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A minimum spanning tree analysis of the Polish stock market

Abstract

Aim/purpose – This article aims to explore the network topology of the stock market in Poland during the COVID-19 pandemic.

Design/methodology/approach – Kruskal's algorithm was used to find the minimum spanning trees (MST) of three undirected correlation networks: MST1 (December 2019 – August 2021), MST2 (February 2020 – April 2020), and MST3 (June 2021 – August 2021). There were123 firms included in all three networks representing three key indexes (WIG20, mWIG40, and sWIG80).

Findings – The comovements of stock prices varied between various periods of the pandemic. The most central firms in Poland were PEO, UNT, SPL, PKO, KGH, CCC, and PZU. WIG20 was the most influential stock index for all networks. During the turbulent period represented by MST2, many of Poland's largest companies have clustered around KGH at the center of the network. In contrast, MST3 is the least compact of the three networks and is characterized by the absence of a single strongly influential node.

Research implications/limitations – Correlation networks are efficient at quantitatively describing the degree of interdependence of a stock. MST finding algorithms are a crucial method of analysis for correlation networks. However, a limitation of the study, inherent to undirected correlation networks, is the inability to determine the direction of influence that stocks have on each other.

Originality/value/contribution – The results of the article contribute to the economic analysis of stock markets in several ways. First, it expands on Gałązka (2011) by including additional centralities and the dynamic aspect of changes in the topology during the

COVID-19 pandemic. Second, it broadens the MST-based empirical research of stock markets by showing the emergence of the star topology during the period of high uncertainty in Poland. Third, it has practical applications for systemic risk assessment and portfolio diversification.

Keywords: network analysis, minimum spanning tree, correlation network, stock market, COVID-19, Poland.
JEL Classification: D85, L14, G10, G32.

1. Introduction

Network analysis has a long and storied history in many scientific disciplines. Its origins can be traced to the 18th century when prominent mathematician Leonhard Euler (1953) provided his seminal solution to the Königsberg bridges problem. The flourishing methodology of modern network analysis can provide important insights into the workings of the ever-increasingly interconnected global financial markets. The current economic crisis is unique compared to the 2007-2008 financial collapse because the financial institutions have handled it remarkably well. Still, the impact on the stock market had been substantial and led to historic volatility in the prices. The use of network methodology provides ample opportunity to expand our understanding of financial markets, economic shocks, and systemic risk.

The financial network analysis used in this article is based on the minimum spanning tree (MST) methodology. This method was first proposed by Mantegna (1999) as a way to analyze the topology of financial markets. To perform an MST analysis, a correlation network must first be constructed. Correlation networks are relatively easy to model and can provide meaningful insights complementing or substituting traditional econometric methods. More specifically, they are simple undirected graphs that always contain an MST. In other words, they have a single connected component, which follows the definition of a complete graph. MST methodology can be used to examine numerous aspects of the financial markets. Examples include the analysis of the interdependence of national stock markets (Coelho et al., 2007; Memon & Yao, 2021; Roy & Sarkar, 2013; Wang et al., 2018; Yin et al., 2017), individual firms listed on a stock market (Balcı et al., 2021; Gałązka, 2011; Jung et al., 2006; Kanno, 2021; Wang et al., 2017), and bond markets (Dias, 2012; Gilmore et al., 2010; Pang et al., 2021).

The COVID-19 pandemic had a massive impact on the workplace, teaching, and academic research (Diab-Bahman & Al-Enzi, 2020; Dwivedi et al., 2020; Galanti et al., 2021; Ziemba & Eisenbardt, 2021). The global crisis severely

affected energy prices (Nyga-Łukaszewska & Aruga, 2020) as well as international trade in goods (Hayakawa & Mukunoki, 2021) and services (Ando & Hayakawa, 2022). Its influence also extended to the stock markets causing immense volatility and uncertainty (Ashraf, 2020; He et al., 2020; Liu et al., 2020). The changes that occurred in the network topology during the pandemic provide a significant research opportunity.

This article aims to explore the network topology of the stock market in Poland during the COVID-19 pandemic. Two research questions are explored:

RQ1: Which companies are the most central in Poland?

RQ2: Has the total weight of the minimum spanning tree changed during the analyzed period in Poland?

To provide answers to both research questions explored in this article, three undirected correlation networks were conducted. An MST was found for each network, which is a subgraph with all the nodes and the smallest possible subset of edges connecting them all. For weighted networks, an MST minimizes the sum of edge weights (also known as distance). In general terms, correlation networks quantify the interdependence of stocks. An MST algorithm allows us to filter the relevant data of a correlation network and provides an easily distinguishable visualization of the entire stock market. It is a relatively novel method for economics and finance but has a storied history in mathematics that can be traced to 1926 and the works of Otakar Borůvka (Graham & Hell, 1985; Kruskal, 1956; Nešetřil et al., 2001; Prim, 1957). One popular application of MSTs in mathematical problems is the journey of a traveling salesman, who has to visit every city taking the shortest route (Kruskal, 1956; Prim, 1957).

This article comprises six sections (including the introduction). The literature review section focuses primarily on previous studies utilizing the MST methodology and non-network financial analysis. The research methodology section explains the steps necessary to create correlation networks. The edges of the networks are filtered and three commonly used centrality measures are calculated for the MSTs (degree, closeness, and betweenness). The research findings section shows that the most central firms in Poland were PEO (Bank Polska Kasa Opieki), UNT (Unimot), SPL (Santander Bank Polska), PKO (Powszechna Kasa Oszczędności Bank Polski), KGH (KGHM Polska Miedź), CCC, and PZU (Powszechny Zakład Ubezpieczeń). The star topology (centered around KGH) emerged during the period of high uncertainty in Poland. The final two sections concern the discussion and the conclusions. The names and symbols of all firms included in the analysis are provided in the Appendix.

2. Literature review

The growing interdependence of financial markets (Szyszka, 2011) and business networks (Tomeczek, 2022) compels economics to adopt multidisciplinary and alternative methods. The modern banking sector's systemic risk measures, while vastly improved, are still not foolproof (Sum, 2016). Allen et al. (2014) explained that the international transmission of liquidity shocks forces foreign subsidiaries of banks to reduce their credit supply during a financial crisis. Modern financial networks might already be too complex to successfully regulate systemic risk which might lead to excessive social costs (Battiston et al., 2016).

