STATISTICS IN TRANSITION new series, December 2021
Vol. 22, No. 4 pp. 139–152, DOI 10.21307/stattrans-2021-042
Received – 23.12.2019; accepted – 27.04.2020

Type II Topp-Leone Frechet distribution: properties and applications

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ABSTRACT

The paper focuses on type II Topp-Leone Frechet distribution. Its properties including hazard rate function, reverse hazard rate function, Mills ratio, quantile function and order statistics have been studied. The maximum likelihood estimation used for estimating the parameters of the proposed distribution has been explained and expressions for the Fisher information matrix and confidence intervals have been provided. The paper discusses the applications of the distribution for modeling several datasets relating to temperature. Finally, the goodness of fit of the proposed distribution has been compared with that of the Frechet distribution.

Key words: Frechet distribution, Topp-Leone distribution, reliability properties, applications.

1. Introduction

Frechet distribution introduced by Mourice Rene Frechet (1927) is defined by its cumulative distribution function (cdf) and probability density function (pdf)

$$G(x;\alpha,\beta)=e^{-\alpha x^{-\beta}}\,;x>0,\alpha>0,\beta>0 \tag{1.1}$$

and $$g(x;\alpha,\beta)=\alpha\beta\, x^{-(\beta+1)}e^{-\alpha x^{-\beta}}\,;x>0,\alpha>0,\beta>0 \tag{1.2}$$

where $\alpha>0$ is a shape parameter $\beta>0$ is a scale parameter. It is an inverse of Weibull distribution introduced by Weibull (1951). Shanker and Shukla (2019) derived a generalization of Weibull distribution and discussed its statistical properties, estimation of parameter and applications. Frechet distribution is the type II extreme value distribution used for modeling extreme data from accelerate life testing, natural calamaties, rainfall, temperature, wind speed and so on. Nadarajah and Kotz (2003a, 2006) introduced exponentiated Frechet distribution and other exponentiated type

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distributions and discussed their statistical properties and parameter estimation. Mubarak (2012) discussed maximum likelihood and least squares estimates for the parameter of Frechet distribution based on progressive type II censoring. The transmuted Frechet distribution and Marshall-Olkin Frechet distribution were discussed by Mahmoud and Mandouh (2013) and Krishna and Jose (2013), respectively. The Bayesian estimation of the shape parameter of Frechet distribution using different prior distribution and various loss functions has been discussed by Nasir and Aslam (2015). The beta exponential Frechet distribution and Weibull- Frechet distribution have been discussed by Mead *et al* (2017) and Afify *et al* (2016), respectively.

The Topp-Leone distribution (TLD) proposed by Topp and Leone (1955) is one of the continuos distribution useful for generating new distribution. The most important characteristics of TLD is to provides closed forms for both the pdf and cdf. The TLD distribution received attention in statistics after the works of Nadarajah and Kotz (2003b) who studied some properties of TLD including moments, central moments and characteristic function. Ghitany *et.al.* (2005) discussed some reliability measures and stochastic orderings of TLD. The goodness of fit tests for the TLD has been studied by Al-Zahrani (2012). Reyad et al (2021) studied the properties, estimation and applications of Frechet Topp-Leone G-family of distributions.

The type-I TLD developed for empirical data with J-shaped histogram such as powered band functions and automatic calculating machine failure. Suppose a continuos random variable X following type-I TLD (TITLD) are given by

$$F_{TLG}(x) = [G(x)]^{\alpha}\,[2 - G(x)]^{\alpha} \tag{1.3}$$

and $$f_{TLG}(x) = 2\alpha\, g(x)[1 - G(x)][G(x)]^{\alpha-1}[2 - G(x)]^{\alpha-1} \tag{1.4}$$

where $g(x) = \dfrac{dG(x)}{dx}$ and $\alpha > 0$ is a shape parameter. It has been observed that the TL random variable with finite support has the same bounds as the cdf $G(x)$ of any other random variable.

By taking $G(x) = \left(1 - e^{-\lambda x}\right)^{\beta}$, where $\lambda > 0$ is a scale parameter and $\beta > 0$ is a shape parameter, as the cdf of generalized exponential distribution proposed by Gupta and Kundu (1999), Sangsanit and Bodhisuwan (2016) introduced the Topp-Leone generalized exponential distribution(TLGED) defined by its pdf and cdf,

$$f_{TLGE}(x;\alpha,\beta,\lambda) = 2\alpha\beta\lambda e^{-\lambda x}\left\{1 - \left(1 - e^{-\lambda x}\right)^{\beta}\right\}\left(1 - e^{-\lambda x}\right)^{\beta\alpha-1}\left\{2 - \left(1 - e^{-\lambda x}\right)^{\beta}\right\}^{\alpha-1} \tag{1.5}$$

and $$F_{TLGE}(x;\alpha,\beta,\lambda) = \left(1 - e^{-\lambda x}\right)^{\beta\alpha}\left\{2 - \left(1 - e^{-\lambda x}\right)^{\beta}\right\}^{\alpha} \tag{1.6}$$

Various statistical properties, estimation of parameters using maximum likelihood estimation and goodness of fit of TLGED have been studied by Sangsanit and Bodhisuwan (2016).

