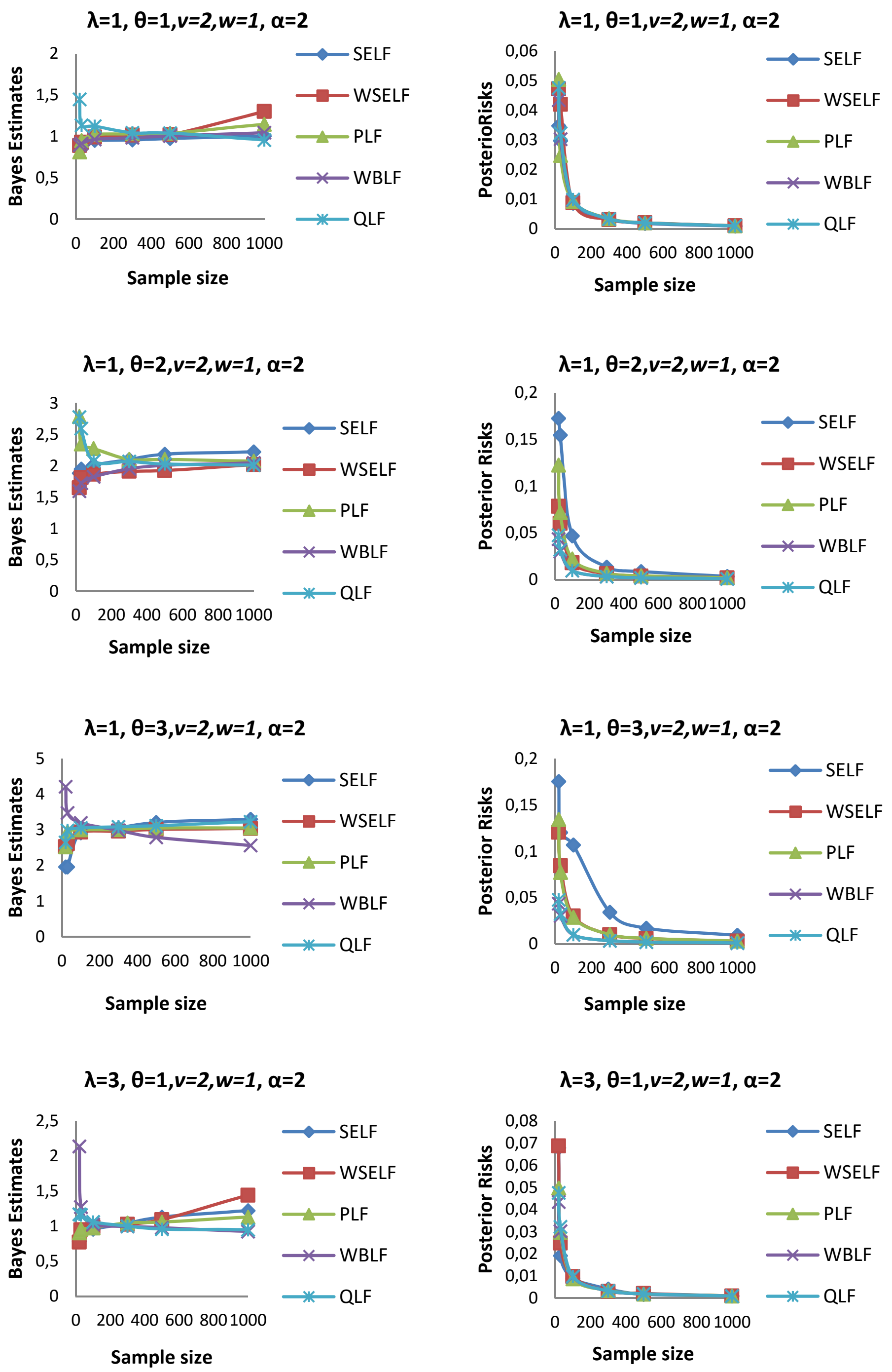

Figure 4.4. Graphs of BEs and PRs for GP

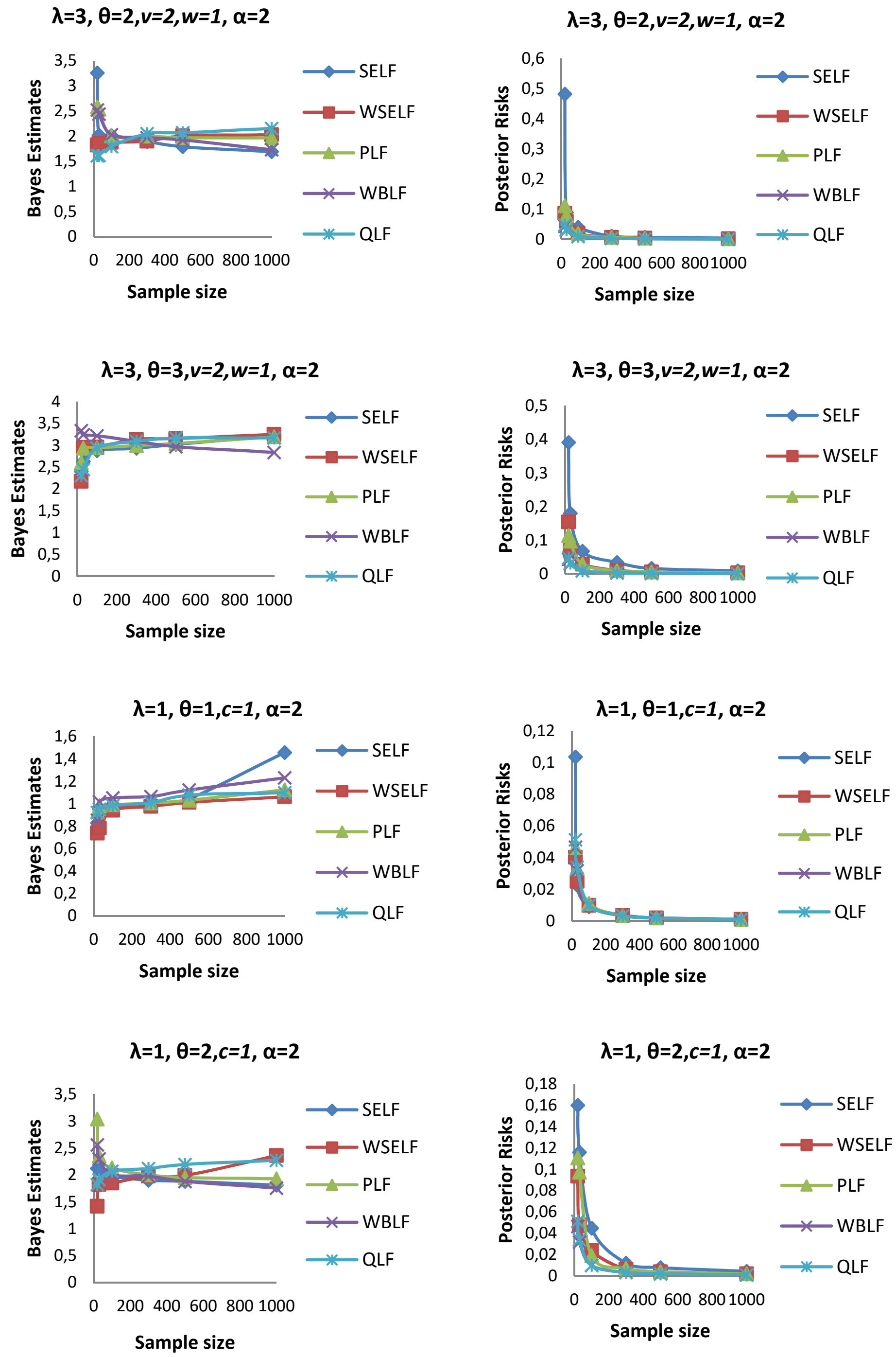

Figure 4.5. Graphs of BEs and PRs for GP

5. Examining a real-life data set

The data set consists of 50 observations of lifetimes of devices as given in Gohary et al. (2013). These are given below for the ready reference of the readers.
0.1, 0.2, 1, 1, 1, 1, 1, 2, 3, 6, 7, 11, 12, 18, 18, 18, 18, 18, 21, 32, 36, 40, 45, 46, 47, 50, 55, 60, 63, 63, 67, 67, 67, 67, 72, 75, 79, 82, 82, 83, 84, 84, 84, 85, 85, 85, 85, 85, 86, 86.
The various values of the BEs, PRs for parameter θ for selected values of α, λ, v, w *and* c are tabulated in Tables 5.1. to 5.6. for two priors and five loss functions. The WBLF loss function highlighted in bold indicate the prefer priors.