Many network analysis methods have been adapted to financial modeling from social network analysis. As such, while financial network analysis is a relatively new methodology, it is growing rapidly and already has considerable tools at its disposal. Tomeczek (2021) identified four major types of financial networks based on what the edges represent: cross-shareholding (tiered structure of equity holders), correlation (correlation between returns on financial instruments), debt (liabilities of financial institutions or firms), and Granger-causality (longitudinal causal effects based on F-statistic or p-value).

There have been multiple global and regional studies exploring the various aspects of financial networks. Analyses of credit contagion in national cross-shareholding financial networks include Kanno (2019) for Japan, Ma et al. (2011) and Li et al. (2014) for China, and Dastkhan and Shams Gharneh (2016) for Iran. Vitali et al. (2011) created a vast static cross-shareholding network of global multinationals with over 600 thousand nodes. In a similar study, Brancaccio et al. (2018) expanded that methodology to include multiple years. Granger causality networks include the influential methodology of Billio et al. (2012) as well as the more recent studies by Tang et al. (2019) and Yun et al. (2019).

The focus of this article is on correlation networks. Dungey et al. (2012) showed that firms in the financial sector are the most interconnected in the correlation network of all S&P 500 firms in the United States. Huang et al. (2009) constructed a threshold-based correlation network for the stock market in China, while Nobi et al. (2014) focused on South Korea. Lee et al. (2019) showed that correlation networks can be efficient at forecasting changes in global stock markets.

MST finding algorithms are a crucial method of analysis for correlation networks. Gilmore et al. (2010) used an MST to analyze the global bond market, while Dias (2012) and Pang et al. (2021) researched European economies. Jang et al. (2011) explored the international impacts of currency crises. Kazemilari et al. (2017) created an international MST of 70 companies in the renewable energy sector. Coelho et al. (2007) and Kwon and Yang (2008) investigated the global interdependence of stock market indices.

A common theme in the MST literature is the global interdependence of national stock markets (Coelho et al., 2007; Kwon & Yang, 2008; Memon & Yao, 2021; Roy & Sarkar, 2013; Wang et al., 2018; Yin et al., 2017). Such studies look into correlations between national stock market indexes to identify the network position of specific countries. Another major theme in the literature is the analysis focusing on firms listed on a specific national stock market. In this case, researchers attempt to calculate the network centralities of individual firms. Examples of such studies include China (Long et al., 2017), Germany (Birch et al., 2016; Wiliński et al., 2013), Greece (Garas & Argyrakis, 2007), Japan (Kanno, 2021), Pakistan (Memon et al., 2020), Poland (Gałązka, 2011), South Korea (Jung et al., 2006; Lee et al., 2012), the United Kingdom (Balcı et al., 2021), and the United States (Gan & Djauhari, 2015; Miccichè et al., 2003; Onnela et al., 2003a; Tumminello et al., 2007; Wang et al., 2017).

MST analysis has practical applications for portfolio selection and systemic risk assessment. Stocks located on the outskirts of the MST provide greater potential for risk diversification (Onnela et al., 2003b). Specifically, these would be leaves representing the firms that have been pushed to the opposite peripheries of the network (a leaf is a node with a degree of one). These results are consistent with the modern portfolio theory of Markowitz (1952, 1991). This finding has important practical applications for investors as MSTs can be a useful tool for portfolio creation (Danko et al., 2022; Danko & Šoltés, 2018). MSTs tend to shrink during a crisis period, as shown by the analysis of the Black Monday crash of 1987 (Onnela et al., 2003a). This shrinkage is also present when looking at intraday prices during periods of high volatility (Lee et al., 2012). The impact of a crisis on a stock market can be quantified by the total weight of an MST and the rapid changes in network topology.

Other innovative applications of MSTs in finance include stock returns prediction based on the book-to-market ratio (Brookfield et al., 2013), systemic risk analysis in the insurance industry (Denkowska & Wanat, 2020), and interbank lending market (Luo et al., 2015). Of course, this methodology can also be used to analyze the interdependence in other international financial markets such as bonds (Dias, 2012; Gilmore et al., 2010) and currencies (Jang et al., 2011; Wang et al., 2012).

To our knowledge, the first study to apply the MST methodology to the Polish stock market is Gałązka (2011), which examined the prices of 252 firms in 2007. This article expands on that study by calculating additional centralities and capturing the dynamic aspect of changes in the topology during a crisis using three MSTs constructed for the uniquely challenging COVID-19 pandemic. Previous non-network studies of the Polish stock market tackle issues such as the association between corporate governance and financial resilience (Gruszczyński, 2006), global

crisis contagion (Konopczak et al., 2010), herding behavior among investors (Goodfellow et al., 2009), the impact of monetary policy announcements (Brzeszczyński et al., 2021), the value of analyst recommendations (Wnuczak, 2021), determinants of asset prices (Rutkowska-Ziarko & Markowski, 2022; Waszczuk, 2013), portfolio selection (Dzicher, 2021; Giemza, 2021; Witkowska et al., 2021), and interdependence with other markets (Li & Majerowska, 2008; Scheicher, 2001).

3. Research methodology

Correlation networks are efficient at quantitatively describing the degree of interdependence of a stock. Easily interpreted higher centrality shows that a stock is important for the entire system, which can guide regulators. Furthermore, these networks are relatively straightforward to model and do not rely on heavily abstracted assumptions. However, a limitation of the study, inherent to undirected networks and correlation networks, is the inability to determine the direction of influence that stocks have on each other. In other words, the results show that some stocks are heavily interlinked in the market, but the research is unable to specify the role that they play (lead-lag effect). Disregarding the direction of the influence provides additional analysis options only suitable to undirected networks such as an MST. A successful application of an MST finding algorithm requires the construction of an undirected correlation network with no parallel edges.