Recently, Elgarhy *et al* (2018) introduced Type-II Topp-Leone generalized family of distribution. Suppose, $g(x)$ and $G(x)$ are the pdf and cdf of the parent distribution. The pdf and cdf of a random variable X following Type-II Topp-Leone distribution (TIITLD) is defined by its cdf and pdf,

$$F(x;\beta)=1-\left[1-\{G(x)\}^2\right]^{\beta};\, x>0, \beta>0 \tag{1.7}$$

$$f(x;\beta)=2\beta\, g(x)G(x)\left[1-\{G(x)\}^2\right]^{\beta-1};\, x>0, \beta>0 \tag{1.8}$$

where $\beta>0$ is a shape parameter.

Since Frechet distribution has been extensively used in the modeling of data related to temperature ,it is hoped and expected that the proposed distribution which is an extension of Frechet distribution using Type II Topp-Leone distribution would provide a better fit for temperature data.

The main motivation of considering Type II Topp-Leone Frechet distribution is that Frechet distribution being two-parameter distribution is very much useful for modeling data relating to temperature and it is expected that the proposed distribution, being three-parameter distribution and based on the concept of Type II Topp-Leone distribution, would provide better fit over Frechet distribution. Some of the important properties of the proposed distribution including shapes of the pdf and cdf, asymptotic behaviour, hazard rate function, reverse hazard rate function, Mills ratio have been studied. Maximum likelihood estimation has been discussed for estimating parameters of the proposed distribuion. Finally, applications of the proposed distribution for modeling datasets relating to minimum temperature of Silchar, Assam have been discussed.

2. Type II Topp-Leone Frechet Distribution

Using the cdf and pdf of Frechet distribution in (1.7) and (1.8), the cdf and the pdf of type II Topp-Leone Frechet distribution (TIITLFD) can be expressed as

$$F(x;\alpha,\beta,\lambda)=1-\left\{1-\left(1-e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda};\, x>0, \alpha>0, \beta>0, \lambda>0 \tag{2.1}$$

$$f(x;\alpha,\beta,\lambda)=2\alpha\beta\,\lambda\, x^{-(\beta+1)}\, e^{-2\alpha x^{-\beta}}\left\{1-\left(e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda-1};\, x>0, \alpha>0, \beta>0, \lambda>0 \tag{2.2}$$

where α and λ are shape parameters and β is a scale parameter. Further,

$$\lim_{x\to-\infty} F(x;\alpha,\beta,\lambda)=0$$

$$\text{and } \lim_{x\to+\infty} F(x;\alpha,\beta,\lambda)=1$$

This shows that TIITLFD is a proper density function.

Graphs of the pdf and the cdf of TIITLFD are shown in fig.1 and fig.2 for varying values of the parameters α, β and λ.