Table 5.1. BEs and PRs for parameter θ when $\alpha = 2, \lambda = 1, v = 1, w = 1, c = 1$

Prior		Loss Functions				
		SELF	WSELF	PLF	WBLF	QLF
GP	BEs	10.22408	10.02361	10.32383	**10.42455**	9.82313
	PRs	2.04964	0.200472	0.19949	0.01923	0.02000
ILP	BEs	11.25167	11.02886	11.36252	**11.47447**	10.80606
	PRs	2.50693	0.222805	0.22171	0.01941	0.020202

Table 5.2. BEs and PRs for parameter θ when $\alpha = 2, \lambda = 1, v = 1, w = 2, c = 2$

Prior		Loss Functions				
		SELF	WSELF	PLF	WBLF	QLF
GP	BEs	8.51671	8.34972	8.59980	**8.683709**	8.18272
	PRs	1.42224	0.16699	0.16618	0.01923	0.02000
ILP	BEs	10.12384	9.92337	10.22359	**10.32432**	9.72289
	PRs	2.02954	0.20047	0.19948	0.01941	0.020202

Table 5.3. BEs and PRs for parameter θ when $\alpha = 2, \lambda = 1, v = 2, w = 1, c = 3$

Prior		Loss Functions				
		SELF	WSELF	PLF	WBLF	QLF
GP	BEs	10.42455	10.22408	10.52431	**10.62502**	10.02361
	PRs	2.08983	0.20047	0.199517	0.01886	0.019607
ILP	BEs	9.20151	9.01931	9.29217	**9.38372**	8.837102
	PRs	1.67659	0.182208	0.18131	0.01941	0.020202

Table 5.4. BEs and PRs for parameter θ when $\alpha = 2, \lambda = 3, v = 1, w = 1, c = 1$

Prior	Loss Functions					
		SELF	WSELF	PLF	WBLF	QLF
GP	BEs	17.49828	17.15517	17.669	**17.84138**	16.81207
	PRs	6.00372	0.343103	0.34143	0.01923	0.02000
ILP	BEs	20.91467	20.50052	21.12073	**21.32883**	20.08637
	PRs	8.66185	0.41415	0.41212	0.01941	0.020202

Table 5.5. BEs and PRs for parameter θ when $\alpha = 2, \lambda = 3, v = 1, w = 2, c = 2$

Prior	Loss Functions					
		SELF	WSELF	PLF	WBLF	QLF
GP	BEs	13.02824	12.77279	13.15535	**13.2837**	12.51733
	PRs	3.32813	0.25545	0.25421	0.01923	0.02000
ILP	BEs	17.32673	16.98362	17.49744	**17.66983**	16.64052
	PRs	5.94486	0.343103	0.34142	0.01941	0.020202

Table 5.6. BEs and PRs for parameter θ when $\alpha = 2, \lambda = 3, v = 2, w = 1, c = 3$

Prior	Loss Functions					
		SELF	WSELF	PLF	WBLF	QLF
GP	BEs	17.84138	17.49828	18.01212	**18.18448**	17.15517
	PRs	6.12144	0.343103	0.34146	0.01886	0.019607
ILP	BEs	14.78955	14.49669	14.93527	**15.08241**	14.20383
	PRs	4.331304	0.29286	0.29142	0.01941	0.020202

5.1. Analysis of real-life data set

From the above tables we can see that WBLF is the best and most preferable loss function under all the priors since it has the lowest PRs followed by QLF, both the loss functions having a minute difference in their PRs. Also, SELF has the highest PRs making it the least preferable loss function. Moreover, the PRs are minimum for the values of hyperparameters (1, 2) of GP making it the most preferable prior and pair to be used. Additionally, WBLF and QLF have same PRs for all values of α, λ. The only difference comes for GP for the values of hyperparameters (2,1), where the value of hyper-parameter 'v' changes and gives a different risk. It can also be noticed that the results of simulation and the results for real life data are identical. These are demonstrated in the graphs below in Fig. 5.1. to 5.3. for various values of v and w.