The methodology of an MST analysis of financial correlation networks is taken from multiple previous studies (Coelho et al., 2007; Dias, 2012; Gałązka, 2011; Gilmore et al., 2010; Jung et al., 2006; Kazemilari et al., 2017; Mantegna, 1999). Let $G = (V, E)$, where G is the graph (network), V is the set of n nodes (vertices) and E is the set of m edges (links). The network is a simple undirected graph (with no parallel edges) and represented by a symmetrical matrix $n \, x \, n$. To start, define the daily log-return of a stock as:

$$R_i(t) = ln \, X_i(t) - ln \, X_i(t - \Delta t) \tag{1}$$

where $X_i(t)$ is the daily close stock price of firm i at time t (the time interval Δt is equal to one day). Next, a Pearson correlation matrix is calculated using the vectors of daily log-returns of stocks:

$$r_{ij} = \frac{\langle R_i R_j \rangle - \langle R_i \rangle \langle R_j \rangle}{\sqrt{\left(\langle R_i^2 \rangle - \langle R_i \rangle^2 \right) \left(\langle R_j^2 \rangle - \langle R_j \rangle^2 \right)}} \tag{2}$$

where brackets indicate a time average, r_{ij} is the correlation between the daily returns of stocks i and j, R_i is the vector of daily log-returns of stock i, and R_j is the vector of daily log-returns of stock j. Coefficients in the Pearson correlation matrix range from –1 to 1. To properly use these data for an MST algorithm, they need to be converted so that there are no negative values and lower values equal stronger correlation (MST algorithms minimize edge weights). Finally, the following equation is used for the data transformation of the correlation coefficient matrix into a distance matrix:

$$d_{ij} = \sqrt{2(1 - r_{ij})} \qquad\qquad (3)$$

where d_{ij} is the distance between nodes i and j and r_{ij} is the correlation between the daily log-returns of stocks i and j. We now have a symmetric matrix $n \; x \; n$ with no negative values. The distance ranges from 0, which indicates perfect correlation, to 2, which indicates perfect negative correlation. The distance matrix is symmetric ($d_{ij} = d_{ji}$). The data can now be used to create an undirected distance network where the MST algorithm can properly minimize edge weights – lower edge weight represents higher correlation. For the MST algorithm, we use a plugin for Gephi based on Kruskal (1956). Another possible MST algorithm is detailed by Prim (1957), but most of the reviewed literature used Kruskal's. These steps are repeated three times to construct three undirected networks corresponding to selected periods (MST1, MST2, and MST3).

MST1 corresponds to the first three waves of the pandemic in Poland, which includes the period when the cases of COVID-19 were still relatively low (December 2019 – August 2020), the period of very high daily numbers of cases (September 2020 – May 2021), and the subsequent period of calm during the summer (June 2021 – August 2021); in total MST1 has 418 daily observations. MST2 corresponds to the period of the initial lockdown and uncertainty of the early pandemic in Poland (February 2020 – April 2020); MST2 has 62 daily observations. MST3 corresponds to the summer months and low daily case numbers in Poland (June 2021 – August 2021); MST3 has 65 daily observations. There are 123 firms included in all three networks, which is important for comparisons between the MSTs (the complete list of firms is available in the appendix). The initial overview concerned 140 largest firms in Poland representing three key indexes (WIG20, mWIG40, and sWIG80); 17 of those firms were excluded from the analysis due to data availability. The most notable of the excluded firms is Allegro.eu, which debuted in October 2020. The period analyzed in the main network (MST1) is the longest and comprises the data from smaller networks (MST2 and MST3). As such, MST2 and MST3 provide the important element of contrasting market expectations (initial uncertainty for the former and general optimism for the latter).

Following the existing literature on correlation networks and MSTs, three commonly used centrality measures were investigated: degree, closeness, and betweenness. Degree measures the local importance of a node while closeness and betweenness measure the systemic importance of a node. Three basic centrality measures were calculated to show which companies, represented by nodes, are the most important for the MST network: degree (Equation 4), closeness (Equation 5), and betweenness (Equation 6). The calculated centralities are represented by equations taken from Brandes and Erlebach (2005). By far, the simplest one is the degree centrality:

$$c_D(v) = d(v) \tag{4}$$

where degree centrality $c_D(v)$ of node v is equal to its degree $d(v)$. Node's degree is simply the number of edges connected to it. Nodes with a value of higher degree centrality have more connections. A higher value of degree centrality means that a node has a stronger local influence over local nodes. In many networks, degree centrality is considered to be too basic compared to measures such as PageRank or eigenvector centrality (which are preferred in large cross--shareholding networks), but for MST networks degree centrality is commonly used as the MST algorithm already filters most of the excess information.

In contrast to local importance (measured by degree centrality), systemic importance is calculated using closeness centrality and betweenness centrality. They are both based on the concept of the shortest path and look at the network position of a node. Closeness centrality measures the length of the shortest paths connecting the node to every other node in the network. Nodes with high closeness centrality are located close to the center of the network. Betweenness centrality represents the access to information passed between nodes. It shows how many of the shortest paths in a graph include the node of interest.

Closeness centrality shows how centrally placed is the node in the network. It measures the node's distance to every other node:

$$c_C(u) = \frac{1}{\sum_{v \in V} d(u,v)} \tag{5}$$

where closeness centrality $c_C(u)$ of node u is equal to the reciprocal of the sum of all shortest paths of node u, with $d(u,v)$ being the shortest path between nodes u and v. In other words, if the distance to every node is relatively short, the closeness centrality will be higher.

Betweenness centrality represents the flow of information that the node has access to:

$$c_B(v) = \sum_{s \neq v \in V} \sum_{t \neq v \in V} \delta_{st}(v) \tag{6}$$

where betweenness centrality $c_B(v)$ of node v depends on $\delta_{st}(v)$, which is the number of shortest paths that pass through node v divided by the number of total shortest paths in the network. High betweenness centrality represents better access to information passed between nodes. Alternatively, it shows how different the network flow would be. If a node with very high betweenness centrality were to be removed, the flow would be drastically altered.

Nodes with very high closeness and betweenness centralities are integral to the networks: if we were to remove them, the resulting graph would be drastically different (depending on the value of those centralities). A node can simultaneously have high closeness centrality and low betweenness centrality. For example, if node v_i in an MST network has only one direct connection to node v_j and v_j is a highly influential node at the center of the network, then v_i has high closeness centrality but its betweenness centrality is equal to zero.

The network visualizations are constructed with Gephi (open-source network modeling software) using the Yifan Hu layout algorithm with some manual adjustments for better visibility. The list of firms listed on the Warsaw Stock Exchange comes from its official website (GPW, 2021). The daily close stock price data are imported using Microsoft Excel's STOCKHISTORY function from the Refinitiv database. A supplemental search is performed in EquityRT (2021) for stocks that were unavailable using STOCKHISTORY. A plugin for Gephi based on Kruskal's (1956) famous algorithm is used to find the MSTs. In practice, an edge weight of 0 would cause problems with network creation since the software would treat it as a missing edge; however, perfect correlation is virtually impossible for the data used (ignoring the main diagonal of the correlation matrix). Closeness and betweenness centralities are calculated using the algorithm of Brandes (2001).