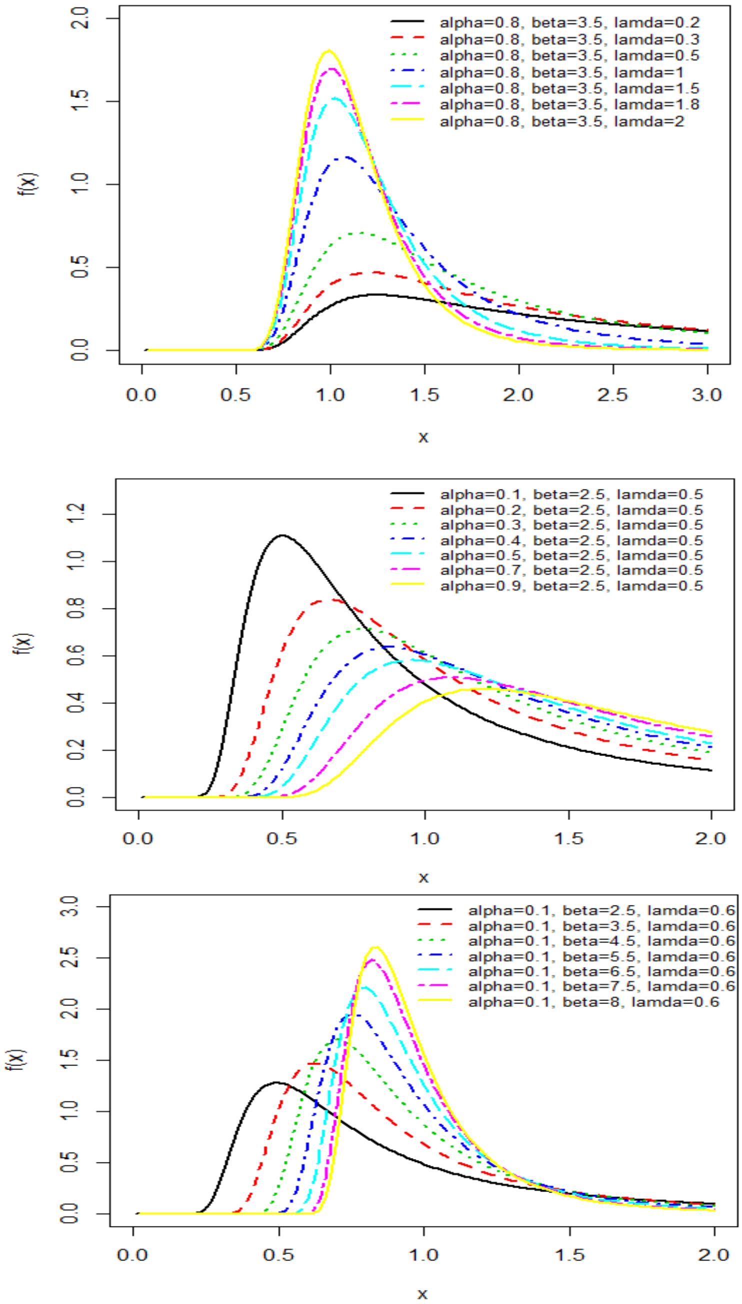

Figure1. Graphs of the pdf of TIITLFD for varying values of parameters

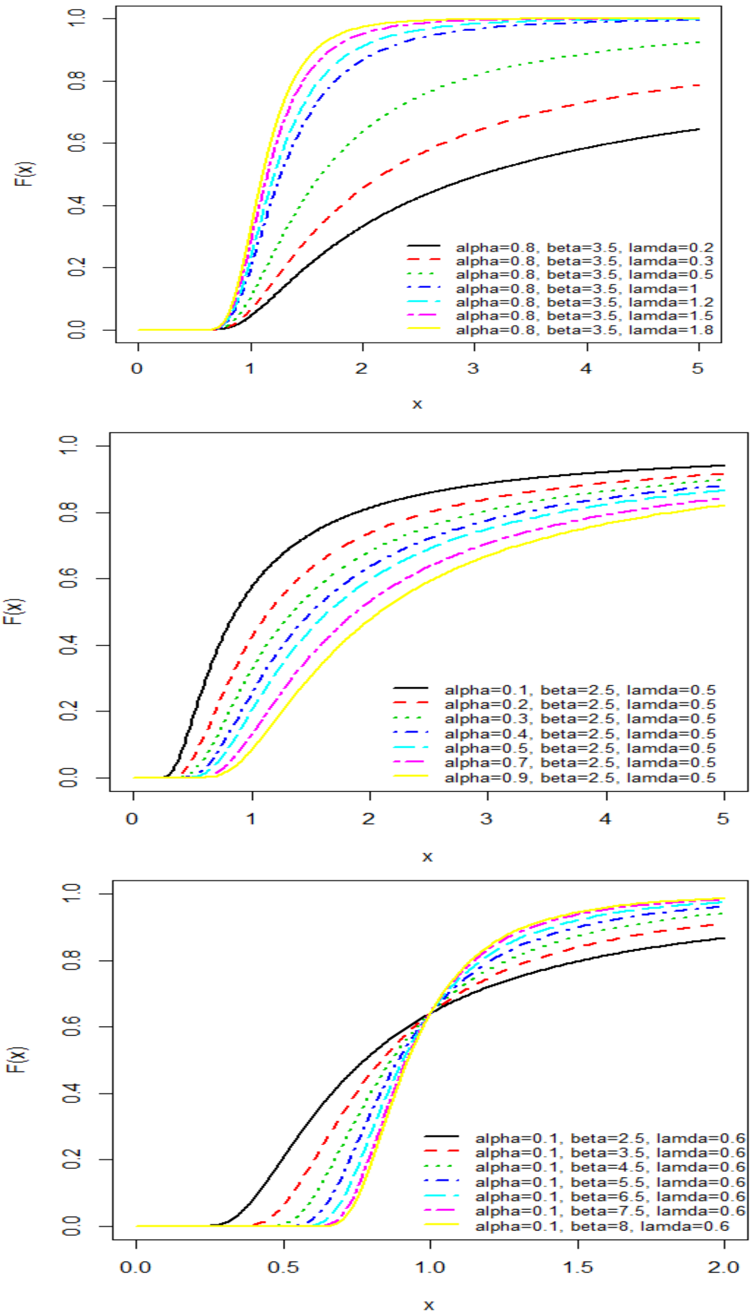

Figure2. Graphs of the cdf of TIITLFD for varying values of parameters

3. Statistical Properties

In this section, statistical properties including asymptotic behaviour, survival function, hazard function, reverse hazard rate and mills ratio of TLLTLFD has been studied.