Posterior distribution graphs

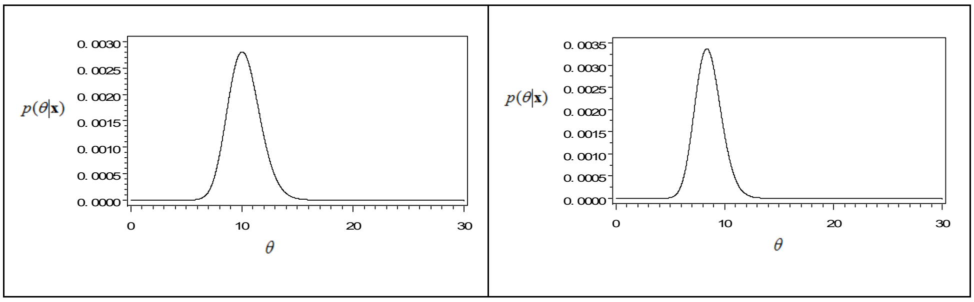

Figure 5.1. Graph of posterior dist. For GP when $v = 1, w = 1$ and $v = 1, w = 2$

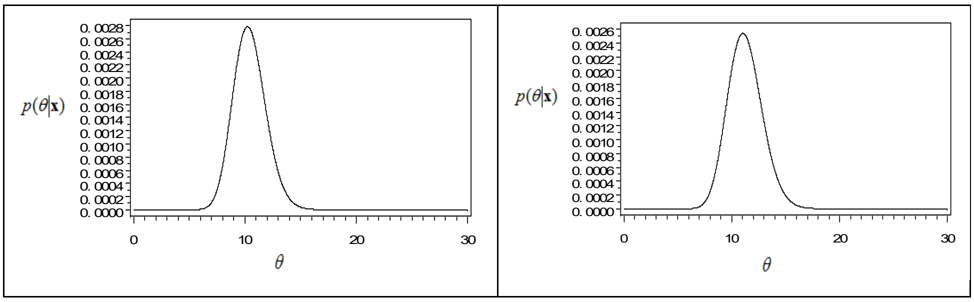

Figure 5.2. Graph of posterior dist. For GP when $v = 2, w = 1$ and ILP for $c = 1$

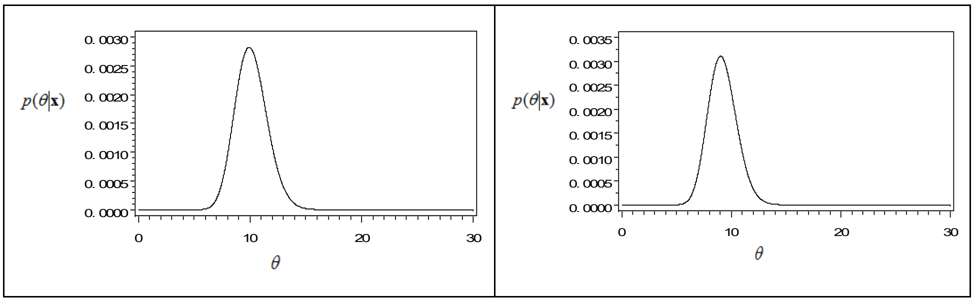

Figure 5.3. Graph of posterior dist. using ILP when $c = 2$ and $c = 3$

6. Concluding remarks

This paper explores important properties of the EGZ distribution under Bayesian using two informative priors: GP and ILP. This is done under five selected loss functions. We observe that the best loss functions are WBLF followed by QLF. The simulated study and real-life data were used for various sample sizes with 10,000 replications. Several values of the scale and shape parameters are considered. α is taken as 2, λ is taken as 1 and 3 and θ is taken as 1, 2 and 3. We observe that as the sample size is extended, PR declines and BE comes nearest to the true value of shape parameter

θ. Also, at various places over-estimation of parameters is noted. We also noted that the loss functions WBLF and QLF have similar PRs for all values of θ, α, λ under both the informative priors: GP and ILP. The only difference comes for GP for the values of hyperparameters (2,1), where the value of hyper-parameter 'v' changes and gives a different risk. In addition, WBLF has minimum PRs as compared to other loss functions followed by QLF under all the priors. Moreover, note that GP is the best prior as compared to all other priors and works best for the pair of hyperparameters (1, 2) since it has minimum PRs. The results for real life data and simulation are identical.

Acknowledgement

We are thankful to the two anonymous referees and handling editor for the constructive comments which have improved the paper. We take all the responsibilities of any errors the manuscript may have.

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