4. Research findings

The total weight of each MST is as follows: 126.94 for MST1 (Figure 1), 91.79 for MST2 (Figure 2), and 130.75 for MST3 (Figure 3). These results show that the comovements of stock prices varied between various periods of the pandemic. The fact that MST2 has the lowest weight is in line with the expectations and previous research that show the emergence of the influential node during financial crises. The investors faced lockdowns and uncertainty which resulted in increased volatility during the three months of the early pandemic. Many of the stocks fell together (e.g., the banking sector) while others rose (e.g., the medical sector). Importantly, the entire market does not have to move in unison for

the MST's weight to fall, as long as there are low-distance edges available for most nodes. In contrast, MST3's weight is higher than MST1's. As the exogenous pressure abated, the Polish stock market continued its recovery. During this summer period, most firms have gained while some have lost but done so at their own pace each day. The weight of an MST quantifies the price comovements of the stock market in a single, easily comparable number (if the networks have the same number of nodes).

Figure 1. Visualization of MST1
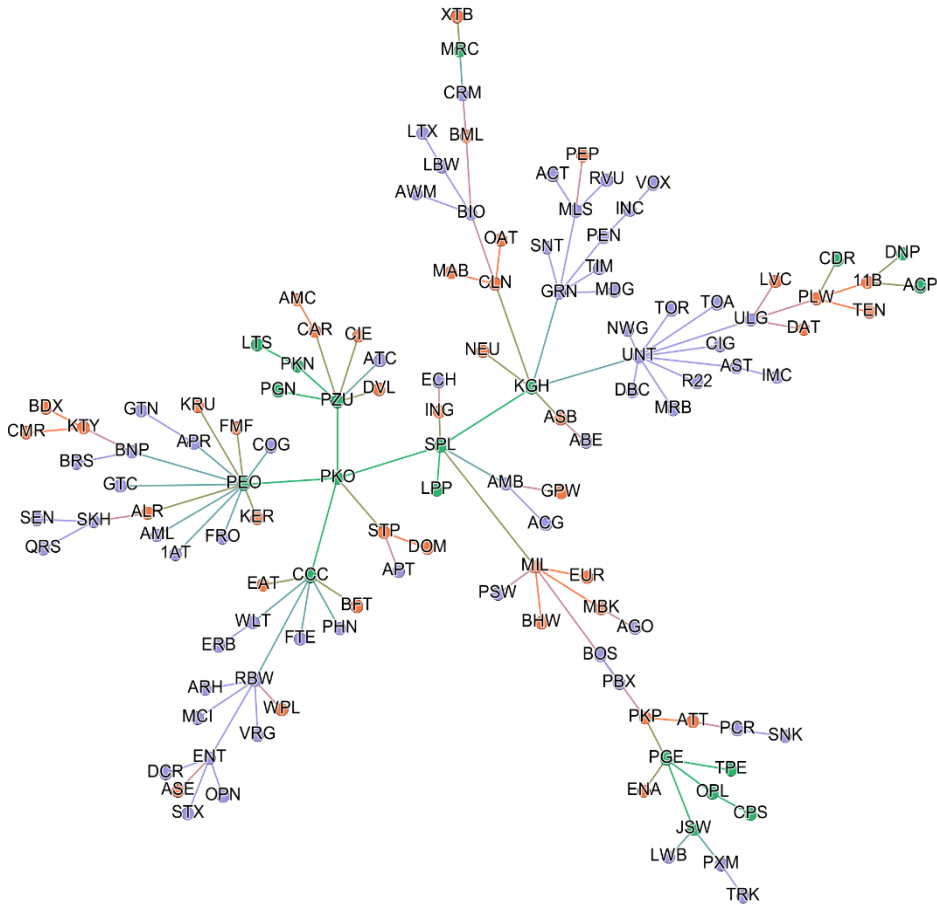
Source: Author's own calculations.

Figure 2. Visualization of MST2
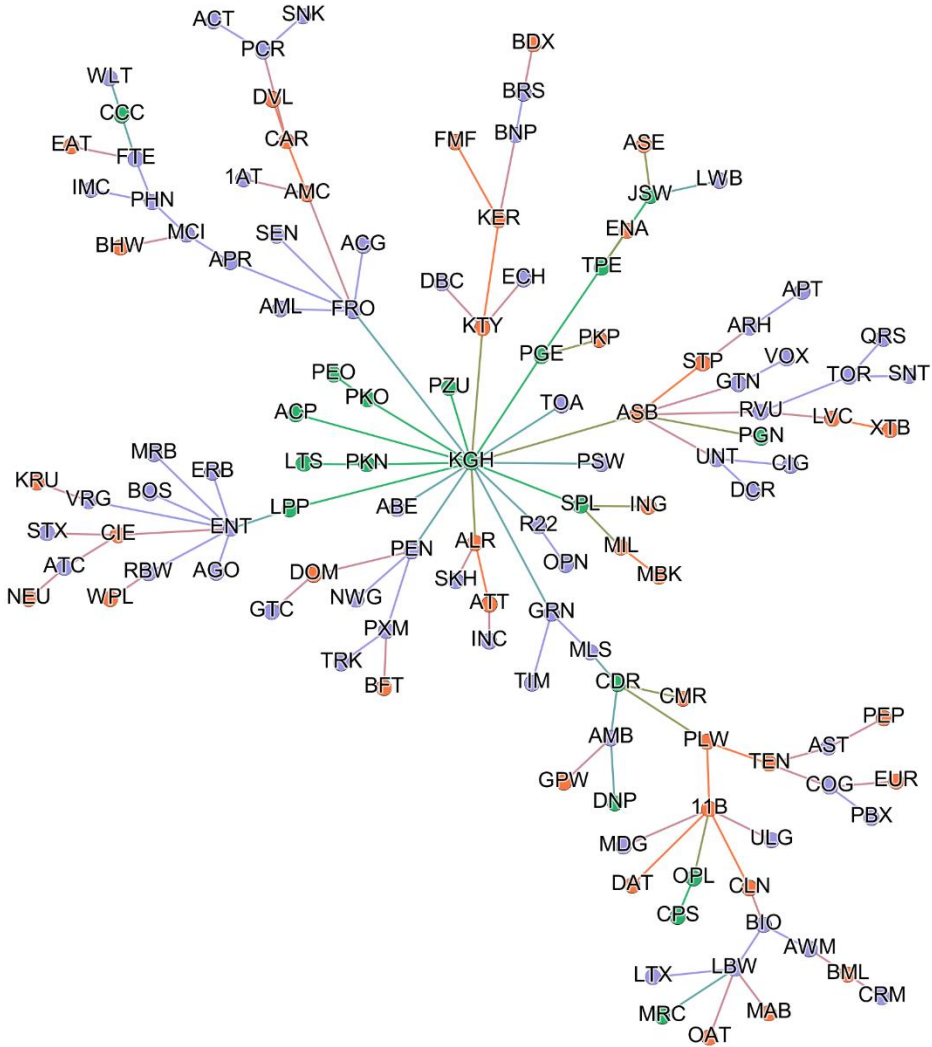
Source: Author's own calculations.