3.1. Asymptotic behavior

The asymptotic behavior of TIITLFD for $x \to 0$ and $x \to \infty$ are

$$\lim_{x \to 0} f(x;\alpha,\beta,\lambda) = \lim_{x \to 0}\left[2\alpha\beta\lambda\, x^{-(\beta+1)} e^{-2\alpha x^{-\beta}}\left\{1-\left(e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda-1}\right] = 0$$

and $$\lim_{x \to \infty} f(x;\alpha,\beta,\lambda) = \lim_{x \to \infty}\left[2\alpha\beta\lambda\, x^{-(\beta+1)} e^{-2\alpha x^{-\beta}}\left\{1-\left(e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda-1}\right] = 0.$$

These results confirm that the proposed distribution has a mode.

3.2. Reliability properties

The survival function (or the reliability function) is the propability that a subject survives longer than the expected time. The survival function of the TIITLFD is given by

$$S(x;\alpha,\beta,\lambda) = 1 - F(x;\alpha,\beta,\lambda) = \left\{1-\left(e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda}.$$

The hazard function (also known as the hazard rate, instantaneous failure rate or force of mortality) is the probability to measure the instant death rate of a subject. Suppose X be a continuous random variable with pdf $f(x)$ and cdf $F(x)$. The hazard rate function of X is defined as

$$h(x) = \lim_{\Delta x \to 0} \frac{P(X < x + \Delta x / X > x)}{\Delta x} = \frac{f(x)}{1-F(x)}$$

The corresponding $h(x)$ of TIITLFD can be obtained as

$$h(x;\alpha,\beta,\lambda) = \frac{2\alpha\beta\lambda x^{-(\beta+1)} e^{-2\alpha x^{-\beta}}}{1-\left(e^{-\alpha x^{-\beta}}\right)^2}$$

The reverse hazard rate is the ratio between the probability density function and its distribution function. The reverse hazard function of TIITLFD is given by

$$h_r(x) = \frac{2\alpha\beta\lambda x^{-(\beta+1)}\left\{1-\left(e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda-1}}{1-\left\{1-\left(e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda}}$$

The mills ratio is the ratio between the cdf and the pdf. The mills ratio of TIITLFD is

$$\frac{1}{h_r(x;\alpha,\beta,\lambda)} = \frac{1-\left\{1-\left(e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda-1}}{2\alpha\beta\lambda x^{-(\beta+1)} e^{-2\alpha x^{-\beta}}\left\{1-\left(e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda-1}}$$

Graphs of the survival function and the hazard function of TIITLFD are shown in fig.3 and fig.4 for varying values of the parameters α, β and λ .

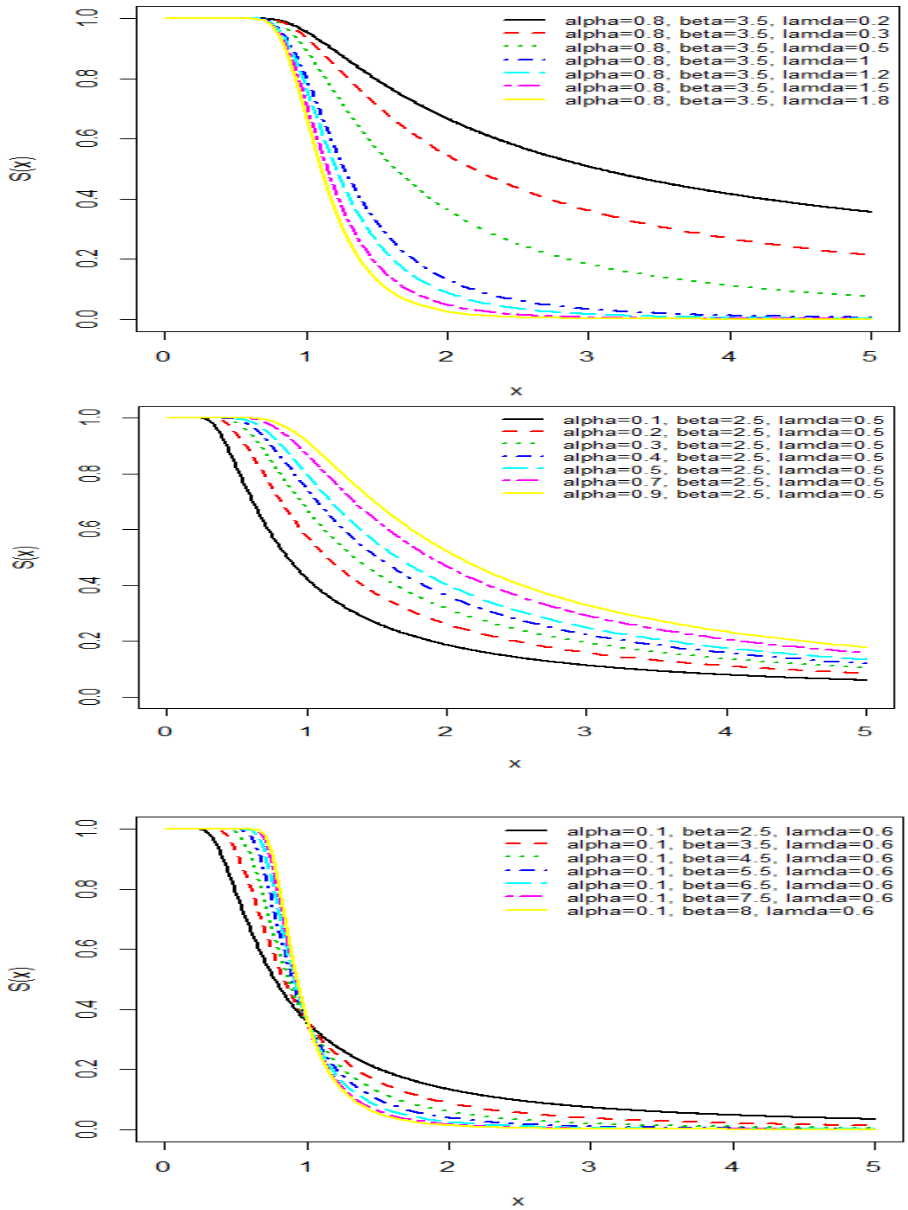

Figure 3. Graphs of survival function of TIITLFD for varying values of parameters