Figure 3. Visualization of MST3
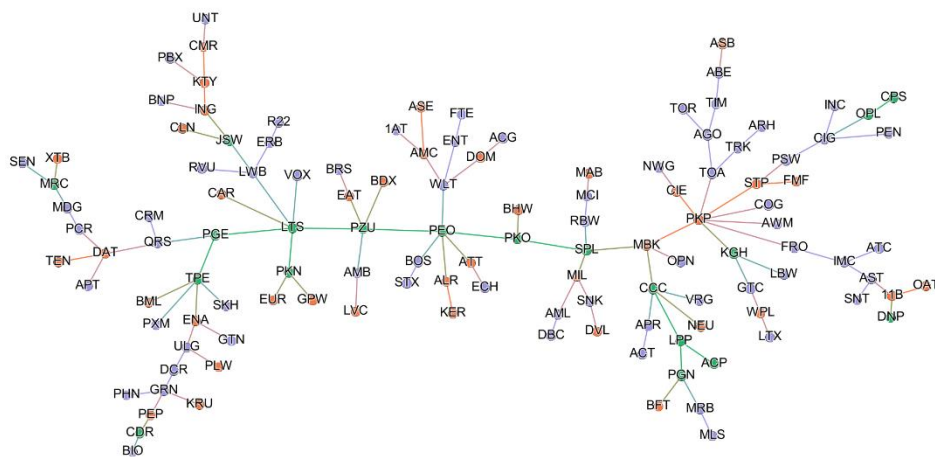

Source: Author's own calculations.

Table 1 shows the rank of the 20 most influential firms according to the selected centrality measures for MST1. Table 2 gives the results for MST2 and MST3. In case of a draw, the firms are listed alphabetically as there are many firms with the same degree centrality in MST networks. Degree centrality is a measure of local influence (direct connections), while closeness and betweenness quantify systemic influence (position in the network). MST1 has the most observations and is the most representative of the entire analyzed period, but MST2 and MST3 provide an opportunity to examine whether the topology has undergone significant changes during the periods of contrasting market sentiment. Table 3 in the appendix lists the names and symbols of all firms included in the analysis.

Table 1. Firms with the highest centrality scores (MST1)

Degree			Closeness		Betweenness	
1			*2*		*3*	
BANK POLSKA KASA OPIEKI	PEO	12	SPL	0.284	SPL	0.671
UNIMOT	UNT	10	PKO	0.270	PKO	0.594
CCC	CCC	7	KGH	0.264	KGH	0.555
POWSZECHNY ZAKŁAD UBEZPIECZEŃ	PZU	7	MIL	0.239	MIL	0.288
GRODNO	GRN	6	PEO	0.228	PEO	0.286
KGHM POLSKA MIEDŹ	KGH	6	CCC	0.226	UNT	0.269
BANK MILLENNIUM	MIL	6	AMB	0.223	CCC	0.240
RAINBOW TOURS	RBW	6	UNT	0.223	BOS	0.207
SANTANDER BANK POLSKA	SPL	6	ING	0.223	PKP	0.182
ENTER AIR	ENT	5	LPP	0.222	GRN	0.157
PGE POLSKA GRUPA ENERGETYCZNA	PGE	5	PZU	0.219	CLN	0.154

Table 1 cont.

1			2		3	
POWSZECHNA KASA OSZCZĘDNOŚCI BANK POLSKI	PKO	5	GRN	0.217	RBW	0.141
BIOTON	BIO	4	CLN	0.217	PZU	0.127
CELON PHARMA	CLN	4	STP	0.214	PGE	0.126
ML SYSTEM	MLS	4	ASB	0.210	ULG	0.125
PLAYWAY	PLW	4	NEU	0.209	BIO	0.111
ULTIMATE GAMES	ULG	4	BOS	0.202	PLW	0.080
11 BIT STUDIOS	11B	3	MBK	0.194	ENT	0.065
AMBRA	AMB	3	BHW	0.193	BNP	0.064
BNP PARIBAS BANK POLSKA	BNP	3	EUR	0.193	MLS	0.049

Source: Author's own calculations.

Table 2. Firms with the highest centrality scores (MST2 and MST3)

| MST2						MST3					
Degree		Closeness		Betweenness		Degree		Closeness		Betweenness	
KGH	17	KGH	0.277	KGH	0.871	PKP	8	PEO	0.192	PEO	0.606
ENT	8	GRN	0.244	GRN	0.378	LTS	6	PKO	0.190	SPL	0.552
11B	6	FRO	0.233	CDR	0.360	PEO	6	SPL	0.187	MBK	0.530
ASB	6	ASB	0.230	MLS	0.357	CCC	5	PZU	0.185	LTS	0.529
FRO	6	LPP	0.228	PLW	0.311	PZU	5	MBK	0.179	PZU	0.516
LBW	5	KTY	0.223	FRO	0.281	TPE	5	LTS	0.176	PKO	0.510
CDR	4	PEN	0.222	ASB	0.229	CIG	4	PKP	0.166	PKP	0.457
KTY	4	PGE	0.222	11B	0.225	DAT	4	WLT	0.164	PGE	0.349
PEN	4	ALR	0.220	LPP	0.192	GRN	4	ALR	0.162	TPE	0.210
ALR	3	SPL	0.220	ENT	0.187	LWB	4	ATT	0.162	LWB	0.169
AMB	3	PKN	0.218	CLN	0.138	MBK	4	BOS	0.162	CCC	0.156
AMC	3	PKO	0.218	BIO	0.126	SPL	4	BHW	0.160	ENA	0.153
BIO	3	R22	0.218	APR	0.124	WLT	4	PGE	0.160	QRS	0.139
CAR	3	ABE	0.217	KTY	0.111	11B	3	MIL	0.159	ULG	0.125
CIE	3	ACP	0.217	MCI	0.110	AGO	3	RBW	0.158	WLT	0.111
COG	3	PSW	0.217	PEN	0.096	AMC	3	AMB	0.157	DAT	0.111
FTE	3	PZU	0.217	PGE	0.095	AST	3	EAT	0.157	TOA	0.110
GRN	3	TOA	0.217	AMC	0.095	ENA	3	BDX	0.157	STP	0.110
JSW	3	MLS	0.216	RVU	0.080	IMC	3	CCC	0.156	JSW	0.110
KER	3	TIM	0.196	TEN	0.080	ING	3	LWB	0.154	FRO	0.109

Source: Author's own calculations.