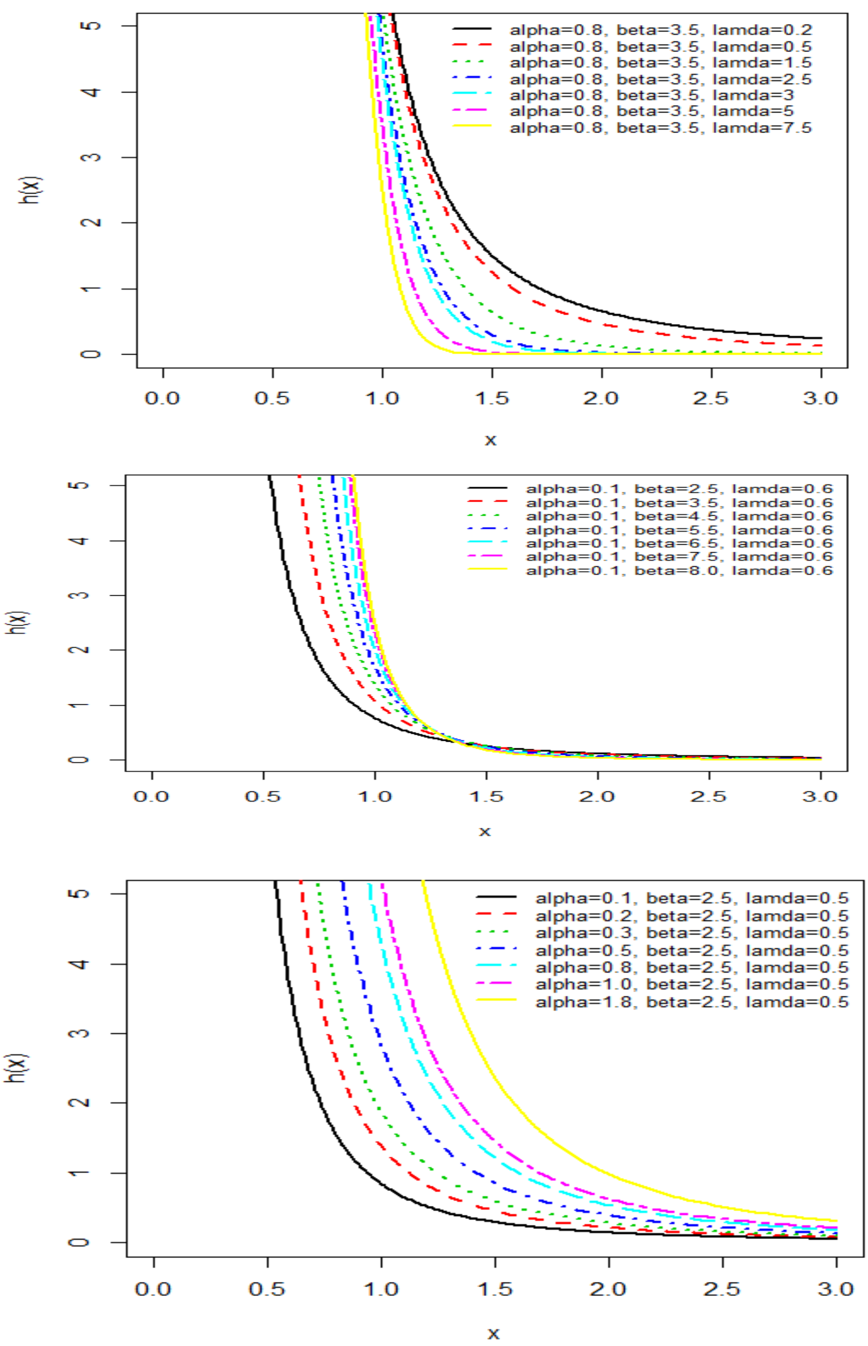

Figure 4. Graphs of hazard function of TIITLFD for varying values of parameters

3.3. Quantile function

The quantile function is defined as

$$Q(u) = F^{-1}(u)$$

Therefore, the corresponding quantile function for TIITLFD can be expressed as

$$Q(u) = F^{-1}(x) = \left[\frac{2\alpha}{-\ln\left\{1-(1-u)^{\frac{1}{\lambda}}\right\}}\right]^{\frac{1}{\beta}}$$

Let U has the uniform $U(0,1)$ distribution. Taking $u = 0.5$, the median of TIITLFD can be obtained as

$$Q(0.5) = F^{-1}(0.5) = \left[\frac{2\alpha}{-\ln\left\{1-(1-0.5)^{\frac{1}{\lambda}}\right\}}\right]^{\frac{1}{\beta}}$$

Thus, the formula for generating random samples from TIITLFD for simulating random variable X is given by

$$X = Q(u) = F^{-1}(u) = \left[\frac{2\alpha}{-\ln\left\{1-(1-u)^{\frac{1}{\lambda}}\right\}}\right]^{\frac{1}{\beta}}$$

4. Distribution of order statistics

Let $x_1, x_2, \ldots, x_n$ be the random samples from TIITLFD(α, β, λ). The pdf of i^{th} order statistics is given by

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!} f_X(x)\left[F_X(x)\right]^{i-1}\left[1-F_X(x)\right]^{n-1}$$

The pdf of i^{th} order statistics $X_{(i)}$ of TIITLFD is given by

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-1)!} 2\alpha\beta\lambda\, x^{-(\beta+1)} e^{-2\alpha x^{-\beta}} \left\{1-\left(e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda-1} \left[1-\left\{1-\left(e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda}\right]^{i-1}$$

$$\times\left[1-1-\left\{1-\left(e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda}\right]^{n-i}$$

Therefore, the pdf of the first order statistic $X_{(1)}$ can be expressed as

$$f_{1:n}(x) = n2\alpha\beta\lambda\, x^{-(\beta+1)} e^{-2\alpha x^{-\beta}} \left\{1-\left(e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda-1} \left[1-1-\left\{1-\left(e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda}\right]^{n-1}$$

The pdf of the highest order statistic $X_{(n)}$ can be expressed as

$$f_{n:n}(x) = n2\alpha\beta\lambda x^{-(\beta+1)} e^{-2\alpha x^{-\beta}} \left[1 - \left\{1 - \left(e^{-\alpha x^{-\beta}}\right)^2\right\}^{\lambda}\right]^{n-1}$$

5. Maximum likelihood estimation

Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from a TIITLFD(α, β, λ). The log-likelihood function can be expressed as

$$\log L = \sum_{i=1}^{n} \log f(x; \alpha, \beta, \lambda) = n(\log 2 + \log\alpha + \log\beta + \log\lambda) - (\beta+1)\sum_{i=1}^{n} x_i$$
$$- 2\alpha\sum_{i=1}^{n} x_i^{-\beta} + (\lambda-1)\sum_{i=1}^{n} \log\left\{1 - \left(e^{-\alpha x_i^{-\beta}}\right)^2\right\}$$

The maximum likelihood estimate (MLE) $(\hat{\alpha}, \hat{\beta}, \hat{\lambda})$ of (α, β, λ) of TIITLFD are the solutions of the following log- likelihood equations

$$\frac{\partial \log L}{\partial \alpha} = \frac{n}{\alpha} - 2\sum_{i=1}^{n} x_i^{-\beta} + 2(\lambda-1)\sum_{i=1}^{n} \frac{x^{-\beta}\left(e^{-\alpha x_i^{-\beta}}\right)^2}{1 - \left(e^{-\alpha x_i^{-\beta}}\right)^2} = 0$$

$$\frac{\partial \log L}{\partial \beta} = \frac{n}{\beta} - \sum_{i=1}^{n} \log x_i + 2\alpha\beta\sum_{i=1}^{n} x_i^{-\beta} \log x_i + 2\alpha(\lambda-1)\sum_{i=1}^{n} \frac{x_i^{-\beta} \log x_i \left(e^{-\alpha x_i^{-\beta}}\right)^2}{1 - \left(e^{-\alpha x_i^{-\beta}}\right)^2} = 0$$

$$\frac{\partial \log L}{\partial \lambda} = \frac{n}{\beta} - \sum_{i=1}^{n} \log\left[1 - \left(e^{-\alpha x_i^{-\beta}}\right)^2\right] = 0$$

These log-likelihood equation can't be solved analytically and required statistical software with iterative numerical techniques. These equations can be solved using R-software.

The 3×3 observed information matrix of TIITLFD can be presented as,

$$\begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \\ \hat{\lambda} \end{pmatrix} \sim \left[\begin{pmatrix} \alpha \\ \beta \\ \lambda \end{pmatrix}, \begin{pmatrix} \frac{\partial^2 \log L}{\partial \alpha^2} & \frac{\partial^2 \log L}{\partial \alpha \partial \beta} & \frac{\partial^2 \log L}{\partial \alpha \partial \lambda} \\ \frac{\partial^2 \log L}{\partial \beta \partial \alpha} & \frac{\partial^2 \log L}{\partial \beta^2} & \frac{\partial^2 \log L}{\partial \beta \partial \lambda} \\ \frac{\partial^2 \log L}{\partial \lambda \partial \alpha} & \frac{\partial^2 \log L}{\partial \lambda \partial \beta} & \frac{\partial^2 \log L}{\partial \lambda^2} \end{pmatrix}\right]$$

The inverse of the information matrix results in the well-known variance-covariance matrix. The 3×3 approximate Fishar information matrix corresponding to the above observed information matrix is given by

$$I^{-1} = -E\begin{bmatrix} \frac{\partial^2 \log L}{\partial \alpha^2} & \frac{\partial^2 \log L}{\partial \alpha \partial \beta} & \frac{\partial^2 \log L}{\partial \alpha \partial \lambda} \\ \frac{\partial^2 \log L}{\partial \beta \partial \alpha} & \frac{\partial^2 \log L}{\partial \beta^2} & \frac{\partial^2 \log L}{\partial \beta \partial \lambda} \\ \frac{\partial^2 \log L}{\partial \lambda \partial \alpha} & \frac{\partial^2 \log L}{\partial \lambda \partial \beta} & \frac{\partial^2 \log L}{\partial \lambda^2} \end{bmatrix}$$

The solution of the Fishar information matrix will yield asymptotic variance and covariance of the ML estimators for $\left(\hat{\alpha}, \hat{\beta}, \hat{\lambda}\right)$. The approximate 100(1-α)% confidence intervals for (α, β, λ) respectively are $\hat{\alpha} \pm Z_{\frac{\alpha}{2}} \frac{\sigma_{\alpha\alpha}}{n}$, $\hat{\beta} \pm Z_{\frac{\alpha}{2}} \frac{\sigma_{\beta\beta}}{n}$ and $\hat{\lambda} \pm Z_{\frac{\alpha}{2}} \frac{\sigma_{\lambda\lambda}}{n}$, where Z_α is the upper $100\alpha^{th}$ percentile of the standard normal distribution.

6. Applications

In this study, monthly mean temperature series of Silchar city, Assam, India from January 1988-July 2018 (30 years) which is collected by India Meteorological Department, Pune, India has been analyzed. For the application purpose , the datasets from January to July has been considered. The data sets are given in Table 1.

Table 1. Minimum temperature data for the district Silchar (Assam, India) from 1988-2018