For MST1, the firms with the highest degree are PEO (12), UNT (10), CCC (7), and PZU (7). The highest closeness and betweenness are observed for SPL, PKO, and KGH. The situation in MST2 is clear-cut as the firm with, by far, the highest degree is KGH (17). KGH also dominates the network as the most central node according to closeness and betweenness. For betweenness centrality,

the gap between KGH (0.871) and the second-ranked GRN (0.378) is especially large. Finally, MST3 is characterized by a relatively low degree centrality of its leading nodes: PKP (8), LTS (6), and PEO (6). The highest-ranked closeness and betweenness firms in MST3 are all commercial banks (PEO, PKO, SPL, and MBK).

Regarding the stock indexes, the constituents of WIG20 are very influential in all three MSTs. The core of MST1 is formed by SPL (WIG20), PKO (WIG20), and KGH (WIG20), with two peripheral clusters centered around PEO (WIG20) and UNT (sWIG80). During the turbulent period represented by MST2, the core of the network is overwhelmingly formed around KGH (WIG20). KGH's very large cluster includes large firms such as PKO (WIG20), PKN (WIG20), and PZU (WIG20), as well as some smaller firms. Finally, while MST3 has no single influential firm at its core, relatively high systemic importance is noted for PEO (WIG20), SPL (WIG20), MBK (mWIG40), LTS (WIG20), PZU (WIG20), and PKO (WIG20). MST3's largest peripheral cluster is centered around PKP (mWIG40).

A crucial result of the study is that MST2 shows the emergence of the star topology centered around KGH. The node has the strongest local influence and systemic influence, by all revenant metrics. MST3 has no trace of star topology as its tree is the least compact. MST1 has two nodes with relatively strong local clusters: PEO and UNT. However, both are pushed to the peripheries of the network and do not perform the function of systemic influencers. Interestingly, firms clustering around UNT are all sWIG80 companies. Video game developers, including CDR, also form a small cluster around PLW. Another example of a small cluster, representing the energy industry, is formed around PGE. Figure 4 provides degree distributions that further illustrate the local dominance of KGH in MST2 and the lack of dominant nodes in MST3.

Figure 4. Degree distributions
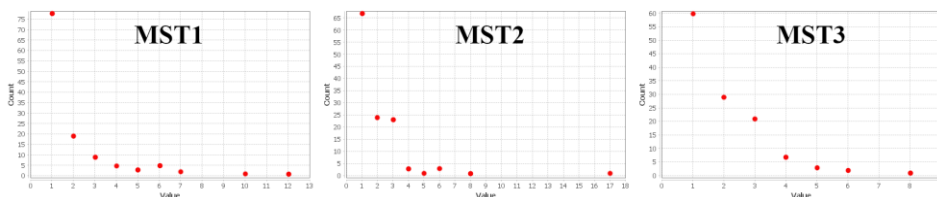
Source: Author's own calculations.

5. Discussion

Correlation networks have numerous advantages (they are compatible with MST algorithms, relatively easy to calculate and interpret, and do not require arbitrary cutoffs for filtering) and one significant limitation (the inability to determine the direction of influence).

MST network identifies central companies that have pushed the market down or pulled it up, which is important for financial economics. The method can also reveal surprising results, as a large company can have its ups and downs separate from most of the key players. This methodology can also help guide regulators and warn of clusters that can be sources of potential volatility in the market. Another key practical application of the MST method is portfolio risk diversification (distant nodes provide opportunity for risk diversification). Various node-level centrality measures (degree, closeness, and betweenness) combined with network-level statistics (total MST weight) are easily interpretable and can be used by economists and regulatory institutions alike. Finally, MST graphs are relatively straightforward to interpret and visually appealing to the general audience.

Government ownership of equity is common for the largest enterprises in Poland. As such, it can partially explain the interdependence between most WIG20 companies, which is especially noticeable in MST2. The following is a short overview of the MSTs' central firms and which industries they represent. The value in brackets stands for the most recently available net income taken from the ORBIS database (BvD, 2022). Starting with the WIG20 firms, PEO ($536 million) is the second largest commercial bank in Poland. PKO ($1.2 billion) is the largest commercial bank and Poland's largest firm by market capitalization. SPL ($308 million) is a subsidiary of Banco Santander, a large multinational bank from Spain. KGH ($1.52 billion) is a Polish multinational mining company specializing in copper and silver. LTS ($791 million) is a large oil company in Poland. Other than SPL, all the listed WIG20 companies are directly or indirectly controlled by the government of Poland. The most important constituent of mWIG40 is PKP (–$55.5 million), which is a railway freight transport operator and a part of the government-controlled PKP Group. Finally, three sWIG80 firms should be highlighted. UNT ($18.8 million) is an independent oil importer and wholesaler. ENT (–$41 million) is a charter airline. GRN ($3.46 million) is a distributor of electrotechnical products.

6. Conclusions

This article uses network methodology to show the changes in the network topology of the stock market during the COVID-19 pandemic in Poland. Three undirected correlation networks are constructed and Kruskal's algorithm is used to find the minimum spanning trees. MST1 corresponds to the first three waves of the pandemic in Poland (December 2019 – August 2021). MST2 corresponds to the period of the initial lockdown and uncertainty (February 2020 – April 2020). MST3 corresponds to the relatively calm summer months (June 2021 – August 2021). There are 123 firms included in all three networks representing three key indices (WIG20, mWIG40, and sWIG80).

The results of the article contribute to the economic analysis of stock markets in several ways. First, it expands on Gałązka (2011) by including additional centralities and the dynamic aspect of changes in the topology during the COVID-19 pandemic. Second, it broadens the MST-based empirical research of stock markets by showing the emergence of the star topology during the period of high uncertainty in Poland. Third, it has practical applications for systemic risk assessment and portfolio diversification.

Despite the numerous advantages mentioned in the article, correlation networks are limited by the inability to determine the direction of influence. Future research can explore additional aspects of the financial market in Poland by utilizing methods based on directed networks (e.g., Granger causality network). Additionally, the fact that financial market maturity might have an impact on network topology warrants further studies. For example, Jung et al. (2006) hypothesized that some chaebol firms in Korea are too large relative to the rest of the stock market, which causes them to have limited price comovements with smaller firms. Another interesting approach would be to compare the centralities of firms during the COVID-19 pandemic to those calculated during any future crises.

Regarding the first research question, the most central firms in Poland during the entire analyzed period were PEO, UNT, SPL, PKO, KGH, CCC, and PZU. WIG20 was the most influential stock index for all networks. During the turbulent period represented by MST2, many of Poland's largest companies have clustered around KGH at the center of the network. In contrast, MST3 is the least compact of the three networks and is characterized by the absence of a single strongly influential node. In general, the financial sector was the most central for the entire analyzed period, but the mining, energy, retail, and transportation industries also played important roles.

With regards to the second research question, MST analysis shows that the comovements of stock prices varied between various periods of the pandemic. The total weight of MST2 (91.79), corresponding to the initial lockdown and uncertainty, is much lower than MST1 (126.94), commensurate with the entire analyzed period, and MST3 (130.75), which represents the recovery during the summer months. The fact that the network representing the high volatility of the initial phase of the financial crisis has the lowest total weight is in line with the expectations and previous research.

The MST network position of firms provides an important signal to investors and regulators. The central firm of the star topology (KGH) and its direct connections show that many of the largest companies in Poland are interdependent when it comes to the stock market during a crisis period.

Disclosure statement

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Appendix

Table 3. List of firms included in all three networks

Symbol	Full name	Index
1	*2*	*3*
ACP	ASSECO POLAND SPÓŁKA AKCYJNA	WIG20
CCC	CCC SPÓŁKA AKCYJNA	WIG20
CDR	CD PROJEKT SPÓŁKA AKCYJNA	WIG20
CPS	CYFROWY POLSAT SPÓŁKA AKCYJNA	WIG20
DNP	DINO POLSKA SPÓŁKA AKCYJNA	WIG20
JSW	JASTRZĘBSKA SPÓŁKA WĘGLOWA SPÓŁKA AKCYJNA	WIG20
KGH	KGHM POLSKA MIEDŹ SPÓŁKA AKCYJNA	WIG20
LPP	LPP SPÓŁKA AKCYJNA	WIG20
LTS	GRUPA LOTOS SPÓŁKA AKCYJNA	WIG20
MRC	MERCATOR MEDICAL SPÓŁKA AKCYJNA	WIG20
OPL	ORANGE POLSKA SPÓŁKA AKCYJNA	WIG20
PEO	BANK POLSKA KASA OPIEKI SPÓŁKA AKCYJNA	WIG20
PGE	PGE POLSKA GRUPA ENERGETYCZNA SPÓŁKA AKCYJNA	WIG20
PGN	POLSKIE GÓRNICTWO NAFTOWE I GAZOWNICTWO SPÓŁKA AKCYJNA	WIG20
PKN	POLSKI KONCERN NAFTOWY ORLEN SPÓŁKA AKCYJNA	WIG20
PKO	POWSZECHNA KASA OSZCZĘDNOŚCI BANK POLSKI SPÓŁKA AKCYJNA	WIG20
PZU	POWSZECHNY ZAKŁAD UBEZPIECZEŃ SPÓŁKA AKCYJNA	WIG20
SPL	SANTANDER BANK POLSKA SPÓŁKA AKCYJNA	WIG20
TPE	TAURON POLSKA ENERGIA SPÓŁKA AKCYJNA	WIG20
11B	11 BIT STUDIOS SPÓŁKA AKCYJNA	mWIG40
ALR	ALIOR BANK SPÓŁKA AKCYJNA	mWIG40
AMC	AMICA SPÓŁKA AKCYJNA	mWIG40
ASB	ASBISC ENTERPRISES PLC	mWIG40
ASE	ASSECO SOUTH EASTERN EUROPE SPÓŁKA AKCYJNA	mWIG40
ATT	GRUPA AZOTY SPÓŁKA AKCYJNA	mWIG40
BDX	BUDIMEX SPÓŁKA AKCYJNA	mWIG40
BFT	BENEFIT SYSTEMS SPÓŁKA AKCYJNA	mWIG40
BHW	BANK HANDLOWY W WARSZAWIE SPÓŁKA AKCYJNA	mWIG40
BML	BIOMED-LUBLIN WYTWÓRNIA SUROWIC I SZCZEPIONEK SPÓŁKA AKCYJNA	mWIG40
CAR	INTER CARS SPÓŁKA AKCYJNA	mWIG40
CIE	CIECH SPÓŁKA AKCYJNA	mWIG40
CLN	CELON PHARMA SPÓŁKA AKCYJNA	mWIG40
CMR	COMARCH SPÓŁKA AKCYJNA	mWIG40
DAT	DATAWALK SPÓŁKA AKCYJNA	mWIG40
DOM	DOM DEVELOPMENT SPÓŁKA AKCYJNA	mWIG40
DVL	DEVELIA SPÓŁKA AKCYJNA	mWIG40
EAT	AMREST HOLDINGS SE	mWIG40
ENA	ENEA SPÓŁKA AKCYJNA	mWIG40
EUR	EUROCASH SPÓŁKA AKCYJNA	mWIG40
FMF	FAMUR SPÓŁKA AKCYJNA	mWIG40
GPW	GIEŁDA PAPIERÓW WARTOŚCIOWYCH W WARSZAWIE SPÓŁKA AKCYJNA	mWIG40
ING	ING BANK ŚLĄSKI SPÓŁKA AKCYJNA	mWIG40
KER	KERNEL HOLDING S.A.	mWIG40

Table 3 cont.