Months	Temperature Data (Minimum Temperature)
JANUARY	11.7, 9.7, 11.6, 11.5, 13.5, 12.2, 12.5, 11.5, 12.4, 11.9, 13.3, 12.3, 12.3, 11.3, 13.5, 12.5, 13.9, 12.5, 11.9, 11.4, 12.8, 12.9, 11.1, 11.7, 12.3, 10.3, 12.1, 14.1, 12.6, 12.3, 11.9
FEBRUARY	12.2, 12.8, 13.8, 16.0, 14.0, 14.9, 13.1, 13.8, 14.8, 13.6, 15.4, 15.6, 13.9, 14.2, 14.6, 15.0, 14.1, 16.4, 16.9, 14.6, 12.6, 13.6, 10.8, 13.4, 12.7, 14.3, 12.8, 13.5, 16.3, 14.6, 14.4
MARCH	15.8, 16.1, 15.1, 19.5, 19.6, 17.3, 18.4, 16.7, 19.6, 19.0, 17.2, 18.3, 18.2, 17.9, 17.6, 17.6, 19.8, 19.7, 18.8, 16.5, 18.4, 17.9, 18.9, 17.5, 18.8, 17.6, 16.4, 17.6, 19.4, 17.3, 16.9
APRIL	19.2, 19.7, 17.5, 20.9, 22.0, 20.3, 21.1, 21.2, 21.8, 20.1, 21.5, 23.6, 21.6, 21.8, 20.8, 22.1, 21.0, 22.0, 21.5, 20.9, 22.1, 21.9, 21.7, 21.2, 20.4, 20.9, 20.8, 21.2, 21.7, 20.6, 19.9
MAY	20.0, 22.7, 21.3, 22.2, 22.5, 22.2, 24.0, 24.2, 23.5, 22.9, 24.2, 23.5, 23.5, 23.7, 22.9, 23.7, 23.8, 22.2, 23.3, 24.1, 23.7, 23.4, 23.4, 23.1, 23.0, 22.1, 23.2, 23.1, 23.2, 23.3, 21.4
JUNE	22.6, 23.3, 22.4, 24.8, 25.0, 24.5, 25.3, 25.3, 24.6, 24.6, 25.3, 25.3, 25.2, 25.4, 25.0, 24.9, 25.3, 25.6, 24.5, 25.1, 25.0, 25.1, 24.7, 25.3, 24.3, 25.1, 25.4, 25.1, 25.4, 24.6, 23.6
JULY	24.3, 23.2, 21.9, 25.9, 25.3, 25.2, 25.7, 25.6, 25.3, 25.7, 25.6, 25.4, 25.6, 25.8, 25.4, 25.9, 25.0, 25.4, 26.0, 25.7, 25.5, 25.9, 25.8, 25.5, 25.7, 24.6, 25.8, 25.7, 25.6, 25.3, 24.6

In order to compare the TIITLFD with Frechet distribution (FD), we consider the criteria like Bayesian information criterion *(BIC),* Akaike Information Criterion *(AIC),* Akaike Information Criterion Corrected *(AICC)* and $-2\log L$. The better distribution corresponds to lesser values of *AIC, BIC, AICC* and $-2\log L$. The formulae for calculating *AIC, BIC* and *AICC* are as follows:

$$AIC = 2K - 2\log L,\ BIC = k\log n - 2\log L,\ AICC = AIC + \frac{2k(k+1)}{(n-k-1)},$$

where *k* is the number of parameters, *n* is the sample size and *-2 logL* is the maximized value of log likelihood function. The ML estimates of the parameters of the considered distributions along with values of $-2\log L, AIC, AICC$ and BIC for the datasets in table 1 are presented in table 2.

Table 2. ML estimates of the parameters of the considered distributions along with values of $-2\log L$,AIC, AICC and BIC

Month	Distribution	ML Estimates of Parameters			$-2\log L$	*AIC*	*AICC*	*BIC*
		α	β	λ				
JANUARY	TIITLFD	855.7040	2.2127	580.9820	83.65	89.65	90.54	93.95
	FD	902968.012	6.4906	-----	109.77	113.77	114.66	116.64
FEBRUARY	TIITLFD	610.0494	1.9935	329.3614	104.71	110.71	115.60	115.01
	FD	237844.5	5.6279	-----	128.31	132.31	133.20	135.18
MARCH	TIITLFD	3225.3562	2.3447	1082.718	99.44	105.44	106.33	109.74
	FD	716761.3	5.5199	-----	139.62	143.62	144.51	146.49
APRIL	TIITLFD	8331.643	2.4482	9238.186	90.04	96.04	96.93	100.34
	FD	437995.8	5.0533	-----	152.98	156.99	157.88	159.85
MAY	TIITLFD	12063.11	3.1993	2436.307	76.37	82.37	83.26	86.67
	FD	312889.7	4.7984	-----	160.60	164.60	165.49	167.47
JUNE	TIITLFD	25189.16	3.3265	7596.683	78.99	68.99	69.89	73.30
	FD	905187.7	5.0160	-----	161.98	165.98	166.86	168.84
JULY	TIITLFD	25713.03	3.3140	5699.862	64.38	70.38	71.27	74.69
	FD	462271.8	4.7767	-----	166.41	170.41	171.30	173.28

It is obvious from above table 2 that TIITLFD provides much better fit than Frechet distribution for data relating to minimum temperature and hence the proposed distribution can be considered an improtant distribution for modeling minimum temperaure data.

6. Concluding remarks

In this paper Type II Topp-Leone Frechet distribution (TIITLFD) has been proposed. Its statistical properties including behaviour of pdf, cdf and hazard rate function have been discussed. The distribution of the order statistics has been given. The maximum likelihood estimation for estimating parameters of the proposed distribution has been discussed. The applications of the proposed distribution for modeling data relating to temperature has been explained and the goodness of fit of the TIITLFD and Frechet distribution has been presented for ready comparison.

Acknowledgements

Authors are grateful to the Editor-In-Chief of the journal and anonymous reviewer for constructive and fruitful comments which improved both the presentation and the quality of the paper.

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