1	2	3
KRU	KRUK SPÓŁKA AKCYJNA	mWIG40
KTY	GRUPA KĘTY SPÓŁKA AKCYJNA	mWIG40
LVC	LIVECHAT SOFTWARE SPÓŁKA AKCYJNA	mWIG40
MAB	MABION SPÓŁKA AKCYJNA	mWIG40
MBK	MBANK SPÓŁKA AKCYJNA	mWIG40
MIL	BANK MILLENNIUM SPÓŁKA AKCYJNA	mWIG40
NEU	NEUCA SPÓŁKA AKCYJNA	mWIG40
OAT	ONCOARENDI THERAPEUTICS SPÓŁKA AKCYJNA	mWIG40
PEP	POLENERGIA SPÓŁKA AKCYJNA	mWIG40
PKP	PKP CARGO SPÓŁKA AKCYJNA	mWIG40
PLW	PLAYWAY SPÓŁKA AKCYJNA	mWIG40
STP	STALPRODUKT SPÓŁKA AKCYJNA	mWIG40
TEN	TEN SQUARE GAMES SPÓŁKA AKCYJNA	mWIG40
WPL	WIRTUALNA POLSKA HOLDING SPÓŁKA AKCYJNA	mWIG40
XTB	X-TRADE BROKERS DOM MAKLERSKI SPÓŁKA AKCYJNA	mWIG40
1AT	ATAL SPÓŁKA AKCYJNA	sWIG80
ABE	AB SPÓŁKA AKCYJNA	sWIG80
ACG	AC SPÓŁKA AKCYJNA	sWIG80
ACT	ACTION SPÓŁKA AKCYJNA	sWIG80
AGO	AGORA SPÓŁKA AKCYJNA	sWIG80
AMB	AMBRA SPÓŁKA AKCYJNA	sWIG80
AML	ALUMETAL SPÓŁKA AKCYJNA	sWIG80
APR	AUTO PARTNER SPÓŁKA AKCYJNA	sWIG80
APT	APATOR SPÓŁKA AKCYJNA	sWIG80
ARH	ARCHICOM SPÓŁKA AKCYJNA	sWIG80
AST	ASTARTA HOLDING N.V.	sWIG80
ATC	ARCTIC PAPER SPÓŁKA AKCYJNA	sWIG80
AWM	AIRWAY MEDIX SPÓŁKA AKCYJNA	sWIG80
BIO	BIOTON SPÓŁKA AKCYJNA	sWIG80
BNP	BNP PARIBAS BANK POLSKA SPÓŁKA AKCYJNA	sWIG80
BOS	BANK OCHRONY ŚRODOWISKA SPÓŁKA AKCYJNA	sWIG80
BRS	BORYSZEW SPÓŁKA AKCYJNA	sWIG80
CIG	CI GAMES SPÓŁKA AKCYJNA	sWIG80
COG	COGNOR HOLDING SPÓŁKA AKCYJNA	sWIG80
CRM	PZ CORMAY SPÓŁKA AKCYJNA	sWIG80
DBC	FIRMA OPONIARSKA DĘBICA SPÓŁKA AKCYJNA	sWIG80
DCR	DECORA SPÓŁKA AKCYJNA	sWIG80
ECH	ECHO INVESTMENT SPÓŁKA AKCYJNA	sWIG80
ENT	ENTER AIR SPÓŁKA AKCYJNA	sWIG80
ERB	ERBUD SPÓŁKA AKCYJNA	sWIG80
FRO	FERRO SPÓŁKA AKCYJNA	sWIG80
FTE	FABRYKI MEBLI FORTE SPÓŁKA AKCYJNA	sWIG80
GRN	GRODNO SPÓŁKA AKCYJNA	sWIG80
GTC	GLOBE TRADE CENTRE SPÓŁKA AKCYJNA	sWIG80
GTN	GETIN HOLDING SPÓŁKA AKCYJNA	sWIG80
IMC	IMC S.A.	sWIG80
INC	INC SPÓŁKA AKCYJNA	sWIG80

Table 3 cont.

1	2	3
LBW	LUBAWA SPÓŁKA AKCYJNA	sWIG80
LTX	LENTEX SPÓŁKA AKCYJNA	sWIG80
LWB	LUBELSKI WĘGIEL BOGDANKA SPÓŁKA AKCYJNA	sWIG80
MCI	MCI CAPITAL ALTERNATYWNA SPÓŁKA INWESTYCYJNA SPÓŁKA AKCYJNA	sWIG80
MDG	MEDICALGORITHMICS SPÓŁKA AKCYJNA	sWIG80
MLS	ML SYSTEM SPÓŁKA AKCYJNA	sWIG80
MRB	MIRBUD SPÓŁKA AKCYJNA	sWIG80
NWG	NEWAG SPÓŁKA AKCYJNA	sWIG80
OPN	OPONEO.PL SPÓŁKA AKCYJNA	sWIG80
PBX	POZNAŃSKA KORPORACJA BUDOWLANA PEKABEX SPÓŁKA AKCYJNA	sWIG80
PCR	PCC ROKITA SPÓŁKA AKCYJNA	sWIG80
PEN	PHOTON ENERGY N.V.	sWIG80
PHN	POLSKI HOLDING NIERUCHOMOŚCI SPÓŁKA AKCYJNA	sWIG80
PSW	PGS SOFTWARE SPÓŁKA AKCYJNA	sWIG80
PXM	POLIMEX MOSTOSTAL SPÓŁKA AKCYJNA	sWIG80
QRS	QUERCUS TOWARZYSTWO FUNDUSZY INWESTYCYJNYCH SPÓŁKA AKCYJNA	sWIG80
R22	R22 SPÓŁKA AKCYJNA	sWIG80
RBW	RAINBOW TOURS SPÓŁKA AKCYJNA	sWIG80
RVU	RYVU THERAPEUTICS SPÓŁKA AKCYJNA	sWIG80
SEN	SERINUS ENERGY PLC	sWIG80
SKH	SKARBIEC HOLDING SPÓŁKA AKCYJNA	sWIG80
SNK	SANOK RUBBER COMPANY SPÓŁKA AKCYJNA	sWIG80
SNT	SYNEKTIK SPÓŁKA AKCYJNA	sWIG80
STX	STALEXPORT AUTOSTRADY SPÓŁKA AKCYJNA	sWIG80
TIM	TIM SPÓŁKA AKCYJNA	sWIG80
TOA	TOYA SPÓŁKA AKCYJNA	sWIG80
TOR	TORPOL SPÓŁKA AKCYJNA	sWIG80
TRK	TRAKCJA SPÓŁKA AKCYJNA	sWIG80
ULG	ULTIMATE GAMES SPÓŁKA AKCYJNA	sWIG80
UNT	UNIMOT SPÓŁKA AKCYJNA	sWIG80
VOX	VOXEL SPÓŁKA AKCYJNA	sWIG80
VRG	VRG SPÓŁKA AKCYJNA	sWIG80
WLT	WIELTON SPÓŁKA AKCYJNA	sWIG80

Note: The list of firms listed on the Warsaw Stock Exchange comes from its official website (GPW, 2021).

Source: Author's own calculations.